UNIVERSITÄT LEIPZIG

(Orthology-)Relations on rooted Median Graphs

34th Winterseminar

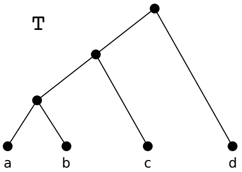
Carmen Bruckmann

Bioinformatics Group, Institute of Computer Science

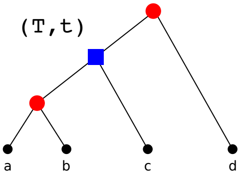
Leipzig University

Bled, Slovenia — February 12, 2019

Introduction and Motivation [1]

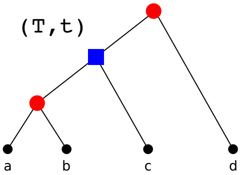


Introduction and Motivation [1]



Introduction and Motivation [1]

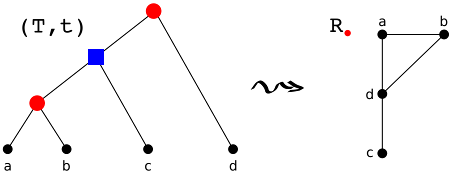
UNIVERSITÄT LEIPZIG



Definition ($R_\bullet$ w.r.t (T, t))

$$(x, y) \in R_\bullet \iff t(lca_T(x, y)) = \bullet$$

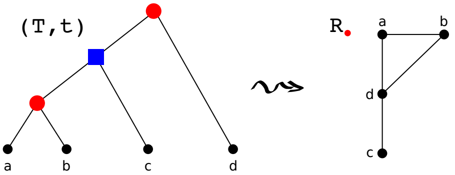
Introduction and Motivation [1]



Definition ($R_{\bullet}$ w.r.t (T,t))

$$(x,y) \in R_{\bullet} \iff t(lca_T(x,y)) = \bullet$$

Introduction and Motivation [1]



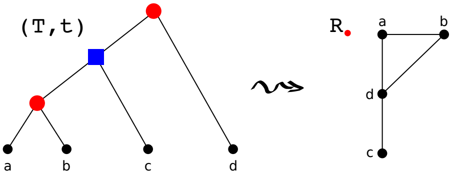
Definition ($R_\bullet$ w.r.t (T, t))

$$(x, y) \in R_\bullet \iff t(lca_T(x, y)) = \bullet$$

Definition (Orthology relation)

R is a *orthology relation* $\iff$ there exists (T, t) with $R_\bullet = R$

Introduction and Motivation [1]



Definition ($R_{\bullet}$ w.r.t (T, t))

$$(x, y) \in R_{\bullet} \iff t(lca_T(x, y)) = \bullet$$

Definition (Orthology relation)

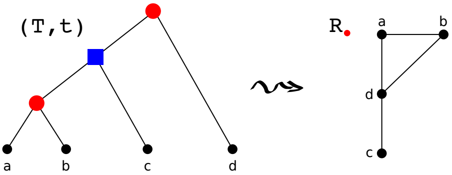
R is a *orthology relation* $\iff$ there exists (T, t) with $R_{\bullet} = R$

Theorem

R is a orthology relation $\iff$ R is "induced P_4-free".

1) Hellmuth M, H.-Rosales M, Huber K, Moulton V,Stadler PF, **Orthology Relations, Symbolic Ultrametrics, and Cographs**, *Journal of Mathematical Biology*, 2012

Introduction and Motivation [1]



Definition ($R_{\bullet}$ w.r.t (T,t))

$$(x,y) \in R_{\bullet} \iff t(lca_T(x,y)) = \bullet$$

Definition (Orthology relation)

R is a *orthology relation* $\iff$ there exists (T,t) with $R_{\bullet} = R$

Theorem

R is a orthology relation $\iff$ R is "induced P_4-free".  P_4

1) Hellmuth M, H.-Rosales M, Huber K, Moulton V,Stadler PF, **Orthology Relations, Symbolic Ultrametrics, and Cographs**, *Journal of Mathematical Biology*, 2012

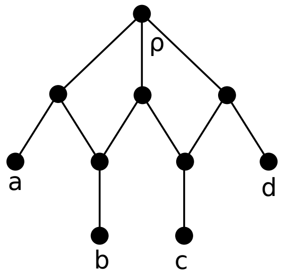
Relations on rooted median graphs

UNIVERSITÄT LEIPZIG

How to deal with induced P_4? Idea: Use median graphs

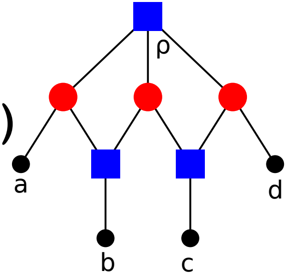
Relations on rooted median graphs

G



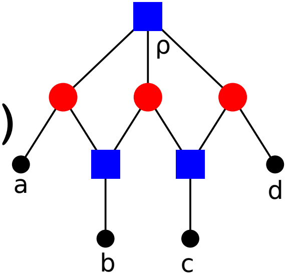
Relations on rooted median graphs

(G,t)



Relations on rooted median graphs

(G,t)

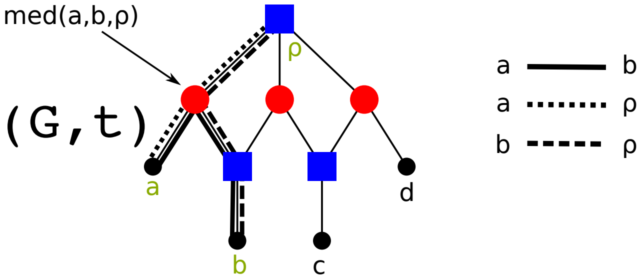


Definition (Median)

$G = (V, E)$, $a, b, c \in V$

A vertex is called *median* w.r.t G, denoted by $med_G(a, b, c)$, if it lies on a shortest path $P_{a,b}, P_{a,c}$, and $P_{b,c}$.

Relations on rooted median graphs

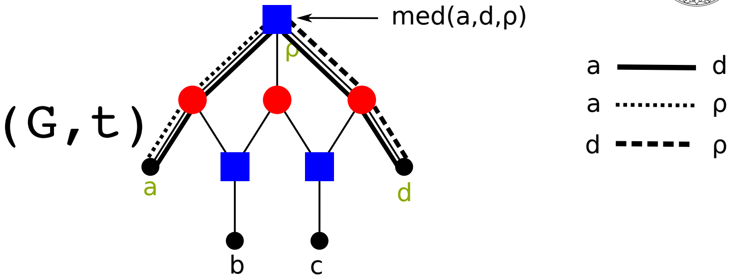


Definition (Median)

$G = (V, E)$, $a, b, c \in V$

A vertex is called *median* w.r.t G, denoted by $med_G(a, b, c)$, if it lies on a shortest path $P_{a,b}$, $P_{a,c}$, and $P_{b,c}$.

Relations on rooted median graphs

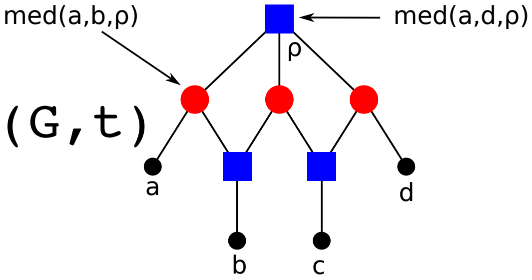


Definition (Median)

$G = (V, E)$, $a, b, c \in V$

A vertex is called *median* w.r.t G, denoted by $med_G(a, b, c)$, if it lies on a shortest path $P_{a,b}$, $P_{a,c}$, and $P_{b,c}$.

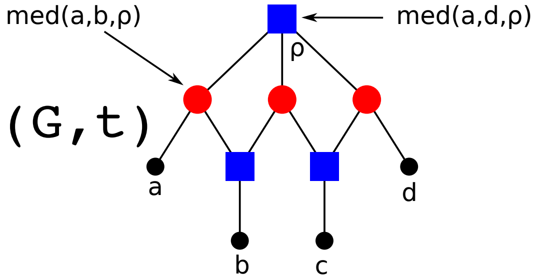
Relations on rooted median graphs



Definition (Median graph)

$G = (V, E)$ is a *median graph* $\iff$ G connected and
$\forall \{a, b, c\} \subseteq V$, with distinct a, b, c $\exists!\ med(a, b, c)$

Relations on rooted median graphs

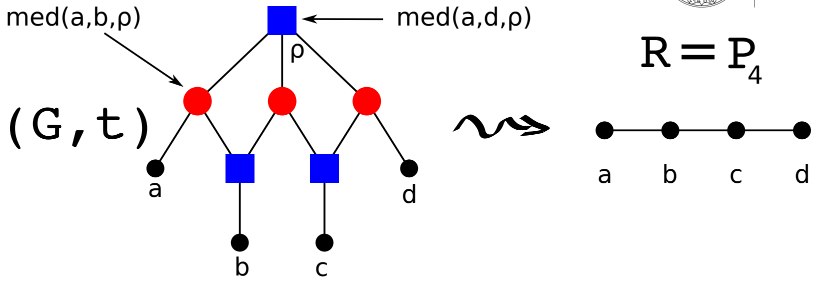


Definition

R is *displayed* by a rooted median graph (G, t), if

$(x, y) \in R \iff t(med_G(x, y, root)) =$ ● $\qquad x, y$ leaves in G

Relations on rooted median graphs



Definition

R is *displayed* by a rooted median graph (G, t), if

$(x, y) \in R \iff t(med_G(x, y, root)) = \bullet$ $\qquad$ x, y leaves in G

Problem:

UNIVERSITÄT LEIPZIG

Every relation R on n vertices can be displayed by a median graph.

Problem:

UNIVERSITÄT LEIPZIG

Every relation R on n vertices can be displayed by a median graph.

- take an n-dimensional hypercube Q_n with coordinates $\in \{0,1\}^n$

Problem:

Every relation R on n vertices can be displayed by a median graph.

- take an n-dimensional hypercube Q_n with coordinates $\in \{0,1\}^n$
- add leaves $l_1, l_2, \ldots, l_n$ at $(1,0,0,\ldots,0)$, $(0,1,0,0,\ldots,0)$, $(0,0,1,0,0,\ldots,0)$ etc.

Problem:

UNIVERSITÄT LEIPZIG

Every relation R on n vertices can be displayed by a median graph.

- take an n-dimensional hypercube Q_n with coordinates $\in \{0, 1\}^n$
- add leaves $l_1, l_2, \ldots, l_n$ at $(1, 0, 0, \ldots, 0)$, $(0, 1, 0, 0, \ldots, 0)$, $(0, 0, 1, 0, 0, \ldots, 0)$ etc.
- choose the root ϱ at $(1, 1, \ldots, 1)$

Problem:

Every relation R on n vertices can be displayed by a median graph.

- take an n-dimensional hypercube Q_n with coordinates $\in \{0,1\}^n$
- add leaves $l_1, l_2, \ldots, l_n$ at $(1,0,0,\ldots,0)$, $(0,1,0,0,\ldots,0)$, $(0,0,1,0,0,\ldots,0)$ etc.
- choose the root ϱ at $(1,1,\ldots,1)$
- $\Rightarrow$ $med(l_i, l_j, \varrho)$ has exactly two '1'
  and each pair of leaves has its own median with ϱ

Problem:

UNIVERSITÄT LEIPZIG

Every relation R on n vertices can be displayed by a median graph.

- take an n-dimensional hypercube Q_n with coordinates $\in \{0,1\}^n$
- add leaves $l_1, l_2, \ldots, l_n$ at $(1,0,0,\ldots,0)$, $(0,1,0,0,\ldots,0)$, $(0,0,1,0,0,\ldots,0)$ etc.
- choose the root ϱ at $(1,1,\ldots,1)$
- $\Rightarrow$ $med(l_i, l_j, \varrho)$ has exactly two '1'
  and each pair of leaves has its own median with ϱ
  $\Rightarrow$ every relation can be displayed

Outlook

UNIVERSITÄT LEIPZIG

- Q_n is not a good biological result

Outlook

UNIVERSITÄT LEIPZIG

- Q_n is not a good biological result
- take a look at subclasses of median graphs such as
  - Buneman graphs
  - cube-free graphs

Acknowledgments

UNIVERSITÄT
LEIPZIG

Thanks to Peter F. Stadler and Marc Hellmuth for their interesting problem and the entertaining discussions,

Acknowledgments

Thanks to Peter F. Stadler and Marc Hellmuth for their interesting problem and the entertaining discussions,

and

THANK YOU
for your attention!