# RNA alignment and folding 

with

## Markov chains

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## RNA simultaneous alignment and folding

Example of an alignment $\mathcal{A}$ of two $R N A$ sequences $A$ and $B$ together with (non-crossing) structures $S=\left\{a_{1}, a_{2}, a_{3}, a_{4}, a_{5}\right\}$ and $T=\left\{b_{1}, b_{2}, b_{3}, b_{4}, b_{5}\right\}$.


## Some of the related pioneering works

| 1985 | Simultaneous solution of the RNA folding, alignment and protosequence problems <br> by: $D$. Sankoff |
| :---: | :---: |
| 1990-1993 | RNA multi-structure landscapes |
|  | A study based on temperature dependent partition functions |
|  | Authors Authors and affiliations |
|  | S. Bonhoeffer, J. S. McCaskill, P. F. Stadler, P. Schuster |
| ~2004: | Alignment of RNA base pairing probability matrices |
|  | Ivo L. Hofacker $\boldsymbol{\triangle}$, Stephan H. F. Bernhart, Peter F. Stadler |
|  | LOCAL SEQUENCE-STRUCTURE MOTIFS IN RNA ROLF BACKOFEN and SEBASTIAN WILL |
|  | Pairwise local structural alignment of RNA sequences with sequence similarity less than $40 \%$. <br> Jakob Hull Havgaard M, Rune B. Lyngsø, Gary D. Stormo, Jan Gorodkin |

## Meanwhile me

## Encoded as unobserved event of a stochastic function



Playing with bro's C64 machine

$\sim 2004 \quad$ procedure bubbleSort( $\mathrm{A}:$ list of sortable items )
$\mathrm{n}=$ length $(\mathrm{A})$

## Prologue: RNA secondary structure probability dotplots

The abstraction making it feasible to visualize the RNA ensemble is:

- Base-pair independency or disconnection

Implicit base-pair dependency or relation hints available via:

- Stacking patterns
- Mutual exclusion rules



## Can we really percept stackings inter-relations?



## Back to the target problem: RNA alignment and folding

Example of an alignment $\mathcal{A}$ of two RNA sequences $A$ and $B$ together with (non-crossing) structures $S=\left\{a_{1}, a_{2}, a_{3}, a_{4}, a_{5}\right\}$ and $T=\left\{b_{1}, b_{2}, b_{3}, b_{4}, b_{5}\right\}$.


- An optimization problem that considers the stability of the predicted structures and the alignment quality
- Objective function:
- f (energy_model (S, $T$ ) + alignment-score (A, B) )


## Sankoff's algorithm for simultaneous RNA alignment and folding

- Sankoff's objective function:
- $\quad-\operatorname{FreeEnergy}(S)-\operatorname{FreeEnergy}(T)+\operatorname{SequenceAlignScore}(A, B)$
- Linear straightforward(?) combination of:
- the free energies of two compatible structures
- the alignment score of the two sequences
- The "golden" standard with efficient dynamic programming recursions
- But computationally expensive $O\left(\mathrm{n}^{\wedge} 6\right)$ so not suitable ...



## PMcomp and PMcomp-like objective function

- Using a light-weight base-pair energy model
- Using precomputed McCaskill base-pair probability dotplots to compute the base-pair scores.
- Estimating energy by combining weights and assuming independency of the pairing evens
- PMcomp coarse model:
- $\quad \Sigma_{i} \operatorname{score}\left(a_{i}\right)+\Sigma_{i} \operatorname{score}\left(b_{i}\right)+\operatorname{SequenceAlignScore}(A, B)$
- PMcomp-like:
- PMcomp, LocARNA, FoldAlignM(?), SPARSE, ...



## Sankoff-Zuker's nearest-neighbor energy model

The free energy depends on the neighboring bases and adjacent loops.


$$
E=E(a, b)+E(b, c)+E(c)
$$



$$
E=E(a, b 1, b 2)+E(b 1)+E(b 2)
$$

## PMcomp base-pair energy model

- An independency between base-pairing probability events
- Efficient computations and flexible pairing, reduces complexity to $\mathrm{O}(\mathrm{n} \wedge 4)$


Score (a, b, c) = ( $\mathrm{P}(\mathrm{a}) \cdot \mathrm{P}(\mathrm{b}) \cdot \mathrm{P}(\mathrm{c})) / \mathrm{P}\left(\mathrm{pO}^{\wedge} 3\right)$

Score (a, b1, b2) ~= (P (a) . P (b1) . P (b2)) / P (p0^3)
p0: a length-dependent estimation of the minimum significant/background probability

## A Markov chain model: non-branching loops

For a sub-structure $S$ with set of basepairs $\{a, b, c\}$ let have:


$$
P(a, b, c)=P(a) . P(b, c \mid a)
$$

nearest-neighbor rule:

$$
\begin{aligned}
= & P(a) \cdot P(b \mid a) \cdot P(c \mid a, b) \\
& =P(a) \cdot P(b \mid a) \cdot P(c \mid b)
\end{aligned}
$$

Basic math for conditional probabilities to the joint probabilities.

$$
\begin{gathered}
P(a, b, c)=P(a) \cdot P(b \mid a) \cdot P(c \mid b) \\
=P(a) \cdot P(b, a) / P(a) \cdot P(c, b) / P(b)
\end{gathered}
$$

## A Markov chain model: multi-loops



$$
P(a, b 1, b 2)=P(a) . P(b 1, b 2 \mid a)
$$

Nearest-neighbor rule not enough for decomposition:

$$
=P(a) \cdot P(b 1 \mid a) \cdot P(b 2 \mid a, b 1)
$$

Assuming multiloop independency:

$$
=P(a) \cdot P(b 1 \mid a) \cdot P(b 2 \mid a)
$$

A better trick:

$$
=P(a) \cdot \min (P(b 1 \mid a), P(b 2 \mid a))
$$

## Alright but how to get chain probabilities?

- Extending the base-pair stacking probabilities (RNAfold -p2) to loops

A: Joint probability pre-computations


B: To sparsify the alignment search space -


C: Using loop-closing aware recursions


Figures: [Will, Otto, Miladi, Möhl, Backofen; SPARSE; Bioinf., 2015], [Otto, Möhl, Heyne, Amit, Landau, Backofen, Will; ExpaRNA-P: BMC Bioinf., 2014]

## Evaluation of the energy model: rnasubopt trna

- In this plot, dots represents suboptimal structures of a tRNA sequence, sorted by free energy.








## Evaluate energy models on RNAstrand dataset



## Pankov: Probabilistic Sankoff's-like alignment with Markov chains

| Name | Size | Last commit |
| :--- | :--- | :--- |
| Bralibase |  | $2019-02-07$ |
| RNAStrand |  | $2019-02-07$ |
| lib |  | $2016-10-06$ |
| src |  | $2016-10-06$ |
| E gitignore | README.md | 54 B |
| E conda-env-pankoff.yml | $8016-09-07$ |  |
| E B | 1 minute ago |  |

README.md

Pankov: Probabilistic Sankoff's-like alignment with Markov chains

Currently protected access, feel free to contact me: bitbucket.org:mmiladi/pankov

Primary evaluation on Bralibase dataset


## "Dent"less Bralibase; same data as the previous slide



## Conclusion and speculation

- Pankov: Our closest attempt to enter the Eutopia
- Flexibility introduced by the base-pairing energy model may have benefits beyond a computational complexity reduction
- It's actually very hard to combine the folding-energy and alignment scoring components that have two different natures
- The conditional probability of loops has the potential to be applied for other purposes as well
- Please reach me if you have ideas


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## Thanks to everyone for sharing your joy of TBI-Bled with me these years!

- The dilemma of TBI-Bled Villa


Find your Pareto front;)

