# Bubble Structures in Digraphs

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Bubbles

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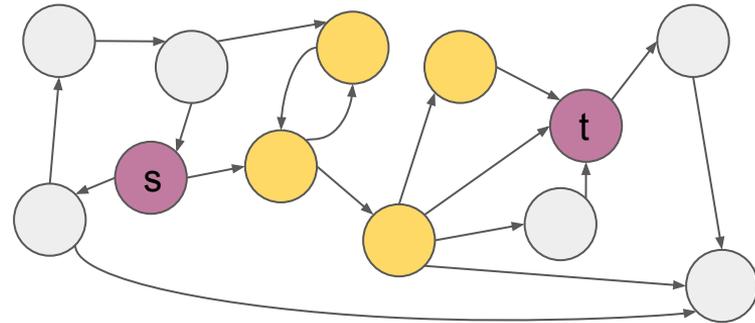
Bubbles

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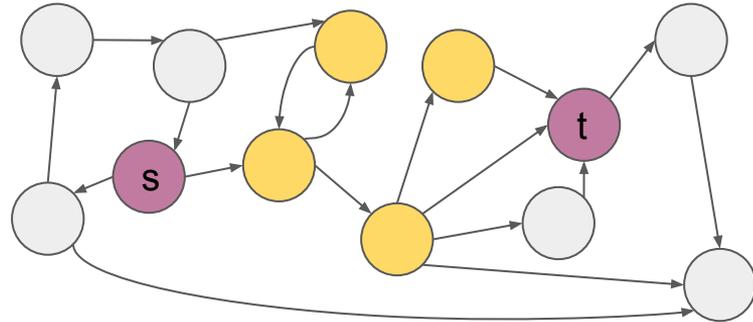
[Philip Göhler. Generalized bubbles in digraphs. 2021.]

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We say that (s, A, t) is a **bubble** in G if for all $x \in A$ there is
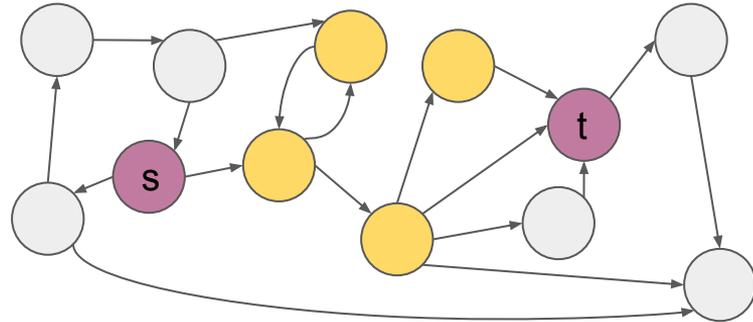
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a path from s to x in $\hat{G}[A]$ that does not pass through t and

a path from x to t in $\hat{G}[A]$ that does not pass through s.
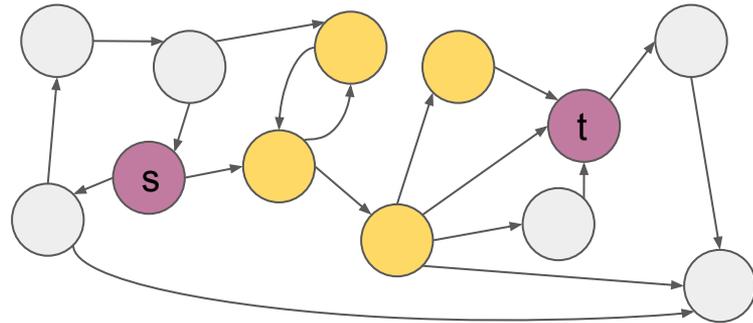
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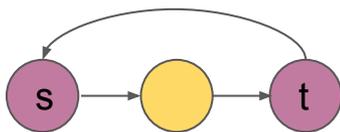
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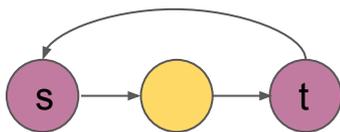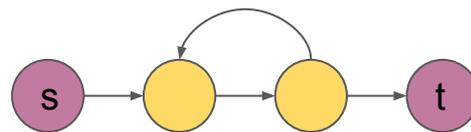
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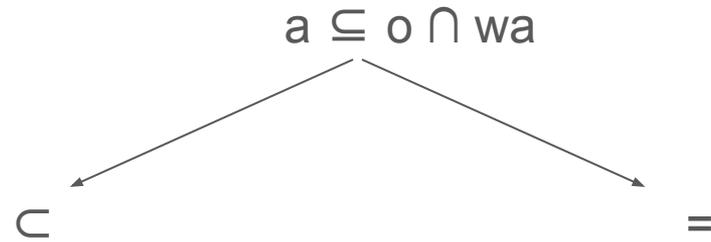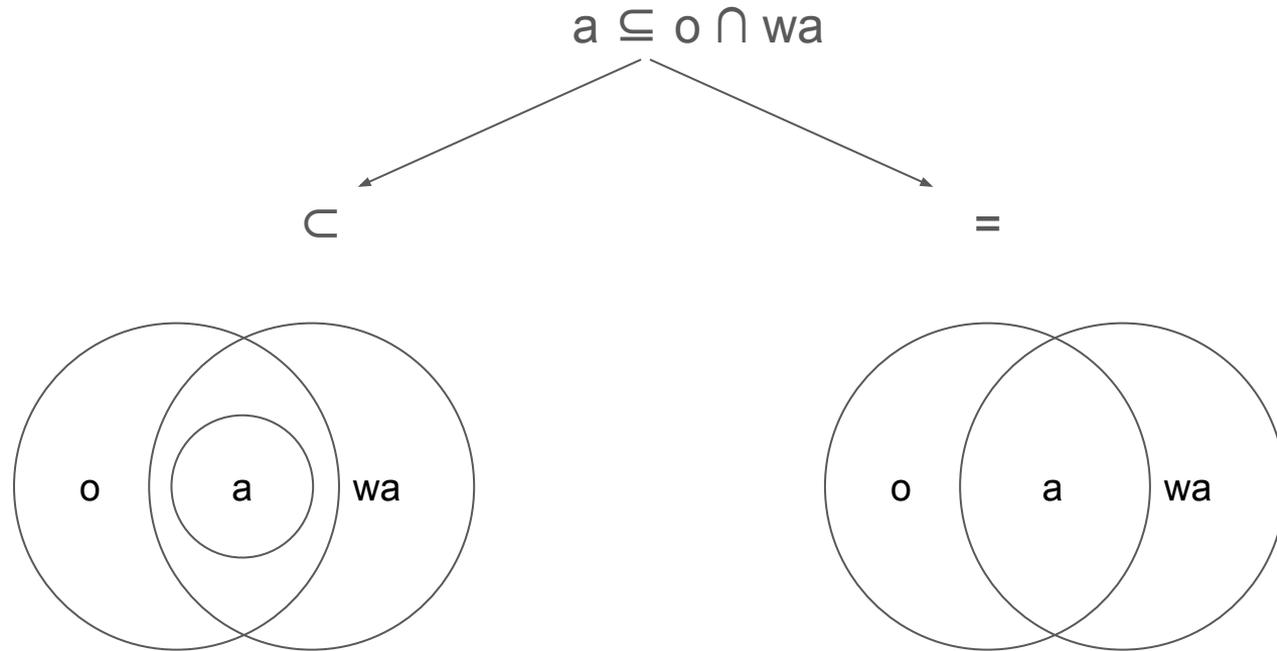
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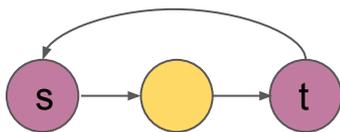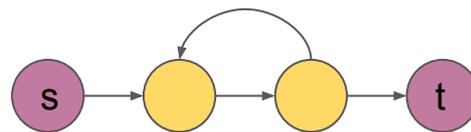
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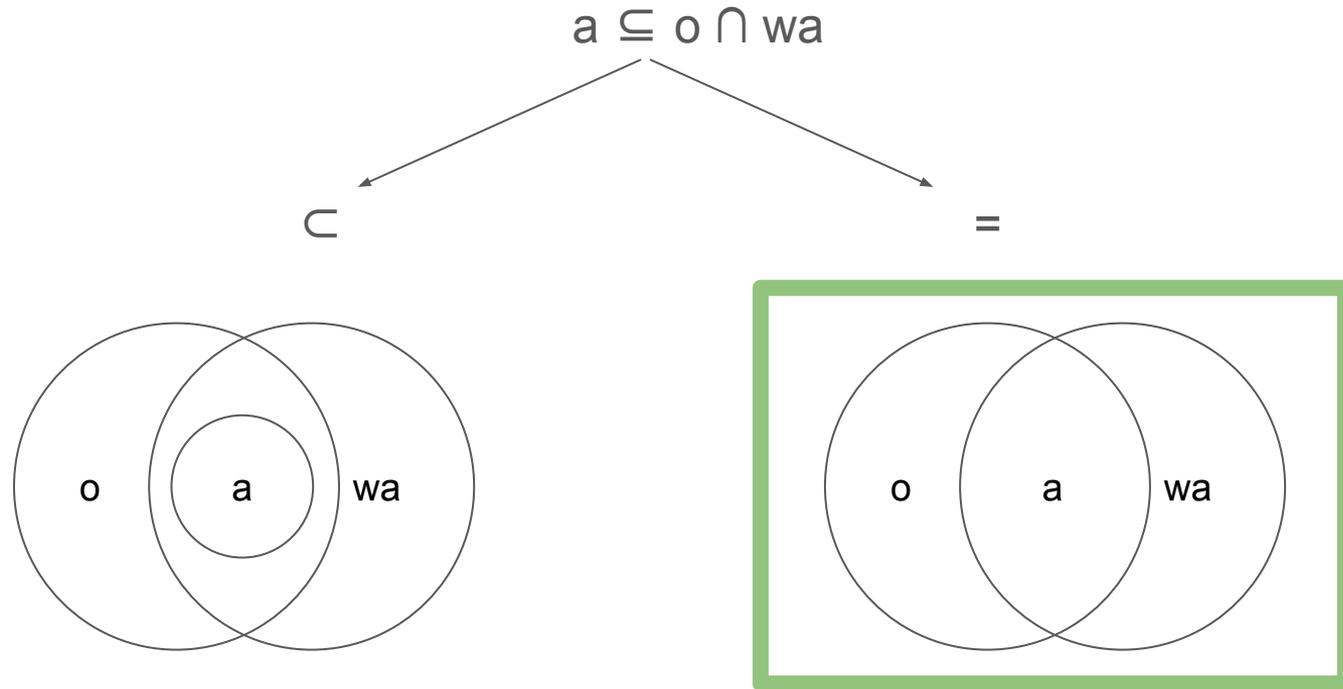
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- algorithmic approaches to find bubbles (with certain properties) efficiently

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I am looking for a PhD position abroad in graph theory starting September 2023 at the earliest :)

Thank you for your attention!