The Dynamics of Adaptive Parties under Spatial Voting

By

John H. Miller[a,b,*] and Peter F. Stadler[b,c]

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[a]Department of Social and Decision Sciences
Carnegie Mellon University, Pittsburgh, PA 15213

[b]The Santa Fe Institute, 1660 Old Pecos Trail, Santa Fe, NM 87501

[c]Institut für Theoretische Chemie, Universität Wien
Währingerstr. 17, A–1090 Vienna, Austria

[*]Mailing Address

Abstract

We explore the dynamics of a model of two-party competition under spatial voting. The parties are allowed to *incrementally* adapt their platforms by following the voting gradient imposed by the preferences of the electorate and platform of the opposition. The emphasis in this model is on the dynamic system formed by these conditions, in particular, we examine the characteristics of the transient paths and the convergence points of the evolving platforms. We find that in a simple spatial model with probabilistic voting, regardless of the initial platforms of each party, platforms eventually converge to a unique, globally stable equilibrium matching the strength-weighted mean of the voters' preferred positions. This result holds even if we allow simple cross-issue weightings, however, if we allow nonlinear weighting functions many dynamic possibilities occur, including multiple equilibria and, perhaps, limit cycles.

1. Introduction

We formally examine a spatial voting model where candidates in a two-party system incrementally adjust their separate platforms to appeal to voters. In our model, candidates are only allowed to make local adjustments to their previously endorsed platform. We assume that they do so by following the local voting gradient imposed on the system by the preferences of the voters and the position of the other candidate. The emphasis in this model is on the dynamic system formed by these conditions, in particular, we examine the characteristics of the transient paths and the convergence points of the evolving platforms.

We find that in a simple spatial model with probabilistic voting, regardless of the initial platforms of each party, platforms eventually converge to a unique, globally stable equilibrium matching the strength-weighted mean of the voters' preferred positions. This result holds even if we allow simple cross-issue weightings, however, if we allow nonlinear weighting functions many dynamic possibilities occur, including multiple equilibria and, we suspect, limit cycles.

The emphasis of our model is on an adaptive dynamic system, whereby global consequences emerge from locally adapting candidates. The inspiration for this model comes from the computational results discovered by Kollman, Miller, and Page (1992). They found that parties following simple locally adaptive rules rapidly converged toward common platforms. While the ideas of probabilistic voting (for a general review see Coughlin, 1990) and locally restricted strategy searches in such models (for example, Coughlin and Nitzan (1981) and Samuelson (1984)) have been widely discussed, here we assume that parties do not start at an identical status quo platform, and that their ability to maximize voter support is limited to climbing the local voting gradient. Such a model serves as a necessary benchmark for spatial voting results when candidates' abilities to optimize their platform positions are limited and where the dynamic behavior of platform modification is of interest.

2. The Basic Model

Consider a model with I issues and V voters. We will use the following notation:

y_i^j Platform position of party j on issue i.

x_{vi} Voter v's preferred position on issue i.

s_{vi} Voter v's strength on issue i. We require for all i that $s_{vi} > 0$ for at least one v. (That is, at least one voter cares about each issue.)

$u_v(y^j)$ Voter v's utility of party j's platform:

$$u_v(y^j) = -\sum_{i=1}^{I} s_{vi}(y_i^j - x_{vi})^2. \tag{1}$$

(This functional form will be generalized in Section 4.)

$P(x)$ Probability of voting for a platform that has a difference x in utility against the opposing platform. If the two parties are distinguished only by their platforms then $P(x)$ will be symmetric about 0.5:

Prob[v votes for 1] $= P(u_v(y^1) - u_v(y^2))$ and

Prob[v votes for 2] $= P(u_v(y^2) - u_v(y^1)) = 1 - P(u_v(y^1) - u_v(y^2))$.

In the extreme case we have

$$P(x) = \begin{cases} 0 & x < 0 \\ 0.5 & x = 0 \\ 1 & x > 0 \end{cases} \tag{2}$$

where a voter always votes for the party that offers the larger utility, and if both parties provide the same utility the voter is indifferent and flips a coin. To add both realism and mathematical tractability, we impose the following additional requirements on $P(x)$. First, $P(x)$ is assumed to be strictly monotonic, implying that the probability of choosing party j always increases as the difference in utility increases and that there is always a positive probability of voting for either party for any difference in utilities, x. Second, we require $P(x)$ to be continuously differentiable. A simple class of functions that fit our requirements is the symmetric sigmoidal functions (such as the error integral). If $P(x)$ is only monotonic, then when $P'(x) = 0$ it is possible to have large areas in issue-space forming equilibria where incremental changes in the platforms do not cause any voters to change their expected votes. With strict monotonicity, platforms will only stabilize if there is an attractor present. Continuity is necessary for the results, and in the extreme case of $P(x)$ above our results no longer hold and the result of Plott (1967) applies.

Given $P(x)$, the expected number of votes for each party, E_j, is

$$\begin{aligned} E_1(y^1, y^2) &= \sum_{v=1}^{V} P(u_v(y^1) - u_v(y^2)) \text{ and} \\ E_2(y^1, y^2) &= \sum_{v=1}^{V} P(u_v(y^2) - u_v(y^1)) = V - E_1(y^1, y^2). \end{aligned} \tag{3}$$

We assume that a party's utility, U_j, is given by the number of votes that it is ahead of its rival:

$$\begin{aligned} U_1(y^1, y^2) &= E_1(y^1, y^2) - E_2(y^1, y^2) = 2E_1(y^1, y^2) - V \text{ and} \\ U_2(y^1, y^2) &= E_2(y^1, y^2) - E_1(y^1, y^2) = V - 2E_1(y^1, y^2) = -U_1(y^1, y^2). \end{aligned} \tag{4}$$

Dynamics

For the dynamics of platform adjustment we follow an approach used in the analysis of biological games (for an introduction to this area, see Hofbauer and Sigmund (1988); Friedman (1991) reviews economic applications of these ideas). Hofbauer and Sigmund (1990), in a related application to continuous strategy spaces called "adaptive dynamics," explore strategic stability around fixed points (unfortunately, our dynamics are not characterized by their results). The dynamics we explore here have not been previously analyzed.

In our model both parties update their platforms according to the following dynamics

$$\begin{aligned} \dot{y}^1 &= \nabla_{y^1} U_1(y^1, y^2) \text{ and} \\ \dot{y}^2 &= \nabla_{y^2} U_2(y^1, y^2). \end{aligned} \tag{5}$$

That is, each party alters its platform so as to *locally* improve its share of votes. The optimization is local rather than global since each party simply follows the gradient of its utility function. Since $P(x)$ is continuously differentiable the vector field $(\nabla_{y^1} U_1, \nabla_{y^2} U_2)$ is continuous every where, and therefore the existence and uniqueness of the trajectory for all initial conditions is guaranteed.

We can rewrite the dynamics in a more explicit way. First, define $\sigma(x) = 2P(x) - 1$. As a consequence of differentiability, monotonicity and symmetry of $P(x)$, σ has the following properties: $\sigma(x) = -\sigma(-x)$, $\sigma(0) = 0$, $\sigma'(x) > 0$, and $\sigma'(x) = \sigma'(-x)$. The utility of the parties can be rewritten as

$$\begin{aligned} U_1(y^1, y^2) &= 2E_1(y^1, y^2) - V \\ &= \sum_v [2P(u_v(y^1) - u_v(y^2)) - 1] \\ &= \sum_v \sigma(u_v(y^1) - u_v(y^2)) \text{ and} \\ U_2(y^1, y^2) &= 2E_2(y^1, y^2) - V \\ &= \sum_v [2P(u_v(y^2) - u_v(y^1)) - 1] \\ &= \sum_v \sigma(u_v(y^2) - u_v(y^1)). \end{aligned} \tag{6}$$

The dynamic equations now yield

$$\begin{aligned} \dot{y}^1 &= \nabla_{y^1} U_1 \\ &= \sum_v \nabla_{y^1} \sigma(u_v(y^1) - u_v(y^2)) \\ &= \sum_v \sigma'(u_v(y^1) - u_v(y^2)) \nabla_{y^1} u_v(y^1) \text{ and} \\ \dot{y}^2 &= \nabla_{y^2} U_2 \\ &= \sum_v \nabla_{y^2} \sigma(u_v(y^2) - u_v(y^1)) \\ &= \sum_v \sigma'(u_v(y^2) - u_v(y^1)) \nabla_{y^2} u_v(y^2). \end{aligned} \tag{7}$$

Using the voter's utility function given by equation (1) we find

$$[\nabla_y u_v(y)]_l = -\sum_{i=1}^{I} s_{vi} \frac{\partial}{\partial y_l} (y_i - x_{vi})^2 = -2 s_{vl} (y_l - x_{vl}). \tag{8}$$

Combining (7) and (8) yields the system of differential equations

$$\begin{aligned} \dot{y}_l^1 &= -2 \sum_v \sigma'(\sum_i s_{vi} [(y_i^2 - x_{vi})^2 - (y_i^1 - x_{vi})^2]) s_{vl} (y_l^1 - x_{vl}) \text{ and} \\ \dot{y}_l^2 &= -2 \sum_v \sigma'(\sum_i s_{vi} [(y_i^1 - x_{vi})^2 - (y_i^2 - x_{vi})^2]) s_{vl} (y_l^2 - x_{vl}). \end{aligned} \tag{9}$$

The terms $\sigma'(\cdot)$ in these equations weight the importance of a given voter for the changes that a party implements in its platform. If $P(x)$ is sigmoidal the most important voters will be the pivotal ones where the utility provided by the two parties are close. The multiplicative factor, $s_{vl}(y_l^j - x_{vl})$, implies that changes of position on issue l are influenced most by those voters who care a lot about issue l and have preferred positions further away from the current platform position.

3. Analysis

Lemma 1. Let $x_i^{\min}$ and $x_i^{\max}$ denote the extreme positions on issue i across the population of voters. Then the box

$$B = \prod_{i=1}^{I} [x_i^{\min}, x_i^{\max}]$$

is a compact, globally attracting, forward invariant set for the dynamical system given in (9).

Proof. If the position of a party on issue l lies outside the box, say, $y_l > x_l^{\max}$, then $\dot{y}_l < 0$. If some voters have different positions on issue l, that is $x_l^{\min} < x_l^{\max}$, then $\dot{y}_l$ can be uniformly bounded from above by $-\delta < 0$ for $y_l \geq x_l^{\max}$, and hence y_l will become smaller than $x_l^{\max}$ after finite time. (The actual time required depends on the initial conditions.) An analogous argument holds for $y_l < x_l^{\min}$. In the degenerate case where all voters have the same position we have $\dot{y}_l = -\alpha(y_l - x_l)$, where $\alpha > 0$, and therefore y_l converges exponentially to x_l. ∎

Lemma 1 simply shows that regardless of the initial platform positions, all platforms eventually end up in the box spanned by the extreme positions of the voters. This lemma insures that the model harbors no absurd dynamical properties.

Lemma 2. For all possible choices of s_{vl} and x_{vl}, the difference between the platforms, $y^1 - y^2$, vanishes for the dynamical equations given in (9) as $t \to \infty$. This convergence is exponential.

Proof. Consider $y_l^1 - y_l^2$ for an arbitrary issue l. Since $\sigma'(x) = \sigma'(-x)$ we have

$$\sigma'(\sum_i s_{vi}[(y_i^1 - x_{vi})^2 - (y_i^2 - x_{vi})^2]) = \sigma'(\sum_i s_{vi}[(y_i^2 - x_{vi})^2 - (y_i^1 - x_{vi})^2])$$

and hence using (9)

$$\begin{aligned}\frac{d}{dt}(y_l^1 - y_l^2) &= \dot{y}_l^1 - \dot{y}_l^2 \\ &= -2\sum_v \sigma'(\sum_i s_{vi}[(y_i^2 - x_{vi})^2 - (y_i^1 - x_{vi})^2]) s_{vl}(y_l^1 - x_{vl} - y_l^2 + x_{vl}) \\ &= -(y_l^1 - y_l^2)\cdot 2\sum_v s_{vl}\sigma'(\sum_i s_{vi}[(y_i^2 - x_{vi})^2 - (y_i^1 - x_{vi})^2])\end{aligned}$$

Since the sum is bounded away from 0 within the (finite) box B, say by $\delta > 0$, we have $|y_l^1 - y_l^2| \le C\exp(-\delta\, t) \to 0$. ∎

Lemma 3. There is a unique equilibrium

$$\hat{y}_l^1 = \hat{y}_l^2 = \frac{\sum_v s_{vl}x_{vl}}{\sum_v s_{vl}} = \bar{x}_l. \tag{11}$$

This equilibrium is globally stable within the manifold $\{y^1 = y^2\}$.

Proof. Lemma 2 implies that there is no equilibrium with $y^1 \neq y^2$. For $y^1 = y^2$ the equations in (9) simplify considerably, since the arguments of the "response" functions, $\sigma'(\cdot)$, vanish and we are left with the linear set of equations

$$\dot{y}_l^j = -2\sigma'(0)\sum_v s_{vl}(y_l^j - x_{vl}).$$

Thus the equations for the different issues decouple on the manifold $\{y^1 = y^2\}$. It is easily checked that $\hat{y}^1$ is a stable rest point within the manifold. The linearity of these equations implies that the rest point is even globally stable within $\{y^1 = y^2\}$. ∎

The above lemmas can be summarized by the following:

Theorem. The adaptive-platform two-party voting model derived in (9) converges to a unique and globally stable equilibrium $\hat{y}^1 = \hat{y}^2 = \bar{x}$, where $\bar{x}$ is the strength-weighted mean of the voters' preferred positions as given by (11).

Thus, two competing parties climbing their local voting gradients will converge to a globally stable equilibrium. While the convergence is guaranteed, the transient paths can be long and display unusual behavior. For example, Figure 1 shows the transient paths, in the space of platform positions, for the two parties with two issues and three randomly generated voter utility functions. In this case, while the two platforms are getting closer to each other over time, they exhibit some unusual cycling behavior. Eventually the distance between the platforms is small enough so that they converge on the predicted equilibrium point indicated by the star to the north-east of the top of the cycle.

Finally, let us return to the dynamics (7), and develop a simple result that will be needed later. Suppose we start with two identical platforms $y^1 = y^2$. In this case (7) reduces to

$$\dot{y}^j = \sum_v \sigma'(0)\nabla_{y^j} u_v(y^j) = \sigma'(0)\nabla W(y^j), \tag{12}$$

that is, the restriction of (7) to the invariant manifold $\{y^1 = y^2\}$ is a gradient system, and all trajectories converge to a (local) maximum of $W(y) = \sum_v u_v(y)$. Thus, parties that begin with identical platforms will converge to a local maximum of the additive social utility function. Note that if the utility functions are non-concave, then multiple equilibria may exist (see Section 4).

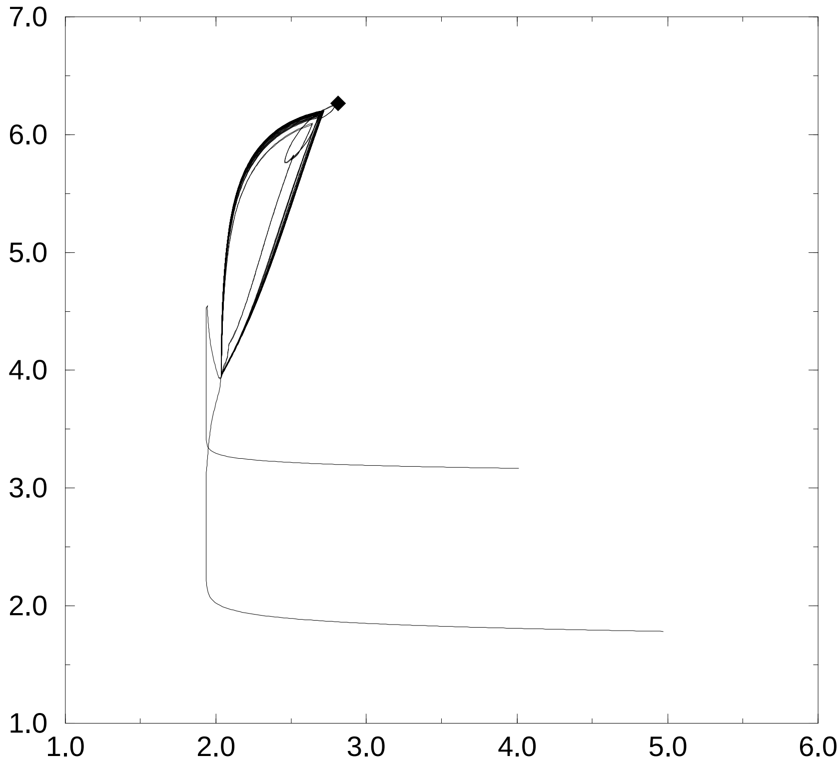

Figure 1: Transient paths for two parties are displayed in platform-position space. Three voters with random strengths, uniformly distributed on $[0, 3)$, and random ideal positions, uniformly distributed on $[0, 8)$, were initially generated. The starting platform positions were randomly generated uniformly on $[0, 8)$, and, in this case, were located in the south-east of the diagram. $P(x) = 1/(1 + e^{-20x})$. The stength-weighted equilibrium point is designated by the star to the north-east of the cycles.

4. Generalizations of the Model

We modify the above model in a variety of directions. First we analyze the situation where parties are restricted in how much they can change their positions on the different issues. We also explore how modifications to the underlying utility functions alter the dynamics.

Limits on Platform Mobility

Often times, parties may be reluctant to change their positions on particular issues. This behavior can be modeled by

$$\begin{aligned} \dot{y}^1 &= A^1 \nabla_{y^1} U_1(y^1, y^2) \text{ and} \\ \dot{y}^2 &= A^2 \nabla_{y^2} U_2(y^1, y^2), \end{aligned} \tag{5'}$$

where A^j are $n \times n$ diagonal matrices with entries between 0 (the party does not change its position on a particular issue at all), and 1 (the party changes its positions in a purely opportunistic way, as in the model described above). It is easy to check that $\{y^1 = y^2\}$ is no longer invariant if $A^i \neq A^j$. In this case the optimizing properties of a party are distorted by its reluctance to change its positions in certain issues. In the following we will assume $A^j_i > 0$ for all issues i and both parties.

Lemma 4. The voting model with dynamics (5′) has a unique rest point corresponding to the weighted mean voter, $(\bar{x}, \bar{x})$, which is locally asymptotically stable.

Proof. The fixed points of (5′) are given by $0 = A^j \nabla_{y^j} U_j(y^1, y^2)$. Multiplying by A_j^{-1} shows that the rest points must fulfill $\nabla_{y^j} U_j(y^1, y^2) = 0$, hence (5′) has the same rest points as (5), that is, $\hat{y}^1 = \hat{y}^2 = \bar{x}$. It is simple to check stability since the Jacobian at $(\bar{x}, \bar{x})$ is of the form

$$J = -2\sigma'(0)\mathrm{diag}(A^1, A^2) \times \mathrm{diag}(s, s)$$

where s is the vector with components $s_i = \sum_{v=1}^{V} s_{vi}$. ∎

Note that we could not prove global stability in this case as we did for model (5). The absence of further equilibria implies that any trajectory that does not converge to $(\bar{x}, \bar{x})$ is periodic or chaotic.

Preference Correlations Among Issues

If voters have preference correlations across the issues, we can generalize (1) in a very natural way to general metric distances in the space of issues:

$$u_v(y^j) = -(y^j - \vec{x}_v) A_v (y^j - \vec{x}_v), \tag{1'}$$

where $\vec{x}_v$ is the vector of voter v's preferred positions, and A_v is a symmetric positive definite matrix that represents the covariance matrix across the issues for voter v.

Lemma 5. The voting model (7) with voter utility function (1′) has a unique rest point that is asymptotically stable. The manifold $\{y^1 = y^2\}$ is invariant and $(\bar{x}, \bar{x})$ is globally stable within $\{y^1 = y^2\}$.

Proof. We know $\nabla u_v(y) = -2A_v(y - \vec{x}_v)$. For the evolution of $\Delta y = y^1 - y^2$ we find

$$\Delta\dot{y} = -2\sum_v \sigma'(u_v(y^1) - u_v(y^2)] \cdot A_v \Delta y = O(y^1, y^2)\Delta y,$$

where $O(y^1, y^2)$ is a symmetric positive definite operator for all y^1 and y^2, hence $\{y^1 = y^2\}$ is (at least locally) attracting. The dynamics on the invariant manifold $\{y^1 = y^2\}$ are given by

$$\dot{y} = -2\sigma'(0)\sum_v A_v(y - \vec{x}_v) = -2\sigma'(0)H(y - \bar{x}),$$

with the (unique) equilibrium point, $\bar{x}$, defined by

$$\bar{x} = H^{-1}\sum_v A_v \vec{x}_v,$$

where $H = \sum_v A_v$ is again a positive definite matrix, and therefore invertible. Thus $\bar{x}$ exists and is unique. Since $-2\sigma'(0)H$ is negative definite, $\bar{x}$ is globally stable within the manifold $\{y^1 = y^2\}$. ∎

One of the old open problems in ODEs is the Jacobi conjecture (see, for example, Meister, 1982). "Let $\dot{x} = f(x)$ be an ODE in $\mathbb{R}^n$ such that 0 is a rest point and the Jacobian is a stable matrix (all eigenvalues have a negative real part) for all x. Then 0 is globally stable." If the Jacobi conjecture is true, then $(\bar{x}, \bar{x})$ is even globally stable under the dynamics (7) with voter utility functions of (1′).

Non-Quadratic (but Concave) Utility Functions

Another generalization of (1) is

$$u_v(y) = -\sum_i \phi_{vi}(y_i - x_{vi}), \tag{1''}$$

where $\phi_{vi}(t) > 0$ for $t \neq 0$, $\phi_{vi}(0) = 0$ and $\phi'_{vi}(t) > 0$ for $t > 0$ and $\phi'_{vi}(t) < 0$ for $t < 0$.

Lemma 6. There is a unique and globally stable rest point $(\bar{x}, \bar{x})$ for the dynamical system (7) with voter utility function $(1'')$ provided $\phi''_{vi}(t) > 0$.

Proof. Here, the explicit dynamics are

$$\begin{aligned} \dot{y}_l^1 &= -\sum_v \sigma'(u_v(y^1) - u_v(y^2))\phi'_{vl}(y_l^1 - x_{vl}) \text{ and} \\ \dot{y}_l^2 &= -\sum_k \sigma'(u_v(y^2) - u_v(y^1))\phi'_{vl}(y_l^2 - x_{vl}), \end{aligned}$$

and thus the dynamics of the difference in the platforms is given by

$$\dot{y}_l^1 - \dot{y}_l^2 = -\sum_v \sigma'(u_v(y^1) - u_v(y^2))[\phi'_{vl}(y_l^1 - x_{vl}) - \phi'_{vl}(y_l^2 - x_{kl})].$$

If $\phi''_{vi} > 0$ the difference of the ϕ'_{vi} terms is positive if and only if $y_l^1 - y_l^2$ is positive; hence, the difference $y^1 - y^2$ decreases and $\{y^1 = y^2\}$ is globally stable. A fixed point must therefore fulfill $\sum_v \phi_{vi}(y_i - x_{vi}) = \Phi_i(y_i) = 0$. Since $\phi''_{vi} > 0$ we know $\Phi'_i > 0$, that is, Φ_i is strictly monotonically increasing. Therefore, there is at most one zero $\bar{x}$, which is defined by $\Phi_i(\bar{x}_i) = 0$, for all issues i. Since the box B is forward invariant, Brouwer's fixed point theorem guarantees the existence of at least one rest point in B and consequently $(\bar{x}, \bar{x})$ is unique. ∎

Non-Concave Utility Functions

Our stability and uniqueness results concerning equilibrium required strictly concave utility functions (a typical assumption for similar dynamic game models). However, some non-concavity in such functions may be quite reasonable, for example, once platforms are a sufficient distance from a voter's preferred position, changes that move the platform even further away are likely to have little impact.

While $\phi''_{vi} > 0$ everywhere on $\mathbb{R}$ is possible, $\phi''_{vi} < 0$ on all $\mathbb{R}$ leads to a contradiction, since we require that ϕ_{vi} has a minimum at 0 hence positive curvature in a neighborhood of 0. We can, however, construct a model where ϕ_{vi} has positive curvature only in a very small neighborhood of the preferred voter positions, and $\phi''_{vi} < 0$ everywhere else. Again $\{y^1 = y^2\}$ is invariant. But now we can have more than one equilibrium. It is instructive to calculate the Jacobian at an equilibrium fulfilling $y^1 = y^2$. We find that the Jacobian is diagonal with entries

$$-\sigma'(0) \sum_v \phi''_{vi}(\hat{y}_l - x_{vl}) \tag{13}$$

which can be both positive and negative.

In the following example we illustrate the behavior of our model in the case where the concavity assumption on u is violated. As an example we consider the following voter utility function:

$$u_v(y^j) = -\gamma^2 \sum_i s_{vi} \left[1 - \exp\left\{ -\frac{(y_i^j - x_{vi})^2}{\gamma^2} \right\} \right]. \tag{1'''}$$

Note that the quadratic utility function (1) agrees with (1''') up to the third order in $(y - x_v)$. While in (1) the utility diverges, it levels off in (1'''). We further observe that the curvature of u_v, given by

$$\frac{\partial u_v(y^j)}{\partial y_k^j} = -2 s_{vk} \left[1 - \frac{2(y_k^j - x_{vk})}{\gamma^2} \right] \exp\left\{ -\frac{(y_i^j - x_{vk})^2}{\gamma^2} \right\},$$

changes sign at

$$|y_k^j - x_{vk}| = \frac{\gamma}{\sqrt{2}}.$$

Consider a simple example with only two issues and two voters. Set $x_1 = (-1, 1)$, and $x_2 = (1, -1)$. Furthermore assume that $s_{vi} = 1$ for both voters and both issues. The mean voter equilibrium of (7) is the origin $(0, 0)$. As a consequence of (13) the origin is a locally stable equilibrium (sink) if $\gamma > \sqrt{2}$ and an unstable equilibrium (source) if $\gamma < \sqrt{2}$. The symmetry of the problem and the forward invariance of the box $[-1, 1] \times [-1, 1]$ (Lemma 1) immediately imply that there cannot be a globally stable equilibrium. Furthermore, any equilibrium on the invariant manifold $\{y^1 = y^2\}$ is unstable.

We could not explicitly solve (7) with utility function $(1''')$ for all equilibria, thus we used numerical integration of the dynamical system. The results are summarized in Figure 2. The figure shows six pairs of platform paths induced by parties starting randomly about $(0, 0)$.

The stable equilibria are of the form

$$\begin{array}{rclcrcl} y^1 & = & (\xi, \xi) & \qquad & y^2 & = & (\xi, -\xi) \\ y^1 & = & (\xi, \xi) & & y^2 & = & (-\xi, \xi) \\ y^1 & = & (-\xi, -\xi) & & y^2 & = & (\xi, -\xi) \\ y^1 & = & (-\xi, -\xi) & & y^2 & = & (-\xi, \xi) \end{array}$$

with $\xi \approx 0.955$ for $\gamma = 1$ (of course, solutions with y^1 and y^2 exchanged also exist). Which of these locally stable equilibria is actually reached depends on the initial conditions. Note that none of the equilibria are symmetric—the parties agree on one issue and dissent on the other.

Conclusions

By employing the tools of evolutionary game dynamics we analyze a two-party spatial-voting system composed of locally-adaptive candidates. Our results indicate that in such a system, the two platforms eventually converge to a globally stable equilibrium point located at the strength-weighted mean

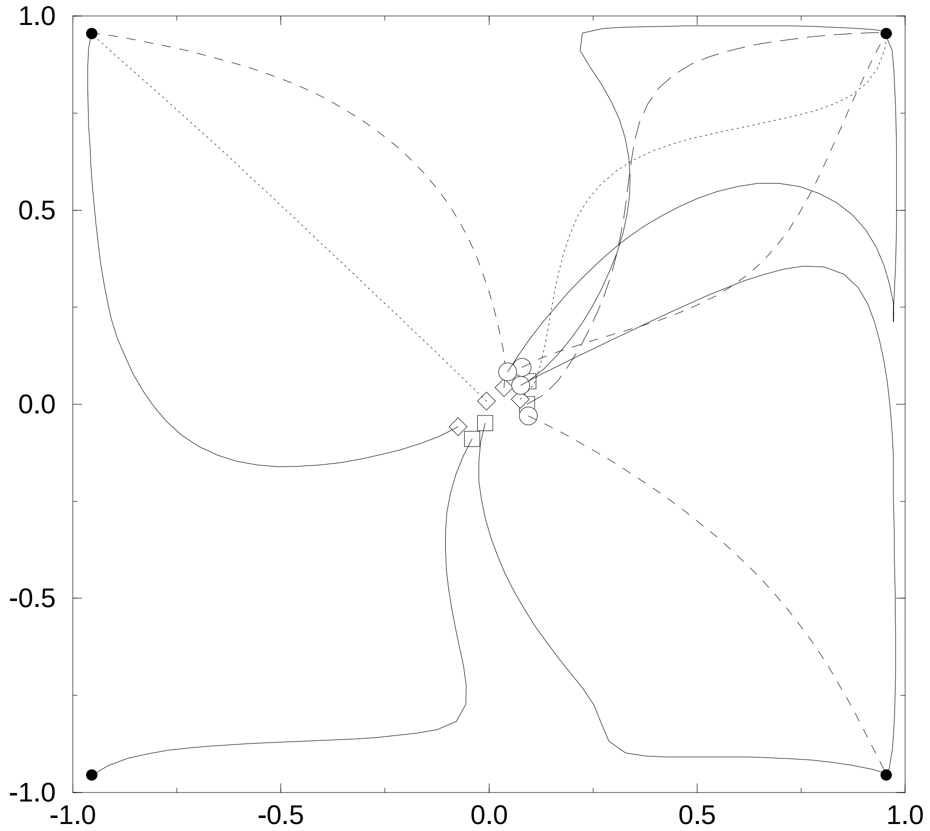

Figure 2: The platform dynamics for the two parties are shown in platform-position space. Two voters with strengths equal to one on all issues and ideal points of $(-1, 1)$ and $(1, -1)$ were used. The mean voter equilibrium of (7) is the origin $(0, 0)$. Voters had utility functions given by $(1''')$ with $\gamma = 1$. Six different initial platform pairs were randomly generated about the origin (each pair has the same marker and line style). Similar markers end up at the same equilibrium points.

of the voters' preferred positions. While the convergence is assured, the transient paths can be long and convoluted.

The fundamental results are relatively invariant to a variety of modifications of the underlying model. However, the incorporation of non-concave utility functions can result in a variety of new behaviors, including multiple equilibria and, we suspect, limit-cycles.

The model provides a necessary benchmark for the behavior of systems with agents that are limited in their abilities to directly solve for equilibria and whose strategic choices are constrained by past actions. While we find that such systems display predictable behavior in the limit, we also note that the transient dynamics can be remarkable, and are worthy of further analysis.

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References

[1] P. Coughlin and S. Nitzan. Directional and local electoral equilibria with probabilistic voting. *Journal of Economic Theory*, 24:226–239, 1981.

[2] P. J. Coughlin. Candidate uncertainty and electoral equilibria. In J. M. Enelow and M. J. Hinich, editors, *Advances in the Spacial Theory of Voting*, pages 145–166, Cambridge, UK, 1990. Cambridge University Press.

[3] D. Friedman. Evolutionary games in economics. *Econometrica*, 59(3):637–666, May 1991.

[4] J. Hofbauer and K. Sigmund. *The Theory of Evolution and Dynamical Systems*. Cambridge University Press, Cambridge, UK, 1988.

[5] J. Hofbauer and K. Sigmund. Adaptive dynamics and evolutionary stability. *Appl. Math. Lett.*, 3(4):75–79, 1990.

[6] K. Kollman, J. H. Miller, and S. E. Page. Adaptive parties in spatial elections. *American Political Science Review*, 86:929–937, 1992.

[7] G. H. Meister. Jacobi problems in differential equations and algebraic geometry. *Rocky Mountain J. Mathematics*, 12:679–705, 1982.

[8] C. R. Plott. A notion of equilibrium and its possibility under majority rule. *The American Economic Review*, 57:787–806, 1967.

[9] L. Samuelson. Electoral equilibria with restricted strategies. *Public Choice*, 43:307–327, 1984.