Modeling Evolutionary Processes:

Evolution from the Viewpoint of a Physicist

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Steps in Evolution: Perspectives from Physics, Biochemistry and Cell Biology – 150 Years after Darwin

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Web-Page for further information:

http://www.tbi.univie.ac.at/~pks

What is information ?

- *Information* is (only) what is understood.
- Information is (only) what creates information.

Carl Friedrich von Weizsäcker, 1912-2007, German physicist and philosopher.

Information in biology

- Understanding of information is interpreted as decoding,
- *maintenance of information* requires reproduction, and

• *creation of information* occurs through adaptation to the environment by means of a Darwinian mechanism of variation and selection.

- 1. Darwin's two pathbreaking ideas
- 2. Dynamics of Darwinian evolution
- 3. RNA evolution in the test tube
- 4. Stochasticity in evolution
- 5. Evolutionary optimization of RNA structure

1. Darwin's two pathbreaking ideas

- 2. Dynamics of Darwinian evolution
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Three necessary conditions for Darwinian evolution are:

- 1. Multiplication,
- 2. Variation, and
- 3. Selection.

Darwin discovered the principle of **natural selection** from empirical observations in nature.

$$f_1 = 10$$

$$f_1 = 10$$

$$s = \frac{f_2 - f_1}{f_1} = 0.1$$

$$f_2 = 11$$

Two variants with a mean progeny of ten or eleven descendants



 $N_1(0) = 9999, N_2(0) = 1; s = 0.1, 0.02, 0.01$

Selection of advantageous mutants in populations of N = 10000 individuals

Charles Darwin drew a tree of life and suggested that all life on Earth descended form **one common ancestor**



Charles Darwin, The Origin of Species, 6th edition. Everyman's Library, Vol.811, Dent London, pp.121-122.



Modern phylogenetic tree: Lynn Margulis, Karlene V. Schwartz. *Five Kingdoms. An Illustrated Guide to the Phyla of Life on Earth.* W.H. Freeman, San Francisco, 1982.

$| T - A - G - G - C - T - A - T - A - A - C - C - \dots - G - C \rightarrow$



Deoxyribonucleic acid - DNA





Punktmutation



Insertion



Deletion



Reconstruction of phylogenies through comparison of molecular sequence data

1. Darwin's two pathbreaking ideas

- 2. Dynamics of Darwinian evolution
- 3. RNA sequences and structures
- 4. Stochasticity in evolution
- 5. Evolutionary optimization of RNA structure



$$f_{m} = \max \{f_{j}; j=1,2,...,n\}$$
$$x_{m}(t) \rightarrow 1 \text{ for } t \rightarrow \infty$$

Reproduction of organisms or replication of molecules as the basis of selection

Selection equation: $[I_i] = x_i \ge 0$, $f_i > 0$

$$\frac{dx_i}{dt} = x_i (f_i - \phi), \quad i = 1, 2, \dots, n; \quad \sum_{i=1}^n x_i = 1; \quad \phi = \sum_{j=1}^n f_j x_j = \overline{f}$$

Mean fitness or dilution flux, $\phi(t)$, is a non-decreasing function of time,

$$\frac{d\phi}{dt} = \sum_{i=1}^{n} f_i \frac{dx_i}{dt} = \overline{f^2} - \left(\overline{f}\right)^2 = \operatorname{var}\{f\} \ge 0$$

Solutions are obtained by integrating factor transformation

$$x_{i}(t) = \frac{x_{i}(0) \cdot \exp(f_{i}t)}{\sum_{j=1}^{n} x_{j}(0) \cdot \exp(f_{j}t)}; \quad i = 1, 2, \cdots, n$$



Chemical kinetics of replication and mutation as parallel reactions

Mutation-selection equation: $[I_i] = x_i \ge 0, f_i > 0, Q_{ii} \ge 0$

$$\frac{dx_i}{dt} = \sum_{j=1}^n f_j Q_{ji} x_j - x_i \phi, \quad i = 1, 2, \dots, n; \quad \sum_{i=1}^n x_i = 1; \quad \phi = \sum_{j=1}^n f_j x_j = \overline{f}$$

Solutions are obtained after integrating factor transformation by means of an eigenvalue problem

$$x_{i}(t) = \frac{\sum_{k=0}^{n-1} \ell_{ik} \cdot c_{k}(0) \cdot \exp(\lambda_{k}t)}{\sum_{j=1}^{n} \sum_{k=0}^{n-1} \ell_{jk} \cdot c_{k}(0) \cdot \exp(\lambda_{k}t)}; \quad i = 1, 2, \dots, n; \quad c_{k}(0) = \sum_{i=1}^{n} h_{ki} x_{i}(0)$$

$$W \div \{f_i Q_{ij}; i, j=1,2,\cdots,n\}; \ L = \{\ell_{ij}; i, j=1,2,\cdots,n\}; \ L^{-1} = H = \{h_{ij}; i, j=1,2,\cdots,n\}$$

$$L^{-1} \cdot W \cdot L = \Lambda = \{\lambda_k; k=0, 1, \cdots, n-1\}$$

Matrix W and Frobenius theorem:

W =
$$\begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{pmatrix}$$

Primitive matrix W:

A nonnegative square matrix $W = \{w_{ij}\}$ is said to be a primitive matrix if there exists k such that $W^k \gg 0$, i.e., if there exists k such that for all i, j, the (i, j) entry of W^k is positive.

Perron-Frobenius theorem applied to the value matrix W

W is primitive: (i) λ_0 is real and strictly positive (ii) $\lambda_0 > |\lambda_k|$ for all $k \neq 0$ (iii) λ_0 is associated with strictly positive eigenvectors (iv) λ_0 is a simple root of the characteristic equation of W (v-vi) etc.

W is irreducible: (i), (iii), (iv), etc. as above (ii) $\lambda_0 \ge |\lambda_k|$ for all $k \ne 0$ Decomposition of matrix W

$$W = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \dots & w_{nn} \end{pmatrix} = Q \cdot F \text{ with}$$

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & \dots & Q_{1n} \\ Q_{21} & Q_{22} & \dots & Q_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{n1} & Q_{n2} & \dots & Q_{nn} \end{pmatrix} \text{ and } F = \begin{pmatrix} f_1 & 0 & \dots & 0 \\ 0 & f_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f_n \end{pmatrix}$$

Uniform error rate model:

$$Q_{ij} = p^{d_H(\mathbf{X}_i, \mathbf{X}_j)} (1-p)^{\left(n-d_H(\mathbf{X}_i, \mathbf{X}_j)\right)}$$

 $d_H(\mathbf{X}_i, \mathbf{X}_j) \ldots$ Hamming distance













The error threshold in replication



Available online at www.sciencedirect.com

Virus Research 107 (2005) 115-116

Preface Antiviral strategy on the horizon

Error catastrophe had its conceptual origins in the middle of the XXth century, when the consequences of mutations on enzymes involved in protein synthesis, as a theory of aging. In those times biological processes were generally perceived differently from today. Infectious diseases were regarded as a fleeting nuisance which would be eliminated through the use of antibiotics and antiviral agents. Microbial variation. although known in some cases, was not thought to be a significant problem for disease control. Variation in differentiated organisms was seen as resulting essentially from exchanges of genetic material associated with sexual reproduction. The problem was to unveil the mechanisms of inheritance. expression of genetic information and metabolism. Few saw that genetic change is occurring at present in all organisms. and still fewer recognized Darwinian principles as essential to the biology of pathogenic viruses and cells. Population geneticists rarely used bacteria or viruses as experimental systems to define concepts in biological evolution. The extent of genetic polymorphism among individuals of the same biological species came as a surprise when the first results on comparison of electrophoretic mobility of enzymes were obtained. With the advent of in vitro DNA recombination. and rapid nucleic acid sequencing techniques, molecular analyses of genomes reinforced the conclusion of extreme inter-individual genetic variation within the same species. Now, due largely to spectacular progress in comparative genomics, we see cellular DNAs, both prokarvotic and eukarvotic, as highly dynamic. Most cellular processes, including such essential information-bearing and transferring events as genome replication, transcription and translation, are increasingly perceived as inherently inaccurate. Viruses, and in particular RNA viruses, are among the most extreme examples of exploitation of replication inaccuracy for survival.

Error catastrophe, or the loss of meaningful genetic information through excess genetic variation, was formulated in quantitative terms as a consequence of quasispecies theory, which was first developed to explain self-organization and adaptability of primitive replicons in early stages of life. Recently, a conceptual extension of error catastrophe that could be defined as "induced genetic deterioration" has emerged as

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a possible antiviral strategy. This is the topic of the current special issue of *Virus Research*.

Virus

www.elsewier.com/locate/virusre-

Research

Few would nowadays doubt that one of the major obstacles for the control of viral disease is short-term adaptability of viral pathogens. Adaptability of viruses follows the same Darwinian principles that have shaped biological evolution over eons, that is, repeated rounds of reproduction with genetic variation, competition and selection, often perturbed by random events such as statistical fluctuations in population size. However, with viruses the consequences of the operation of these very same Darwinian principles are felt within very short times. Short-term evolution (within hours and days) can be also observed with some cellular pathogens, with subsets of normal cells, and cancer cells. The nature of RNA viral pathogens begs for alternative antiviral strategies, and forcing the virus to cross the critical error threshold for maintenance of genetic information is one of them.

The contributions to this volume have been chosen to reflect different lines of evidence (both theoretical and experimental) on which antiviral designs based on genetic deterioration inflicted upon viruses are being constructed. Theoretical studies have explored the copying fidelity conditions that must be fulfilled by any information-bearing replication system for the essential genetic information to be transmitted to progeny. Closely related to the theoretical developments have been numerous experimental studies on quasispecies dynamics and their multiple biological manifestations. The latter can be summarized by saving that RNA viruses, by virtue of existing as mutant spectra rather than defined genetic entities, remarkably expand their potential to overcome selective pressures intended to limit their replication. Indeed, the use of antiviral inhibitors in clinical practice and the design of vaccines for a number of major RNA virus-associated diseases, are currently presided by a sense of uncertainty. Another line of growing research is the enzymology of copying fidelity by viral replicases, aimed at understanding the molecular basis of mutagenic activities. Error catastrophe as a potential new antiviral strategy received an important impulse by the observation that ribavirin (a licensed antiviral nucleoside analogue) may be exerting, in some systems, its antiviral activity through enhanced mutagenesis. This has encouraged investigations on new mutagenic base analogues, some of them used in anticancer chemotherapy. Some chapters summarize these important biochemical studies on cell entry pathways and metabolism of mutagenic agents, that may find new applications as antiviral agents.

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This volume intends to be basically a progress report, an introduction to a new avenue of research, and a realistic appraisal of the many issues that remain to be investigated. In this respect. I can envisage (not without many uncertainties) at least three lines of needed research; (i) One on further understanding of quasispecies dynamics in infected individuals to learn more on how to apply combinations of virus-specific mutagens and inhibitors in an effective way, finding synergistic combinations and avoiding antagonistic ones as well as severe clinical side effects. (ii) Another on a deeper understanding of the metabolism of mutagenic agents, in particular base and nucleoside analogues. This includes identification of the transporters that carry them into cells, an understanding of their metabolic processing, intracellular stability and alterations of nucleotide pools, among other issues. (iii) Still another line of needed research is the development of new mutagenic agents specific for viruses, showing no (or limited) toxicity for cells. Some advances may come from links with anticancer research, but others should result from the designs of new molecules, based on the structures of viral polymerases. I really hope that the reader finds this issue not only to be an interesting and useful review of the current situ-

Preface / Virus Research 107 (2005) 115-116

ation in the field, but also a stimulating exposure to the major problems to be faced.

The idea to prepare this special issue came as a kind invitation of Ulrich Desselberger, former Editor of Virus Research, and then taken enthusiastically by Luis Enjuanes, recently appointed as Editor of Virus Research. I take this opportunity to thank Ulrich, Luis and the Editor-in-Chief of Virus Research, Brian Mahy, for their continued interest and support to the research on virus evolution over the years.

My thanks go also to the 19 authors who despite their busy schedules have taken time to prepare excellent manuscripts, to Elsevier staff for their prompt responses to my requests, and, last but not least, to Ms. Lucia Horrillo from Centro de Biologia Molecular "Severo Ochoa" for her patient dealing with the correspondence with authors and the final organization of the issue.

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SECOND EDITION

ORIGIN AND EVOLUTION OF VIRUSES



Edited by ESTEBAN DOMINGO COLIN R. PARRISH JOHN J. HOLLAND



Molecular evolution of viruses



A fitness landscape showing an error threshold

SELF-REPLICATION WITH ERRORS A MODEL FOR POLYNUCLEOTIDE REPLICATION ** Jörg SWETINA and Peter SCHUSTER * Institut für Theoretische Chemie und Strahlenchemie der Universitüt, Währinge alle 17, A-1090 Wien, Au

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Biophysical Chemistry 16 (1982) 329-345 Elsevier Biomedical Pres

Key words: Polynucleotide replication; Quasi-species; Point mutation; Mutant class; Stochastic replication

A model for polynucleotide replication is presented and analyzed by means of perturbation theory. Two basic as allow handling of sequences up to a chain length of p = 80 explicitly: point mutations are restricted to a two-digit model and individual sequences are subsumed into mutant classes. Perturbation theory is in excellent agreement with the exact results for long enough sequences ($\nu > 20$).

tical weights.

constraints ($\phi = 0$) and competitors (n = 1).

1. Introduction

Eigen [8] proposed a formal kinetic equation (eq. 1) which describes self-replication under the constraint of constant total population size:

 $\frac{dx_i}{dx} = \dot{x}_i = \sum w_{ij} x_j - \frac{x_i}{c} \phi; i = 1, ..., n^+$ (1)

By x, we denote the population number or concentration of the self-replicating element I, i.e., $x_i = [I_i]$. The total population size or total concentration $c = \sum_i x_i$ is kept constant by proper adjustment of the constraint ϕ : $\phi = \sum_i \sum_j w_{ij} x_i$. Characteristically, this constraint has been called 'constant organization'. The relative values of diagonal

* Dedicated to the late Professor B.L. Jones who was among the first to do rigorous mathematical analysis on the prob-lems described here.

** This paper is considered as part II of Model Studies on RNA replication. Part I is by Gassner and Schuster [14]. 7 All summations throughout this paper run from 1 to n unless specified differently: $\Sigma_i = \sum_{i=1}^n$ and $\sum_{i,i=j} = \sum_{i=1}^{j-1} + \sum_{i=j+1}^n$. respectively

0301_4677 /82 /0000_0000 /\$02 75 @ 1982 Elsevier Biomedical Press

1.0-10 min Uniform distribution Ouasispecies y:05 (w_{ij}) and off-diagonal $(w_{ij}, i \neq j)$ rates, as we shall see in detail in section 2, are related to the accuracy of the replication process. The specific prop erties of eq. 1 are essentially based on the fact that it leads to exponential growth in the absence of $\Sigma I_{(1)}$ The non-linear differential equation, eq. 1 - the non-linearity is introduced by the definition of ϕ at constant organization - shows a remarkable feature: it leads to selection of a defined ensemble of self-replicating elements above a certain accu-ΣI(25) racy threshold. This ensemble of a master and its most frequent mutants is a so-called 'quasi-species' ΣI(24),ΣI(26) [9]. Below this threshold, however, no selection takes place and the frequencies of the individual elements are determined exclusively by their statis-ΣI(2) SI(23) SI(27) Rigorous mathematical analysis has been performed on eq. 1 [7,15,24,26]. In particular, it was SI(22) SI(28) shown that the non-linearity of eq. 1 can be re-∑I(3) moved by an appropriate transformation. The ei-ΣI(21),ΣI(29) 2Iu genvalue problem of the linear differential equation obtained thereby may be solved approximately by the conventional perturbation technique 0 1.00 0.95 0.90 a 0.05 0.00 0.10 Mutation rate p = 1 - q - q

> Single peak fitness landscape: $f_0 = f$ and $f_1 = f_2 = \dots = f_N = 1$

Quasispecies as a function of the mutation rate p

$$f_0 = \boldsymbol{\sigma} = 10$$

 $\sigma = \frac{f_0}{(1 - x_0) \sum_{i=1}^{N} f_i x_i}$

 I_0 ... master sequence; $N = \kappa^n$


Fitness landscapes showing error thresholds

Hamming distance $d_{H}(I_k, I_0)$





Error threshold: Individual sequences n = 10, $\sigma = 2$ and d = 0, 1.0, 1.85

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Three necessary conditions for Darwinian evolution are:

- 1. Multiplication,
- 2. Variation, and
- 3. Selection.

Variation through mutation and recombination operates on the genotype whereas the phenotype is the target of selection.

One important property of the Darwinian scenario is that variations in the form of mutations or recombination events occur uncorrelated with their effects on the selection process.

All conditions can be fulfilled not only by cellular organisms but also by nucleic acid molecules in suitable cell-free experimental assays.



D.R.Mills, R.L.Peterson, S.Spiegelman, *An extracellular Darwinian experiment with a self-duplicating nucleic acid molecule*. Proc.Natl.Acad.Sci.USA **58** (1967), 217-224

Application of serial transfer to RNA evolution in the test tube



Reproduction of the original figure of the serial transfer experiment with $Q\beta$ RNA

D.R.Mills, R,L,Peterson, S.Spiegelman, An extracellular Darwinian experiment with a self-duplicating nucleic acid molecule. Proc.Natl.Acad.Sci.USA 58 (1967), 217-224

Fig. 9. Serial transfer experiment. Each 0.25 ml standard reaction mixture contained 40 μ g of Q β replicase and ³³P-UTP. The first reaction (0 transfer) was initiated by the addition of 0.2 μ g ts-1 (temperature-sensitive RNA) and incubated at 35 °C for 20 min, whereupon 0.02 ml was drawn for counting and 0.02 ml was used to prime the second reaction (first transfer), and so on. After the first 13 reactions, the incubation periods were reduced to 15 min (transfers 14-29). Transfers 30-38 were incubated for 10 min. Transfers 39-52 were incubated for 7 min, and transfers 53-74 were incubated for 5 min. The arrows above certain transfers (0, 8, 14, 29, 37, 53, and 73) indicate where 0.001-0.1 ml of product was removed and used to prime reactions for sedimentation analysis on sucrose. The inset examines both infectious and total RNA. The results show that biologically competent RNA ceases to appear after the 4th transfer (Mills *et al.* 1967).



self-sustained replication

Tracey A. Lincoln, Gerald F. Joyce, Science 323, 1229-1232, 2009



Exponential growth levels off when the reservoir is exhausted (l.h.s.). RNA production in serial transfer experiments (r.h.s.)

Tracey A. Lincoln, Gerald F. Joyce, Science 323, 1229-1232, 2009



RNA evolution of recombinant replicators

Tracey A. Lincoln, Gerald F. Joyce, Science 323, 1229-1232, 2009

WILEY-VCH

Directed Molecular Evolution of Proteins

or How to Improve Enzymes for Biocatalysis

Edited by Susanne Brakmann and Kai Johnsson





Application of molecular evolution to problems in biotechnology



5' - end

N₁



A symbolic notation of RNA secondary structure that is equivalent to the conventional graphs

RNA sequence: GUAUCGAAAUACGUAGCGUAUGGGGAUGCUGGACGGUCCCAUCGGUACUCCA

RNA folding: Structural biology, spectroscopy of biomolecules, understanding molecular function Iterative determination of a sequence for the given secondary structure

> Inverse Folding Algorithm

Inverse folding of RNA:

Biotechnology, design of biomolecules with predefined structures and functions

RNA structure of minimal free energy:

Sequence, structure, and design



UUUAGCCAGCGCGAGUCGUGCGGACGGGGUUAUCUCUGUCGGGCUAGGGCGC GUGAGCGCGGGGCACAGUUUCUCAAGGAUGUAAGUUUUUGCCGUUUAUCUGG UUAGCGAGAGAGGAGGCUUCUAGACCCAGCUCUCUGGGUCGUUGCUGAUGCG CAUUGGUGCUAAUGAUAUUAGGGCUGUAUUCCUGUAUAGCGAUCAGUGUCCG GUAGGCCCUCUUGACAUAAGAUUUUUCCAAUGGUGGGAGAUGGCCAUUGCAG

The inverse folding algorithm searches for sequences that form a given RNA secondary structure under the minimum free energy criterion.





GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG



GGCUAUCGUACGUUUACCCGAAAGUCUACGUUGGACCCAGGCAUUGGACG

GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG





GGCUAUCGUACGUUUACCCGAAAGUCUACGUUGGACCCAGGCAUUGGACG

GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG







GGCUAUCGUACGUUUACCCGAAAGUCUACGUUGGACCCAGGCAUUGGACG

GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGG<mark>U</mark>CCCAGGCAUUGGACG

GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG



GGCUAUCGUAUGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACUCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGCUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCCAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUGUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCUGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCUGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCACUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG GGCUAUCGUACGUUUACCCCAAAAGUCUACGUUGGACCCAGGCAUUGGACG

PAN

			=	Dearbert	
Total Hamming Distance:	150000	11.647973	23.140715	4.810480	
Nonzero Hamming Distance:	99875	16.949991	30.757651	5.545958	
Degree of Neutrality:	50125	0.334167	0.006961	0.083434	
Number of Structures:	1000	52.31	85.30	9.24	
1 ((((((((((((((((((((((((((((((((((((())))).))).))	50125	0.334167	
2(((((())))))))))))	2856	0.019040	
3 ((((((((((((()))))))))).))	2799	0.018660	
4 ((((((((((((((((())))).))).))	2417	0.016113	
5 ((((((((((((((()).)))))))))))).))	2265	0.015100	
6 ((((((((((((().)))))))))))))))))))))))))))))))))))))).))	2233	0.014887	
7 (((((((())))))))).))	1442	0.009613	
8 ((((((.((()))))))))))).))	1081	0.007207	
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Shadow Surrounding of an DN	Λ structure in	shape space	~	AUGGUC	
AUGC alphabet, chain length n=	50	i snape space.	с С	C A A	



THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

By CHARLES DARWIN, M.A.,

FELLOW OF THE BOYAL, GEOLOGICAL, LINNÆAN, ETC., SOCHETIES; AUTHOR OF 'JOURNAL OF RESEARCHES DURING H. M. S. EEAGLE'S VOYAGE BOUND THE WORLD.'

LONDON: JOHN MURRAY, ALBEMARLE STREET. 1859.

The right of Translation is reserved.

This preservation of favourable individual differences and variations, and the destruction of those which are injurious, I have called Natural Selection, or the Survival of the Fittest. Variations neither useful nor injurious would not be affected by natural selection, and would be left either a fluctuating element, as perhaps we see in certain polymorphic species, or would ultimately become fixed, owing to the nature of the organism and the nature of the conditions.

Charles Darwin. The Origin of Species. Sixth edition. John Murray. London: 1872



THE NEUTRAL THEORY OF MOLECULAR EVOLUTION

MOTOO KIMURA National Institute of Genetics, Japan

Motoo Kimuras population genetics of neutral evolution.

Evolutionary rate at the molecular level. *Nature* **217**: 624-626, 1955.

The Neutral Theory of Molecular Evolution. Cambridge University Press. Cambridge, UK, 1983.



CAMBRIDGE UNIVERSITY PRESS Cambridge London New York New Rochelle Melbourne Sydney Fig. 3.1. Behavior of mutant genes following their appearance in a finite population. Courses of change in the frequencies of mutants destined to fixation are depicted by thick paths. N_e stands for the effective population size and v is the mutation rate.



The average time of replacement of a dominant genotype in a population is the reciprocal mutation rate, 1/v, and therefore independent of population size.

Is the Kimura scenario correct for frequent mutations?

Bulletin of Mathematical Biology Vol. 50, No. 6, pp. 635-660, 1988. Printed in Great Britain. 0092-8240/88\$3.00+0.00 Pergamon Press plc Society for Mathematical Biology

STATIONARY MUTANT DISTRIBUTIONS AND EVOLUTIONARY OPTIMIZATION

 PETER SCHUSTER and JÖRG SWETINA Institut für theoretische Chemie und Strahlenchemie der Universität Wien, Währingerstraße 17, A 1090 Wien, Austria

Molecular evolution is modelled by erroneous replication of binary sequences. We show how the selection of two species of equal or almost equal selective value is influenced by its nearest neighbours in sequence space. In the case of perfect neutrality and sufficiently small error rates we find that the Hamming distance between the species determines selection. As the error rate increases the fitness parameters of neighbouring species become more and more important. In the case of almost neutral sequences we observe a critical replication accuracy at which a drastic change in the "quasispecies", in the stationary mutant distribution occurs. Thus, in frequently mutating populations fitness turns out to be an ensemble property rather than an attribute of the individual.

In addition we investigate the time dependence of the mean excess production as a function of initial conditions. Although it is optimized under most conditions, cases can be found which are characterized by decrease or non-monotonous change in mean excess productions.

1. Introduction. Recent data from populations of RNA viruses provided direct evidence for vast sequence heterogeneity (Domingo *et al.*, 1987). The origin of this diversity is not yet completely known. It may be caused by the low replication accuracy of the polymerizing enzyme, commonly a virus specific, RNA dependent RNA synthetase, or it may be the result of a high degree of selective neutrality of polynucleotide sequences. Eventually, both factors contribute to the heterogeneity observed. Indeed, mutations occur much more frequently than previously assumed in microbiology. They are by no means rare events and hence, neither the methods of conventional population genetics (Ewens, 1979) nor the neutral theory (Kimura, 1983) can be applied to these virus populations. Selectively neutral variants may be close with respect to Hamming distance and then the commonly made assumption that the mutation backflow from the mutants to the wilde type is negligible does not apply.

A kinetic theory of polynucleotide evolution which was developed during the past 15 years (Eigen, 1971; 1985; Eigen and Schuster, 1979; Eigen *et al.*, 1987; Schuster, 1986); Schuster and Sigmund, 1985) treats correct replication and mutation as parallel reactions within one and the same reaction network



Neutral network

 $\lambda = 0.01, s = 367$

$$d_{\rm H} = 1$$

 $\lim_{p \to 0} x_1(p) = x_2(p) = 0.5$

518 514 550 546

Neutral network

 $\lambda = 0.01$, s = 877

Pairs of genotypes in neutral replication networks

 $d_{\rm H} = 2$ $\lim_{p \to 0} x_1(p) = a$ $\lim_{p \to 0} x_2(p) = 1 - a$

$d_{\rm H}$ 3

 $\lim_{p \to 0} x_1(p) = 1, \lim_{p \to 0} x_2(p) = 0 \text{ or}$ $\lim_{p \to 0} x_1(p) = 0, \lim_{p \to 0} x_2(p) = 1$

Random fixation in the sense of Motoo Kimura









Neutral network: Individual sequences

 $n = 10, \sigma = 1.1, d = 1.0$

······ ACAUGCGAA	
······ AUAUACGAA	
······ ACAUGCGCA	
······ GCAUACGAA	
······ ACAUGCUAA	
······ ACAUGCGAG	
······ ACACGCGAA	
······ ACGUACGAA	
······ ACAUAGGAA	
······ ACAUACGAA	

Consensus sequence of a quasispecies of two strongly coupled sequences of Hamming distance $d_H(X_{i,j},X_j) = 1$.



Neutral network: Individual sequences

 $n = 10, \sigma = 1.1, d = 1.0$



Consensus sequence of a quasispecies of two strongly coupled sequences of Hamming distance $d_H(X_{i_i}, X_i) = 2$.



Selection-mutation matrix W

$$\mathbf{W} = \begin{pmatrix} f & \mathbf{O}(\varepsilon^2) & \varepsilon & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & f & \varepsilon & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) \\ \varepsilon & \varepsilon & f & \varepsilon & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & f & \varepsilon & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & f & \varepsilon & \varepsilon \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & f & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & f & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & f & \mathbf{O}(\varepsilon^2) \\ \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \mathbf{O}(\varepsilon^2) & \varepsilon & \mathbf{O}(\varepsilon^2) & f \end{pmatrix}$$

Adjacency matrix A

/0	0	1	0	0	0	0
0	0	1	0	0	0	0
1	1	0	1	0	0	0
0	0	1	0	1	0	0
0	0	0	1	0	1	1
0	0	0	0	1	0	0
0/	0	0	0	1	0	0/
	$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$ \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} $	$ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} $	$ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \end{pmatrix} $

Eigenvalues of W and A

$\lambda_0 = f + 2\varepsilon ,$	$\lambda_0 = 2 ,$
$\lambda_1 = f + \sqrt{2} \varepsilon ,$	$\lambda_1 = \sqrt{2},$
$\lambda_{2,3,4} = f$,	$\lambda_{2,3,4} = 0,$
$\lambda_5 = f - \sqrt{2}\varepsilon ,$	$\lambda_5 = -\sqrt{2}$,
$\lambda_6 = f - 2\varepsilon ,$	$\lambda_6 = -2$.

Largest eigenvector of W and A

 $\xi_0 \ = \ (0.1, 0.1, 0.2, 0.2, 0.2, 0.1, 0.1) \ .$

Computation of sequences in the core of a neutral network

1. Darwin's two pathbreaking ideas

- 2. Dynamics of Darwinian evolution
- 3. RNA evolution in the test tube
- 4. Stochasticity in evolution
- 5. Evolutionary optimization of RNA structure

X₀

Evolution of RNA molecules as a Markow process and its analysis by means of the relay series














Figure 1. Replication as a multitype branching process.

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POLYNUCLEOTIDE EVOLUTION AND BRANCHING PROCESSES*

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The theory of multitype branching processes is applied to the kinetics of polynucleotide replication. The results obtained are compared with the solutions of the deterministic differential equations of conventional chemical kinetics.

RNA replication and mutation as a multitype branching process



1. Darwin's two pathbreaking ideas

- 2. Dynamics of Darwinian evolution
- 3. RNA evolution in the test tube
- 4. Stochasticity in evolution
- 5. Evolutionary optimization of RNA structure



random individuals. The primer pair used for genomic DNA amplification is 5'-TCTCCCTGGATTCT-CATTTA-3' (forward) and 5'-TCTTTGTCTTCTGT TCCACC-3' (reverse). Reactions were performed in 25 µl using 1 unit of Tag DNA polymerase with each primer at 0.4 µM; 200 µM each dATP, dTTP, dGTP, and dCTP; and PCR buffer [10 mM tris-HCI (pH 8.3) 50 mM KCl₂,1.5 mM MgCl₂] in a cycle condition of 94°C for 1 min and then 35 cycles of 94°C for 30 s. 55°C for 30 s, and 72°C for 30 s followed by 72°C for 6 min. PCR products were purified (Qiagen), digested with Xmn L and senarated in a 2% agarose gel.

- 32. A nonsense mutation may affect mRNA stability and result in degradation of the transcript [L. Maguat, Am. J. Hum. Genet. 59, 279 (1996)]
- 33. Data not shown: a dot blot with poly (A)+ RNA from 50 human tissues (The Human BNA Master Blot. 7770-1, Clontech Laboratories) was hybridized with a probe from exons 29 to 47 of MYO15 using the same condition as Northern blot analysis (13).
- 34. Smith-Magenis syndrome (SMS) is due to deletions of 17p11.2 of various sizes, the smallest of which includes MYO15 and perhaps 20 other genes ((6): K-S Chen, L. Potocki, J. R. Lupski, MRDD Res. Rev. 2 122 (1996)] MYO15 expression is easily detected in the pituitary gland (data not shown). Haploinsufficiency for MYO15 may explain a portion of the SMS

Continuity in Evolution: On the Nature of Transitions

Walter Fontana and Peter Schuster

To distinguish continuous from discontinuous evolutionary change, a relation of nearness between phenotypes is needed. Such a relation is based on the probability of one phenotype being accessible from another through changes in the genotype. This nearness relation is exemplified by calculating the shape neighborhood of a transfer RNA secondary structure and provides a characterization of discontinuous shape transformations in RNA. The simulation of replicating and mutating RNA populations under selection shows that sudden adaptive progress coincides mostly, but not always, with discontinuous shape transformations. The nature of these transformations illuminates the key role of neutral genetic drift in their realization.

A much-debated issue in evolutionary biology concerns the extent to which the history of life has proceeded gradually or has been punctuated by discontinuous transitions at the level of phenotypes (1). Our goal is to make the notion of a discontinuous transition more precise and to understand how it arises in a model of evolutionary adaptation.

We focus on the narrow domain of RNA secondary structure, which is currently the simplest computationally tractable, yet realistic phenotype (2). This choice enables the definition and exploration of concepts that may prove useful in a wider context. RNA secondary structures represent a coarse level of analysis compared with the three-dimensional structure at atomic resolution. Yet, secondary structures are empir-

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ically well defined and obtain their biophysical and biochemical importance from being a scaffold for the tertiary structure. For the sake of brevity, we shall refer to secondary structures as "shapes." RNA combines in a single molecule both genotype (replicatable sequence) and phenotype (selectable shape), making it ideally suited for in vitro evolution experiments (3, 4).

phenotype such as short stature. Moreover, a few

SMS natients have sensorineural hearing loss, pos-

sibly because of a point mutation in MYO15 in trans

X-7 Liu et al ihid 17 268 (1997): E Gibson et al

vrinths) obtained from human fetuses at 18 to 22

weeks of development in accordance with guidelines

established by the Human Research Committee at

the Brigham and Women's Hospital. Only samples

without evidence of degradation were pooled for

poly (A)* selection over oligo(dT) columns. First-

strand cDNA was prepared using an Advantage RT-

for-PCR kit (Clontech Laboratories). A portion of the

first-strand cDNA (4%) was amplified by PCR with

Advantage cDNA polymerase mix (Clontech Labora-

tories) using human MYO15-specific oligonucleotide

primers (forward, 5'-GCATGACCTGCCGGCTAAT

GGG-3': reverse, 5'-CTCACGGCTTCTGCATGGT-

GCTCGGCTGGC-3'). Cycling conditions were 40 s

at 94°C; 40 s at 66°C (3 cycles), 60°C (5 cycles), and

55°C (29 cycles); and 45 s at 68°C. PCB products

were visualized by ethidium bromide staining after

fractionation in a 1% agarose gel. A 688-bp PCR

Nature 374, 62 (1995): D. Weil et al. ibid. p. 60.

37. RNA was extracted from cochlea (membranous lab

36. K. B. Avraham et al., Nature Genet. 11, 369 (1995);

to the SMS 17n11.2 deletion.

35. R. A. Fridell, data not shown

To generate evolutionary histories, we used a stochastic continuous time model of an RNA population replicating and mutating in a capacity-constrained flow reactor under selection (5, 6). In the laboratory, a goal might be to find an RNA aptamer binding specifically to a molecule (4). Although in the experiment the evolutionary end product was unknown, we thought of its shape as being specified implicitly by the imposed selection criterion. Because our intent is to study evolutionary histories rather than end products, we defined a target shape in advance and assumed the replication rate of a sequence to be a function of because, in contrast to sequences, there are

product is expected from amplification of the human MYO15 cDNA. Amplification of human genomic DNA with this primer pair would result in a 2903-bp fragment

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9 March 1998; accepted 17 April 1998

the similarity between its shape and the target. An actual situation may involve more than one best shape, but this does not affect our conclusions.

An instance representing in its qualitative features all the simulations we performed is shown in Fig. 1A. Starting with identical sequences folding into a random shape, the simulation was stopped when the population became dominated by the target, here a canonical tRNA shape. The black curve traces the average distance to the target (inversely related to fitness) in the population against time. Aside from a short initial phase, the entire history is dominated by steps, that is, flat periods of no apparent adaptive progress, interrupted by sudden approaches toward the target structure (7). However, the dominant shapes in the population not only change at these marked events but undergo several fitness-neutral transformations during the periods of no apparent progress. Although discontinuities in the fitness trace are evident, it is entirely unclear when and on the basis of what the series of successive phenotypes itself can be called continuous or discontinuous.

A set of entities is organized into a (topological) space by assigning to each entity a system of neighborhoods. In the present case, there are two kinds of entities: sequences and shapes, which are related by a thermodynamic folding procedure. The set of possible sequences (of fixed length) is naturally organized into a space because point mutations induce a canonical neighborhood. The neighborhood of a sequence consists of all its one-error mutants. The problem is how to organize the set of possible shapes into a space. The issue arises

Evolution *in silico*

W. Fontana, P. Schuster, Science 280 (1998), 1451-1455



Replication rate constant:

$$f_{k} = \gamma / [\alpha + \Delta d_{S}^{(k)}]$$
$$\Delta d_{S}^{(k)} = d_{H}(S_{k}, S_{\tau})$$

Selection constraint:

Population size, N = # RNA molecules, is controlled by the flow

$$N(t) \approx \overline{N} \pm \sqrt{\overline{N}}$$

Mutation rate: $p = 0.001 / site \times replication$

The flowreactor as a device for studies of evolution *in vitro* and *in silico*





Probability of a single trajectory to reach the target structure



In silico optimization in the flow reactor: Evolutionary Trajectory

28 neutral point mutations during a long quasi-stationary epoch



entry	GGUAUGGGCGUUGAAUAGUAGGGUUUAAACCAAUCGGCCAACGAUCUCGUGUGCGCAUUUCAUAUCCCGUACAGAA
8	.(((((((((((((()))))))))((((((
exit	GGUAUGGGCGUUGAAUAAUAGGGUUUAAACCAAUCGGCCAACGAUCUCGUGUGCGCAUUUCAUAUCCCAUACAGAA
entry	GGUAUGGGCGUUGAAUAAUAGGGUUUAAACCAAUCGGCCAACGAUCUCGUGUGCGCAUUUCAUAUACCAUACAGAA
9	.((((((((((((((((((((()))))))))))))))))
exit	UGGAUGGACGUUGAAUAACAAGGUAUCGACCAAACAACCAAC
entry	UGGAUGGACGUUGAAUAACAAGGUAUCGACCAAACAACCAAC
10	((((((((((((((((((((((((((((((((((((
exit	UGGAUGGACGUUGAAUAACAAGGUAUCGACCAAACAACCAAC

Transition inducing point mutations change the molecular structure Neutral point mutations leave the molecular structure unchanged

Neutral genotype evolution during phenotypic stasis





Phenylalanyl-tRNA as target structure



A sketch of optimization on neutral networks

Table 8. Statistics of the optimization trajectories. The table shows the results of sampled evolutionary trajectories leading from a random initial structure, S_I , to the structure of tRNA^{phe}, S_T , as the target^a. Simulations were performed with an algorithm introduced by Gillespie [55–57]. The time unit is here undefined. A mutation rate of p = 0.001 per site and replication were used. The mean and standard deviation were calculated under the assumption of a log-normal distribution that fits well the data of the simulations.

Alphabet	Population size, N	Number of runs, n _R	f Real time from start to target		Number of replications [10 ⁷]	
			Mean value	σ	Mean value	σ
AUGC	1 000	120	900	+1380 -542	1.2	+3.1 -0.9
	2 000	120	530	+880 -330	1.4	+3.6 -1.0
	3 000	1199	400	+670 - 250	1.6	+4.4 - 1.2
	10 000	120	190	+230 - 100	2.3	+5.3 -1.6
	30 000	63	110	+97 -52	3.6	+6.7 - 2.3
	100 000	18	62	+50 - 28	_	_
GC	1 000	46	5160	+15700 - 3890	_	_
	3 000	278	1910	+5180 - 1460	7.4	+35.8 - 6.1
	10 000	40	560	+1620 - 420	_	-

^a The structures S_I and S_T were used in the optimization:

Is the degree of neutrality in **GC** space much lower than in **AUGC** space ?

Statistics of RNA structure optimization: P. Schuster, Rep.Prog.Phys. 69:1419-1477, 2006

Total Hamming Distance: Nonzero Hamming Distance: Degree of Neutrality: Number of Structures:	Number 150000 99875 50125 1000	Mean Value 11.647973 16.949991 0.334167 52.31	Variance 23.140715 30.757651 0.006961 85.30	Std.Dev. 4.810480 5.545958 0.083434 9.24	CAGO GAGC CAGC GA C
<pre>1 ((((((((((((((((())))))))))))))))))))</pre>))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))	50125 2856 2799 2417 2265 2233	0.334167 0.019040 0.018660 0.016113 0.015100 0.014887	GC-AUACGU AG-UC
Total Hamming Distance: Nonzero Hamming Distance: Degree of Neutrality: Number of Structures:	Number 50000 45738 4262 1000	Mean Value 13.673580 14.872054 0.085240 36.24	Variance 10.795762 10.821236 0.001824 6.27	Std.Dev. 3.285691 3.289565 0.042708 2.50	و محمو محمو
<pre>1 ((((((((((((((((())))))))))))))) 2 ((((((((((</pre>)))))))))))))))))))))))))))))))))))))).)))).)))).)))).)))).))	4262 1940 1791 1752 1423	0.085240 0.038800 0.035820 0.035040 0.028460	eeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeeee

Shadow – Surrounding of an RNA structure in shape space – AUGC and GC alphabet

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