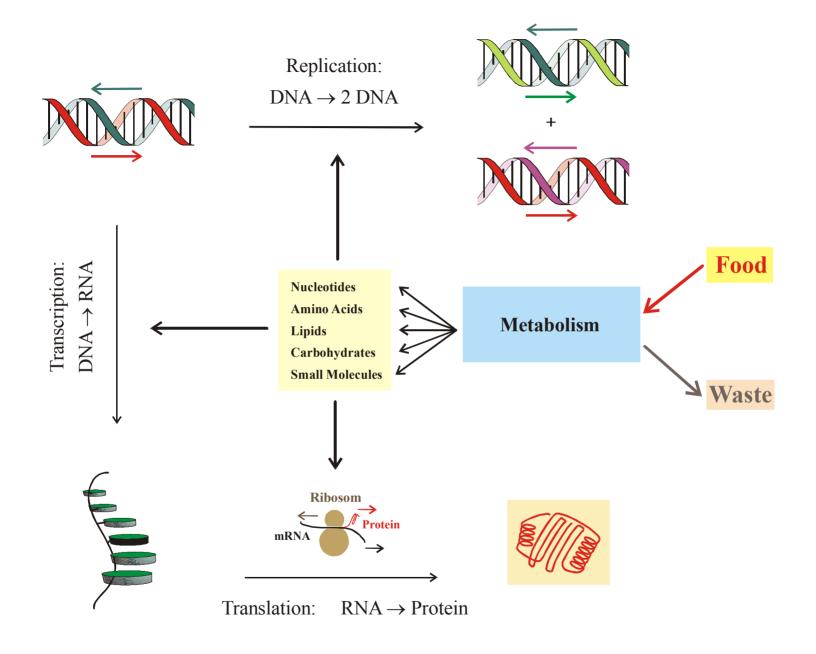
RNA – Das Zaubermolekül

Peter Schuster Institut für Theoretische Chemie und Molekulare Strukturbiologie der Universität Wien

Dies Academicus

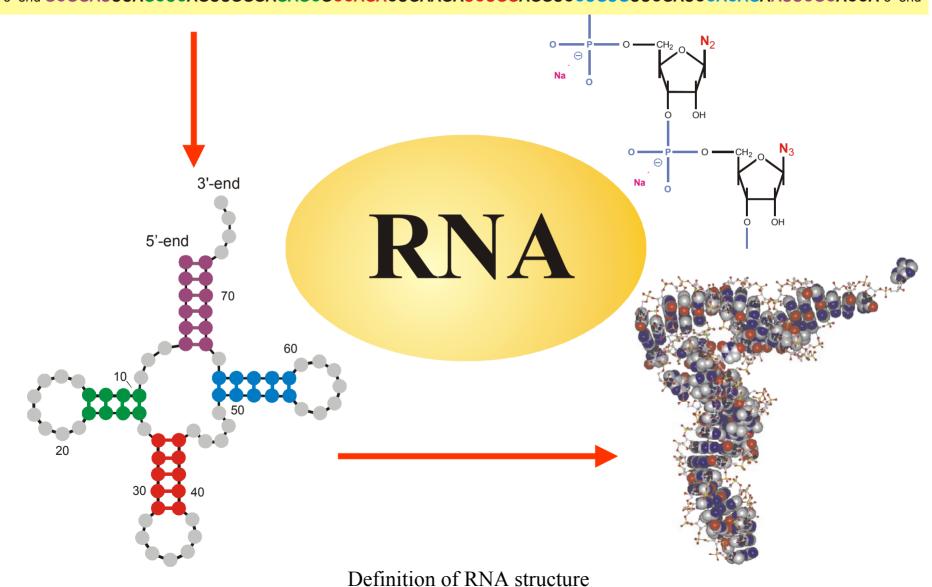
Leipzig, 02.12.2002



A conventional simplified sketch of cellular metabolism



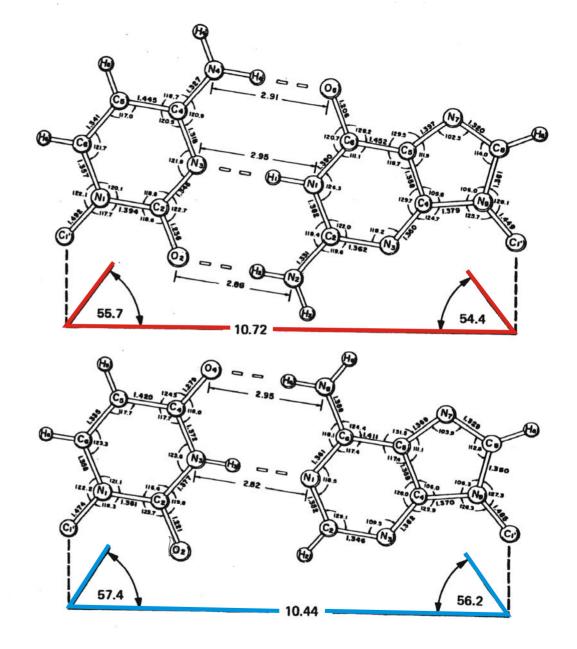
5'-end GCGGAUUUAGCUCAGUUGGGAGACCCCAGACUGAAGAUCUGGAGGUCCUGUGUUCGAUCCACAGAAUUCGCACCA 3'-end

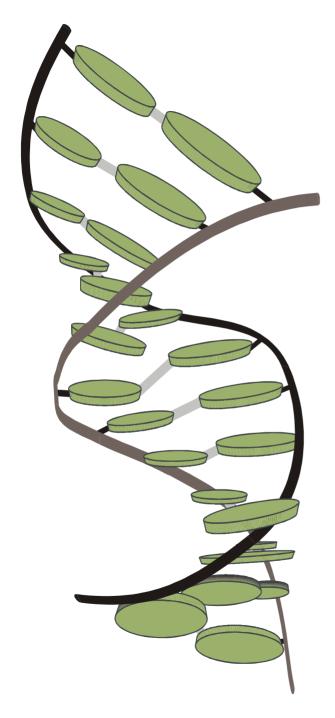


Canonical or Watson-Crick base pairs:

cytosine – guanine G©C

uracil – adenine A=U





The three-dimensional structure of a short double helical stack

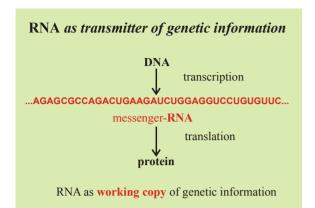
$$G = U$$

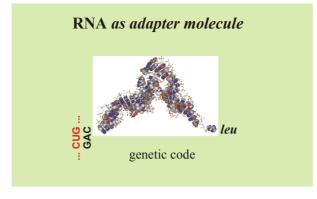
$$A = V$$

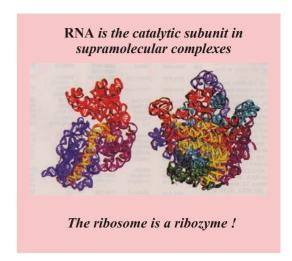
$$A =$$

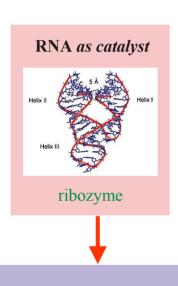
Canonical Watson-Crick base-pair

Wobble base-pairs









RNA

RNA is modified by epigenetic control

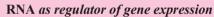
RNA editing

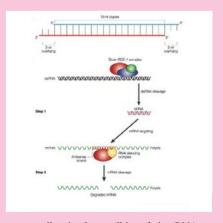
Alternative splicing of messenger RNA

RNA as carrier of genetic information

RNA viruses and retroviruses

RNA as information carrier in evolution *in vitro* and evolutionary biotechnology





gene silencing by small interfering RNAs

The RNA world as a precursor of the current DNA + protein biology

Functions of RNA molecules

Evolution of RNA molecules based on Qβ phage

D.R.Mills, R.L.Peterson, S.Spiegelman, *An extracellular Darwinian experiment with a self-duplicating nucleic acid molecule*. Proc.Natl.Acad.Sci.USA **58** (1967), 217-224

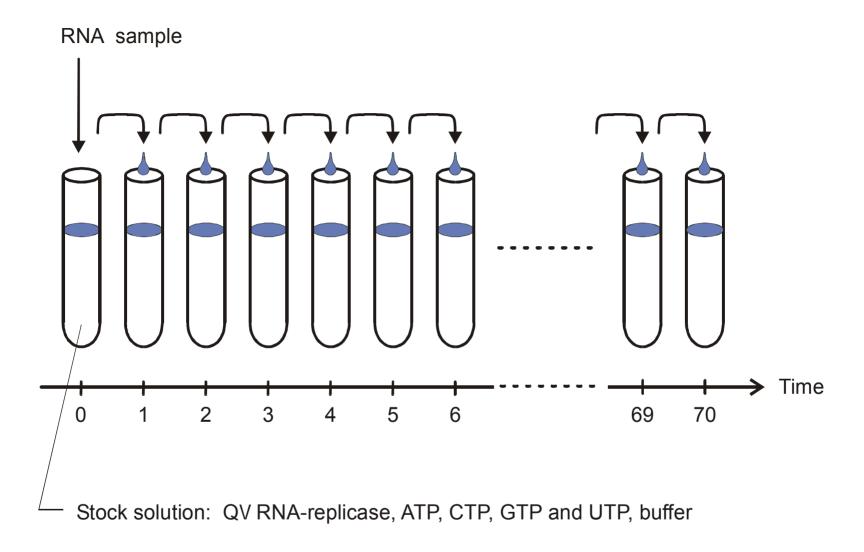
S.Spiegelman, *An approach to the experimental analysis of precellular evolution*. Quart.Rev.Biophys. **4** (1971), 213-253

C.K.Biebricher, *Darwinian selection of self-replicating RNA molecules*. Evolutionary Biology **16** (1983), 1-52

G.Bauer, H.Otten, J.S.McCaskill, *Travelling waves of* in vitro *evolving RNA*. *Proc.Natl.Acad.Sci.USA* **86** (1989), 7937-7941

C.K.Biebricher, W.C.Gardiner, *Molecular evolution of RNA* in vitro. Biophysical Chemistry **66** (1997), 179-192

G.Strunk, T.Ederhof, *Machines for automated evolution experiments* in vitro based on the serial transfer concept. Biophysical Chemistry **66** (1997), 193-202



The serial transfer technique applied to RNA evolution in vitro

Reproduction of the original figure of the serial transfer experiment with Qβ RNA

D.R.Mills, R,L,Peterson, S.Spiegelman, *An extracellular Darwinian experiment with a self-duplicating nucleic acid molecule*. Proc.Natl.Acad.Sci.USA **58** (1967), 217-224

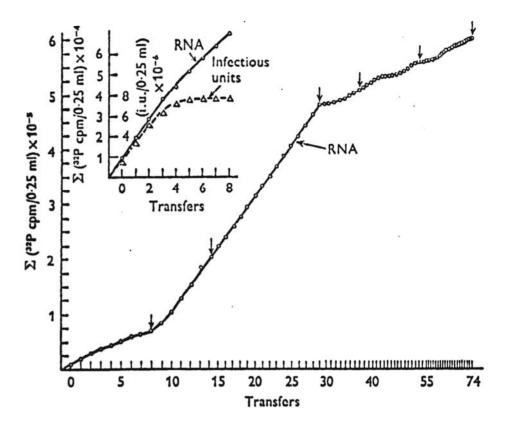
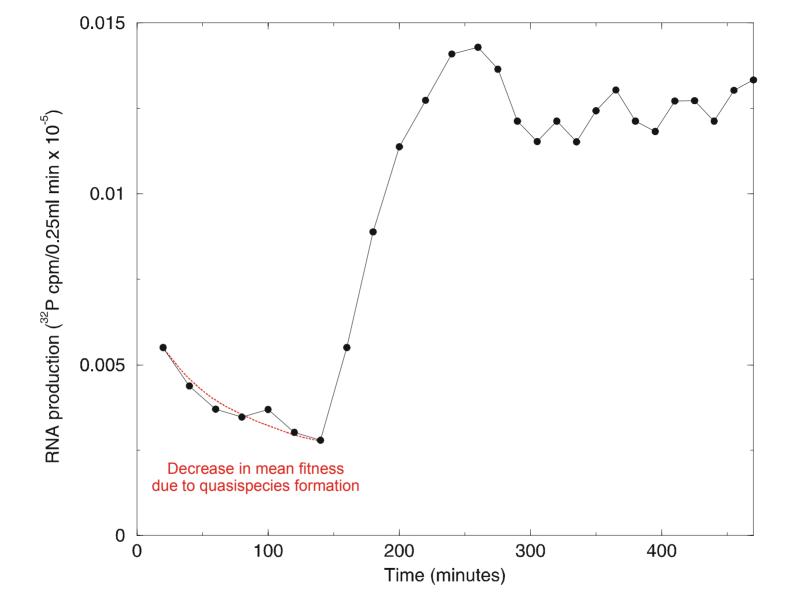


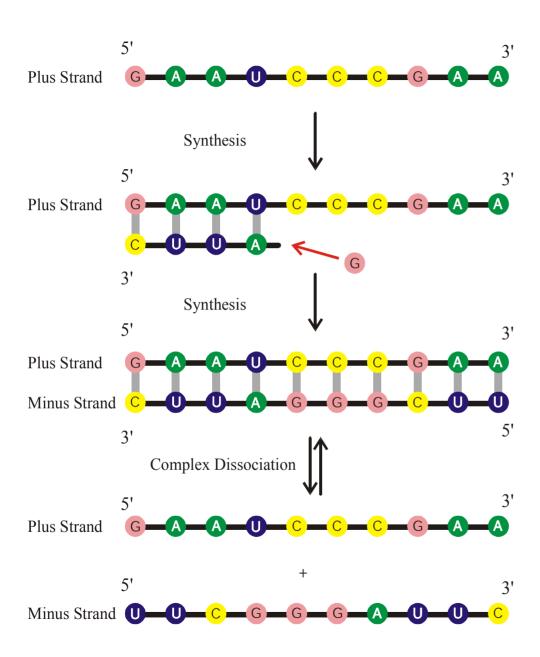
Fig. 9. Scrial transfer experiment. Each 0.25 ml standard reaction mixture contained 40 μ g of Q β replicase and 32 P-UTP. The first reaction (0 transfer) was initiated by the addition of 0.2 μ g ts-1 (temperature-sensitive RNA) and incubated at 35 °C for 20 min, whereupon 0.02 ml was drawn for counting and 0.02 ml was used to prime the second reaction (first transfer), and so on. After the first 13 reactions, the incubation periods were reduced to 15 min (transfers 14–29). Transfers 30–38 were incubated for 10 min. Transfers 39–52 were incubated for 7 min, and transfers 53–74 were incubated for 5 min. The arrows above certain transfers (0, 8, 14, 29, 37, 53, and 73) indicate where 0.001–0.1 ml of product was removed and used to prime reactions for sedimentation analysis on sucrose. The inset examines both infectious and total RNA. The results show that biologically competent RNA ceases to appear after the 4th transfer (Mills et al. 1967).



The increase in RNA production rate during a serial transfer experiment

No new principle will declare itself from below a heap of facts.

Sir Peter Medawar, 1985



Complementary replication as the simplest copying mechanism of RNA Complementarity is determined by Watson-Crick base pairs:

GCC and A=U

$$(A) + I_{1} \xrightarrow{f_{1}} I_{1} + I_{1}$$

$$(A) + I_{2} \xrightarrow{f_{2}} I_{2} + I_{2}$$

$$(A) + I_{i} \xrightarrow{f_{i}} I_{i} + I_{i}$$

$$(A) + I_{i} \xrightarrow{f_{i}} I_{n} + I_{n}$$

$$(A) + I_{m} \xrightarrow{f_{m}} I_{m} + I_{m}$$

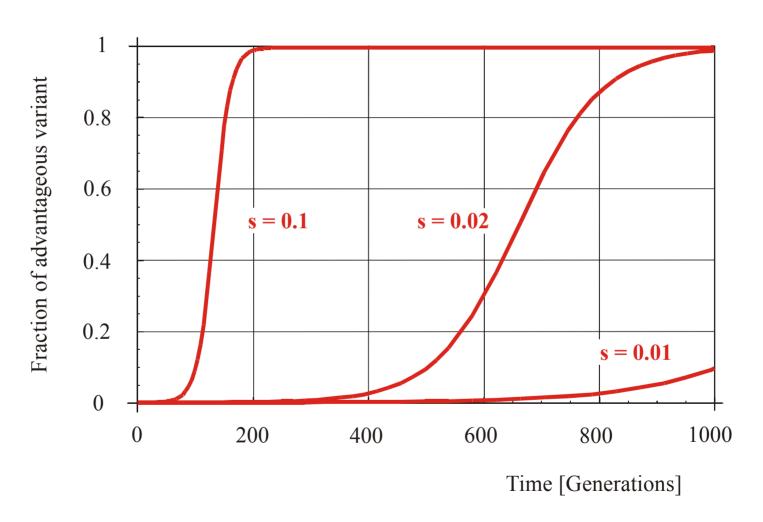
$$(A) + I_{m} \xrightarrow{f_{n}} I_{n} + I_{m}$$

$$(A) + I_{m} \xrightarrow{f_{n}} I_{n} + I_{n}$$

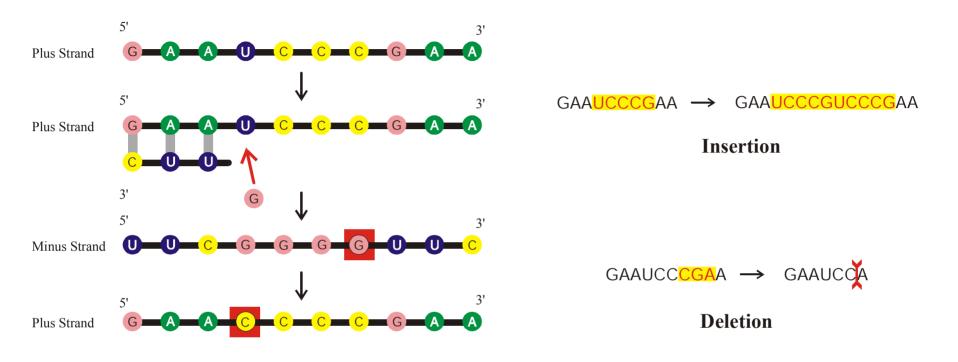
$$(A) + I_{m} \xrightarrow{f_{n}} I_{n} + I_{m}$$

Reproduction of organisms or replication of molecules as the basis of selection

$$\mathbf{s} = (f_2 - f_1) / f_1; f_2 > f_1; x_1(0) = 1 - 1/N; x_2(0) = 1/N$$



Selection of advantageous mutants in populations of N = 10~000 individuals

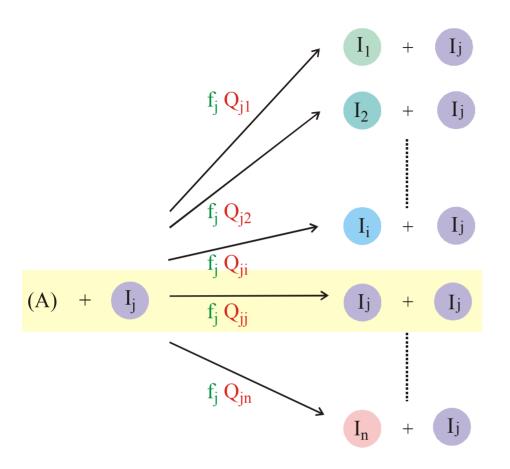


Point Mutation

Mutations in nucleic acids represent the mechanism of variation of genotypes.

Theory of molecular evolution

- M.Eigen, *Self-organization of matter and the evolution of biological macromolecules*. Naturwissenschaften **58** (1971), 465-526
- C.J.Thompson, J.L.McBride, On Eigen's theory of the self-organization of matter and the evolution of biological macromolecules. Math. Biosci. 21 (1974), 127-142
- B.L.Jones, R.H.Enns, S.S.Rangnekar, *On the theory of selection of coupled macromolecular systems*. Bull.Math.Biol. **38** (1976), 15-28
- M.Eigen, P.Schuster, *The hypercycle. A principle of natural self-organization. Part A: Emergence of the hypercycle*. Naturwissenschaften **58** (1977), 465-526
- M.Eigen, P.Schuster, *The hypercycle. A principle of natural self-organization. Part B: The abstract hypercycle*. Naturwissenschaften **65** (1978), 7-41
- M.Eigen, P.Schuster, *The hypercycle. A principle of natural self-organization. Part C: The realistic hypercycle*. Naturwissenschaften **65** (1978), 341-369
- J.Swetina, P.Schuster, *Self-replication with errors A model for polynucleotide replication*. Biophys.Chem. **16** (1982), 329-345
- J.S.McCaskill, *A localization threshold for macromolecular quasispecies from continuously distributed replication rates*. J.Chem.Phys. **80** (1984), 5194-5202
- M.Eigen, J.McCaskill, P.Schuster, *The molecular quasispecies*. Adv.Chem.Phys. 75 (1989), 149-263
- C. Reidys, C.Forst, P.Schuster, *Replication and mutation on neutral networks*. Bull.Math.Biol. **63** (2001), 57-94



$$dx_i / dt = \sum_j f_j Q_{ji} x_j - x_i \Phi$$

$$\Phi = \Sigma_j f_j x_i$$
; $\Sigma_j x_j = 1$; $\Sigma_i Q_{ij} = 1$

$$[I_i] = x_i & 0$$
; $i = 1,2,...,n$;

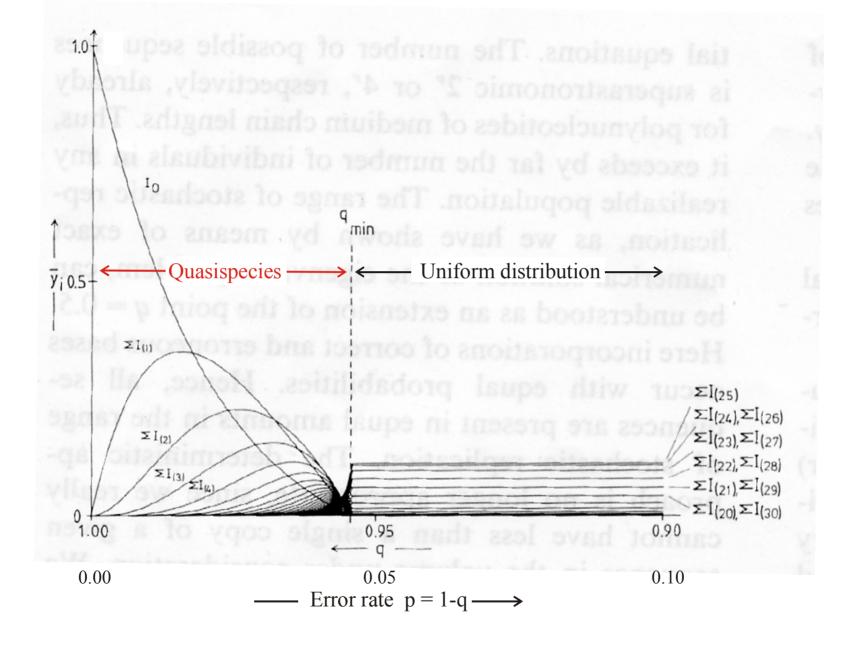
$$[A] = a = constant$$

$$Q_{ij} = (1-p)^{\ell-d(i,j)} p^{d(i,j)}$$

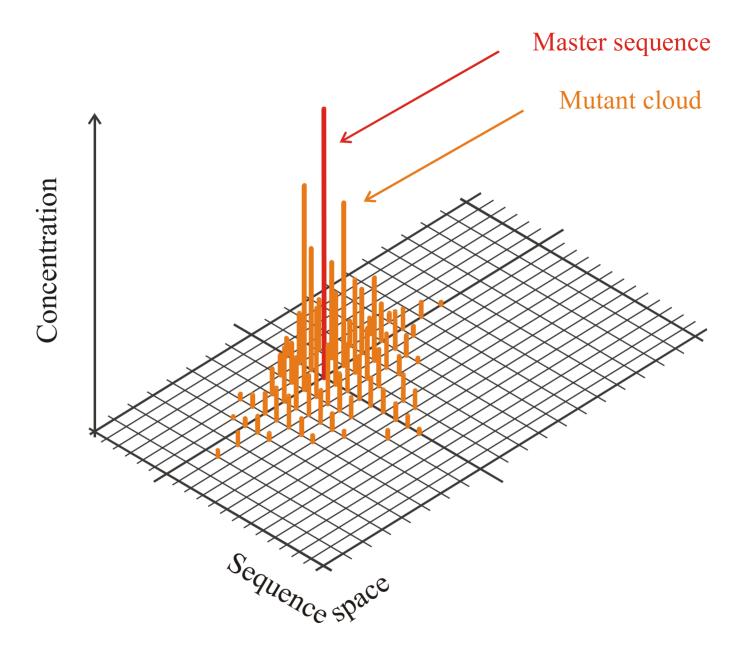
p Error rate per digit

l Chain length of the polynucleotide

 $\begin{array}{c} \text{d}(i,j) \text{ Hamming distance} \\ \text{between } I_i \text{ and } I_j \end{array}$



Quasispecies as a function of the replication accuracy q



In the case of non-zero mutation rates (p>0 or q<1) the Darwinian principle of optimization of mean fitness can be understood only as an **optimization heuristic**. It is valid only on part of the concentration simplex. There are other well defined areas were the mean fitness decreases monotonously or were it may show non-monotonous behavior. The volume of the part of the simplex where mean fitness is non-decreasing in the conventional sense decreases with inreasing mutation rate p.

In systems with recombination a similar restriction holds for Fisher's "universal selection equation". Its global validity is restricted to the one-gene (single locus) model.

Theory of genotype – phenotype mapping

P. Schuster, W.Fontana, P.F.Stadler, I.L.Hofacker, *From sequences to shapes and back: A case study in RNA secondary structures*. Proc.Roy.Soc.London **B 255** (1994), 279-284

W.Grüner, R.Giegerich, D.Strothmann, C.Reidys, I.L.Hofacker, P.Schuster, *Analysis of RNA sequence structure maps by exhaustive enumeration. I. Neutral networks*. Mh.Chem. **127** (1996), 355-374

W.Grüner, R.Giegerich, D.Strothmann, C.Reidys, I.L.Hofacker, P.Schuster, *Analysis of RNA sequence structure maps by exhaustive enumeration. II. Structure of neutral networks and shape space covering*. Mh.Chem. **127** (1996), 375-389

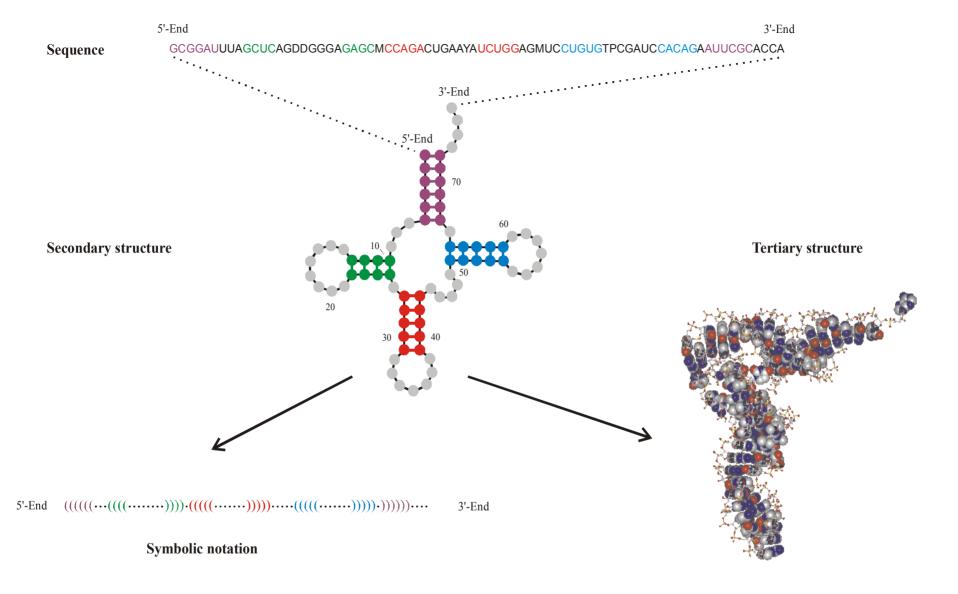
C.M.Reidys, P.F.Stadler, P.Schuster, *Generic properties of combinatory maps*. Bull.Math.Biol. **59** (1997), 339-397

I.L.Hofacker, P. Schuster, P.F.Stadler, *Combinatorics of RNA secondary structures*. Discr.Appl.Math. **89** (1998), 177-207

C.M.Reidys, P.F.Stadler, *Combinatory landscapes*. SIAM Review 44 (2002), 3-54

Genotype-phenotype relations are highly complex and only the most simple cases can be studied. One example is the folding of RNA sequences into RNA structures represented in course-grained form as secondary structures.

The RNA genotype-phenotype relation is understood as a mapping from the space of RNA sequences into a space of RNA structures.



The **RNA secondary structure** is a listing of **GC**, **AU**, and **GU** base pairs. It is understood in contrast to the full 3D-or **tertiary structure** at the resolution of atomic coordinates. RNA secondary structures are biologically relevant. They are, for example, conserved in evolution.

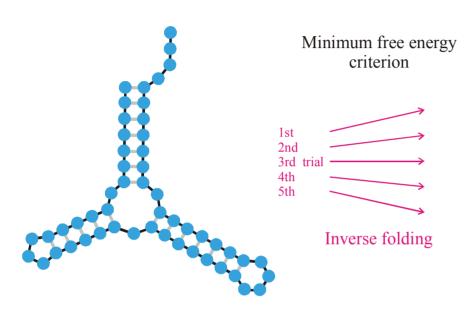
RNA Minimum Free Energy Structures

Efficient algorithms based on dynamical programming are available for computation of secondary structures for given sequences. Inverse folding algorithms compute sequences for given secondary structures.

M.Zuker and P.Stiegler. Nucleic Acids Res. 9:133-148 (1981)

Vienna RNA Package: http://www.tbi.univie.ac.at (includes inverse folding, suboptimal structures, kinetic folding, etc.)

I.L.Hofacker, W. Fontana, P.F.Stadler, L.S.Bonhoeffer, M.Tacker, and P. Schuster. *Mh.Chem.* **125**:167-188 (1994)

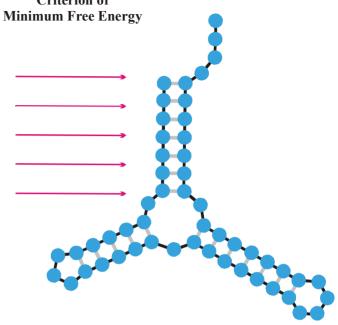


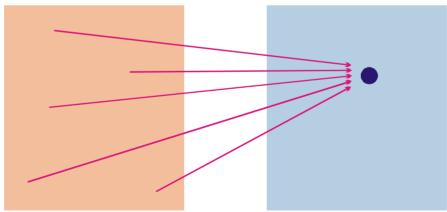
UUUAGCCAGCGCGAGUCGUGCGGACGGGGUUAUCUCUGUCGGGCUAGGGCGC
GUGAGCGCGGGGCACAGUUUCUCAAGGAUGUAAGUUUUUUGCCGUUUAUCUGG
UUAGCGAGAGAGGAGGCUUCUAGACCCAGCUCUCUGGGUCGUUGCUGAUGCG
CAUUGGUGCUAAUGAUAUUAGGGCUGUAUUCCUGUAUAGCGAUCAGUGUCCG
GUAGGCCCUCUUGACAUAAGAUUUUUCCAAUGGUGGGAGAUGGCCAUUGCAG

The **inverse folding algorithm** searches for sequences that form a given RNA secondary structure under the minimum free energy criterion.

Criterion of

UUUAGCCAGCGCGAGUCGUGCGGACGGGGUUAUCUCUGUCGGGCUAGGGCGC GUGAGCGCGGGCACAGUUUCUCAAGGAUGUAAGUUUUUGCCGUUUAUCUGG UUAGCGAGAGAGGCUUCUAGACCCAGCUCUCUGGGUCGUUGCUGAUGCG CAUUGGUGCUAAUGAUAUUAGGGCUGUAUUCCUGUAUAGCGAUCAGUGUCCG GUAGGCCCUCUUGACAUAAGAUUUUUCCAAUGGUGGGAGAUGGCCAUUGCAG





Sequence Space

Shape Space

 S_1 : CGTCGTTACAATTTAGGTTATGTGCGAATTCACAAATTGAAAATACAAGAG..... S_2 : CGTCGTTACAATTTAAGTTATGTGCGAATTCCCCAAATTAAAAACACAAGAG.....

Hamming distance $d_H(S_1,S_2) = 4$

(i)
$$d_H(S_1,S_1) = 0$$

(ii)
$$d_H(S_1,S_2) = d_H(S_2,S_1)$$

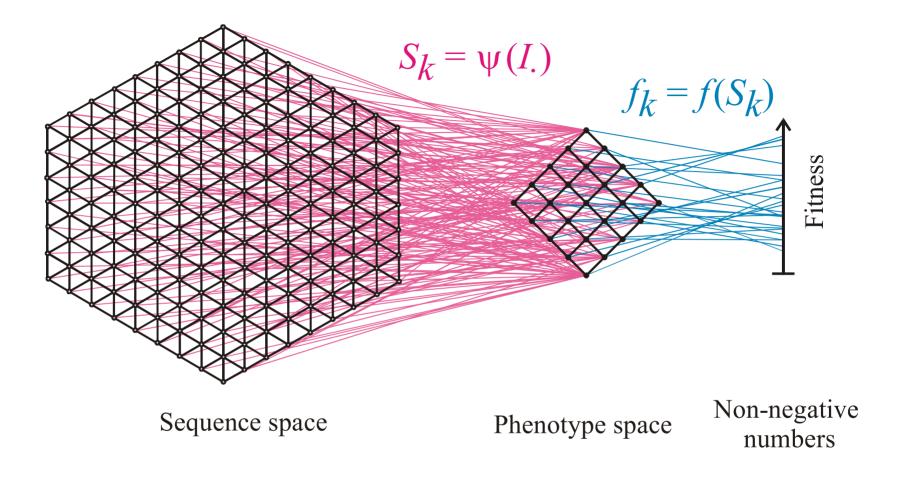
(iii)
$$d_H(S_1,S_3) \leftarrow d_H(S_1,S_2) + d_H(S_2,S_3)$$

The Hamming distance induces a metric in sequence space

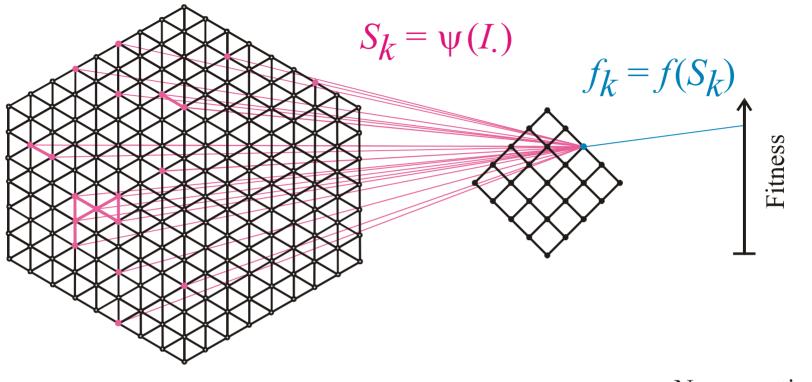
The **RNA model** considers RNA sequences as genotypes and simplified RNA structures, called secondary structures, as phenotypes.

The **mapping** from genotypes into phenotypes is many-to-one. Hence, it is redundant and not invertible.

Genotypes, i.e. RNA sequences, which are mapped onto the same phenotype, i.e. the same RNA secondary structure, form **neutral networks**. Neutral networks are represented by graphs in sequence space.



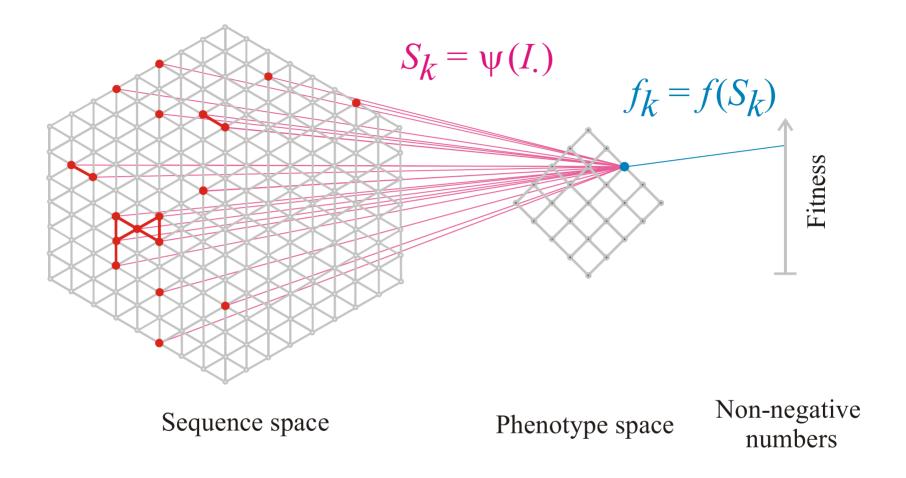
Mapping from sequence space into phenotype space and into fitness values



Sequence space

Phenotype space

Non-negative numbers



The pre-image of the structure S_k in sequence space is the **neutral network G_k**

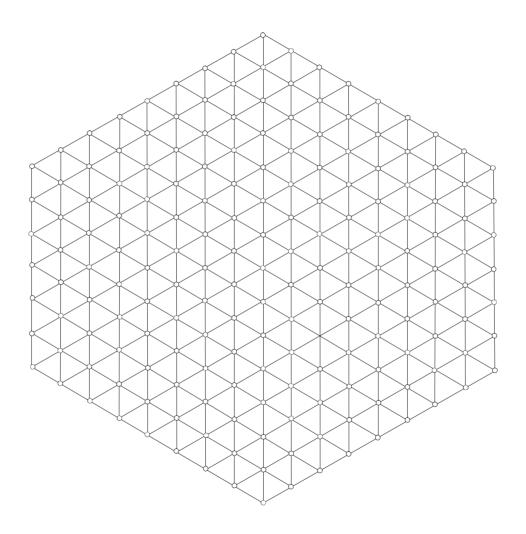
Neutral networks are sets of sequences forming the same structure. G_k is the pre-image of the structure S_k in sequence space:

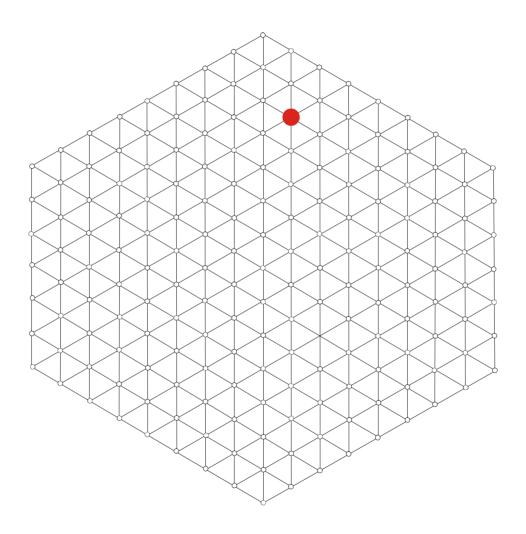
$$G_k = m^{-1}(S_k)$$
 $\{m_j \mid m(I_j) = S_k\}$

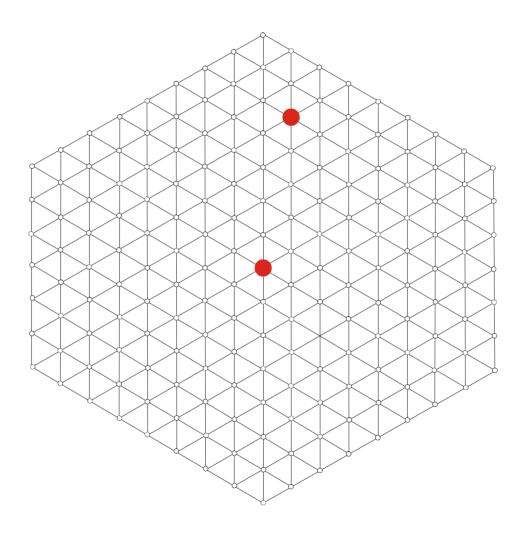
The set is converted into a graph by connecting all sequences of Hamming distance one.

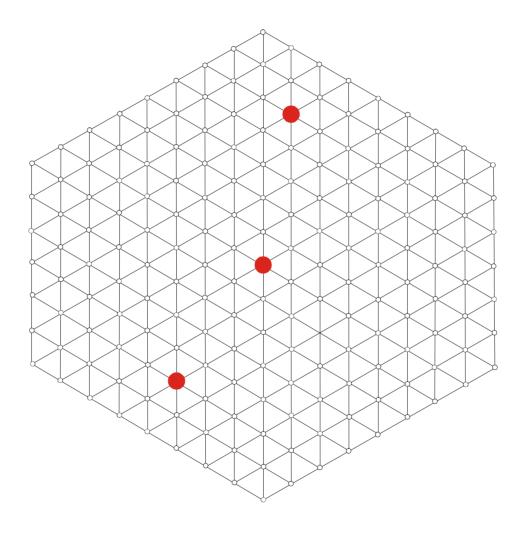
Neutral networks of small RNA molecules can be computed by exhaustive folding of complete sequence spaces, i.e. all RNA sequences of a given chain length. This number, N=4ⁿ, becomes very large with increasing length, and is prohibitive for numerical computations.

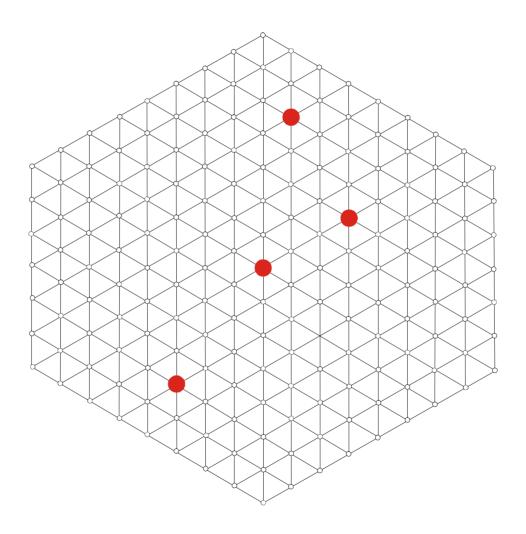
Neutral networks can be modelled by **random graphs** in sequence space. In this approach, nodes are inserted randomly into sequence space until the size of the pre-image, i.e. the number of neutral sequences, matches the neutral network to be studied.

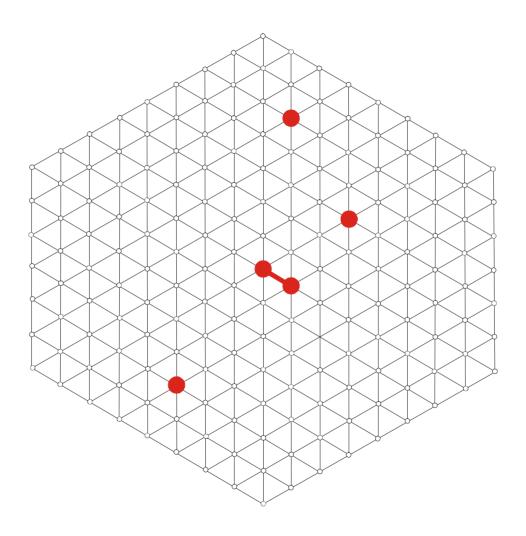


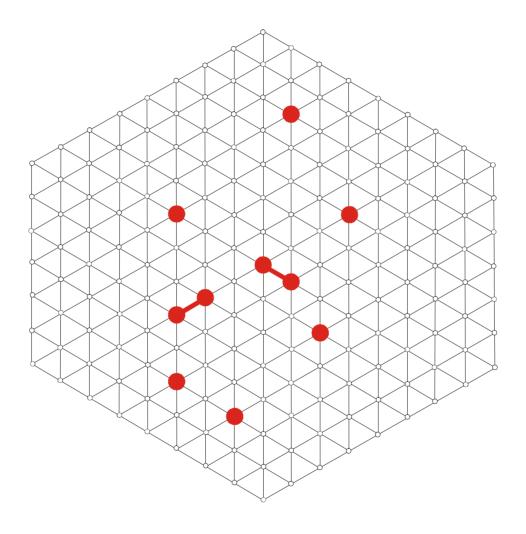


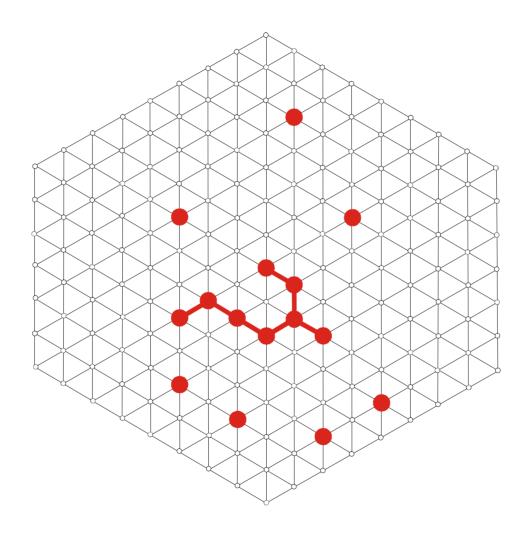


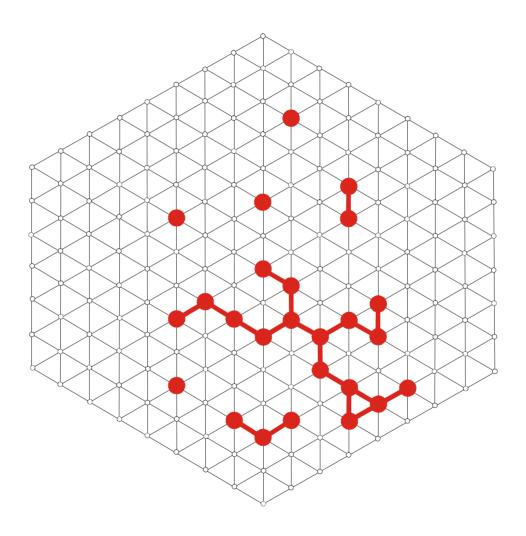


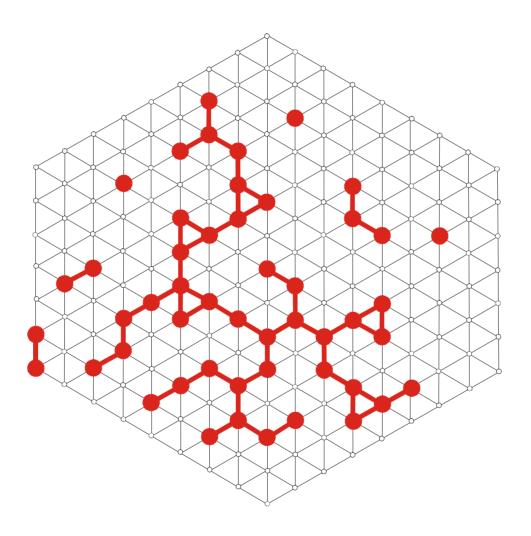


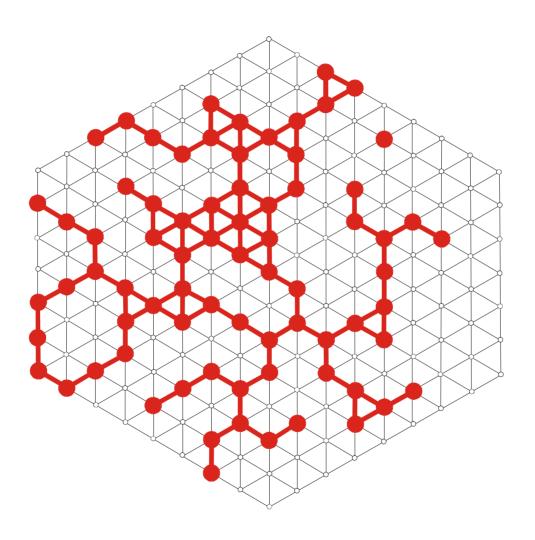


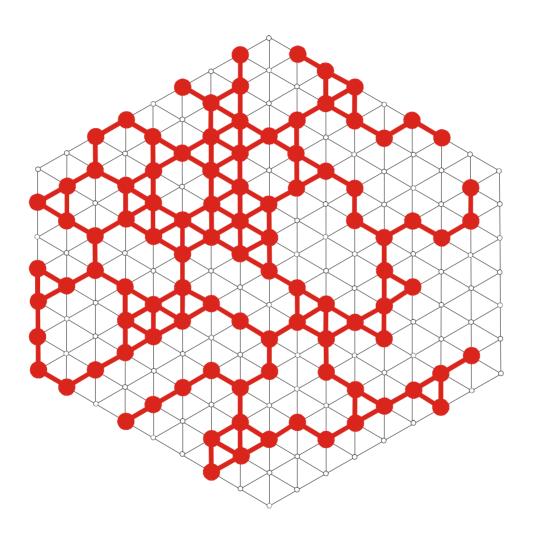


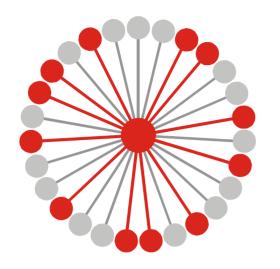












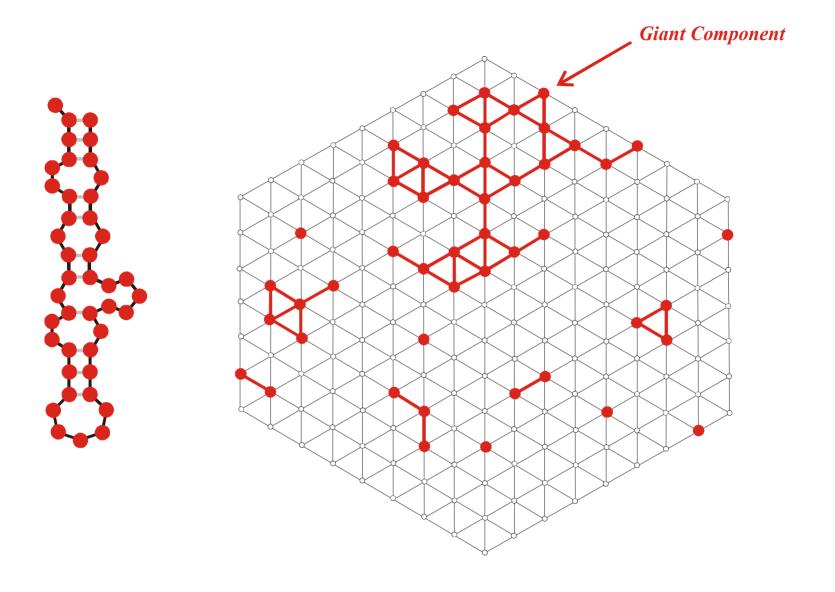
$$G_k = m^{-1}(S_k) \cup OI_j \mid m(I_j) = S_k q$$

$$\lambda_j = 12 / 27$$
, $\bar{\lambda}_k = \frac{O \hat{j} \in |G_k|}{|G_k|} \hat{j}(k)$

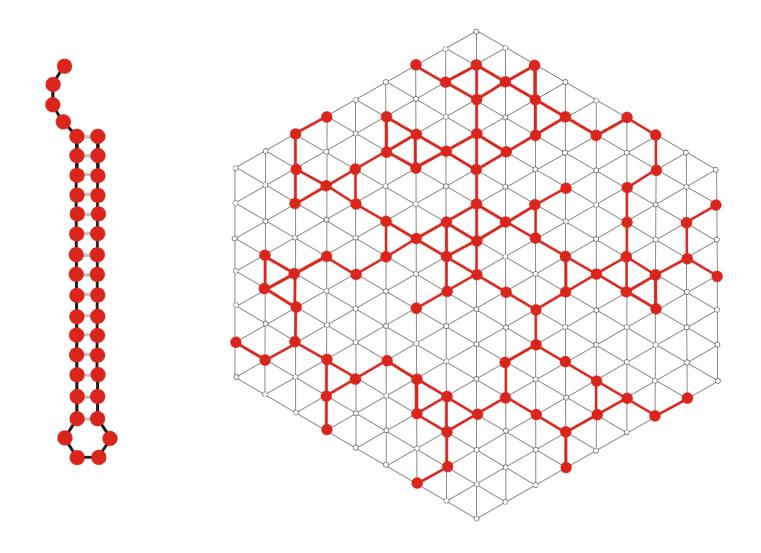
Connectivity threshold:

$$\lambda_{\rm cr} = 1 - \kappa^{-1/(\kappa - 1)}$$

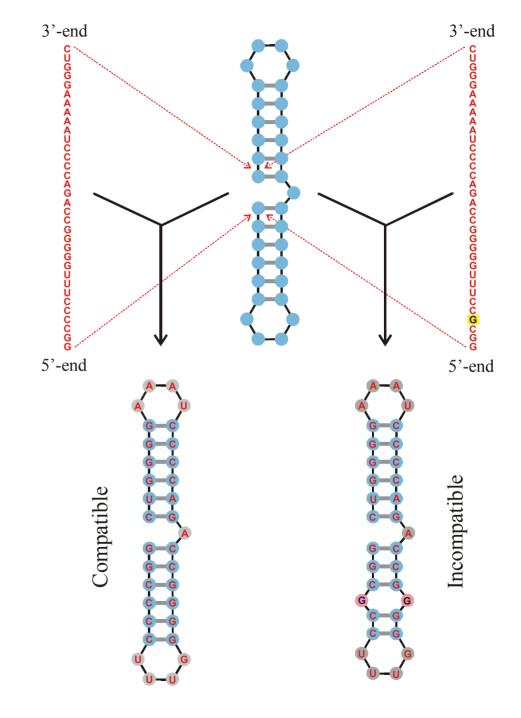
Mean degree of neutrality and connectivity of **neutral networks**



A multi-component neutral network

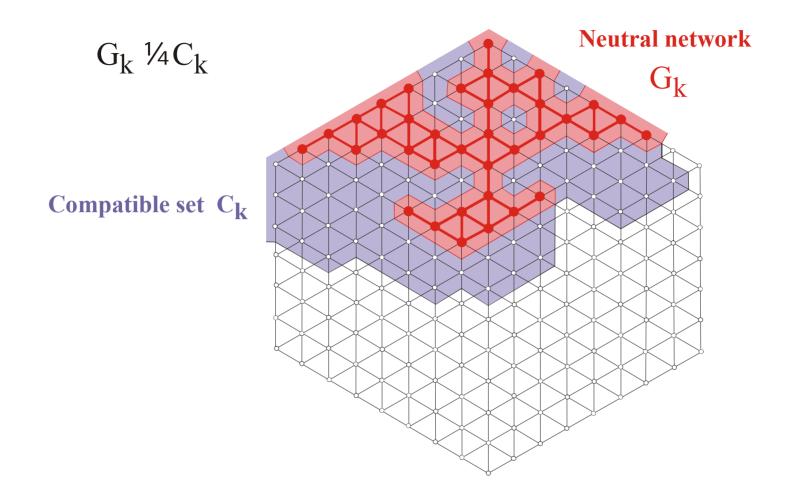


A connected neutral network



Compatibility of sequences with structures

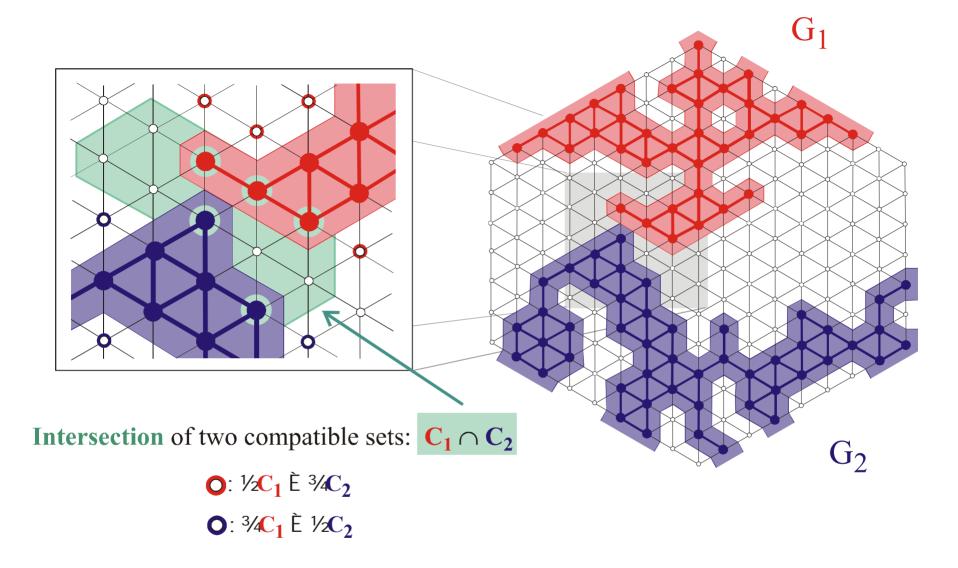
A sequence is compatible with its minimum free energy structure and all its suboptimal structures.



The compatible set C_k of a structure S_k consists of all sequences which form S_k as its minimum free energy structure (neutral network G_k) or one of its suboptimal structures.

3'- end 5'- end $\label{eq:minimum} \mbox{Minimum free energy conformation S}_0$ Suboptimal conformation S₁

A sequence at the **intersection** of two neutral networks is compatible with both structures



The intersection of two compatible sets is always non empty: $C_1 \ \P \ C_2 \ ^3\!/\!\mu$



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S0092-8240(96)00089-4

GENERIC PROPERTIES OF COMBINATORY MAPS: NEUTRAL NETWORKS OF RNA SECONDARY STRUCTURES¹

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Random graph theory is used to model and analyse the relationships between sequences and secondary structures of RNA molecules, which are understood as mappings from sequence space into shape space. These maps are non-invertible since there are always many orders of magnitude more sequences than structures. Sequences folding into identical structures form neutral networks. A neutral network is embedded in the set of sequences that are compatible with the given structure. Networks are modeled as graphs and constructed by random choice of vertices from the space of compatible sequences. The theory characterizes neutral networks by the mean fraction of neutral neighbors (λ). The networks are connected and percolate sequence space if the fraction of neutral nearest neighbors exceeds a threshold value ($\lambda > \lambda^*$). Below threshold ($\lambda < \lambda^*$), the networks are partitioned into a largest "giant" component and several smaller components. Structures are classified as "common" or "rare" according to the sizes of their pre-images, i.e. according to the fractions of sequences folding into them. The neutral networks of any pair of two different common structures almost touch each other, and, as expressed by the conjecture of shape space covering sequences folding into almost all common structures, can be found in a small ball of an arbitrary location in sequence space. The results from random graph theory are compared to data obtained by folding large samples of RNA sequences. Differences are explained in terms of specific features of RNA molecular structures. © 1997 Society for Mathematical Biology

THEOREM 5. INTERSECTION-THEOREM. Let s and s' be arbitrary secondary structures and C[s], C[s'] their corresponding compatible sequences. Then,

$C[s] \cap C[s'] \neq \emptyset$.

Proof. Suppose that the alphabet admits only the complementary base pair [XY] and we ask for a sequence x compatible to both s and s'. Then $j(s,s') \cong D_m$ operates on the set of all positions $\{x_1,\ldots,x_n\}$. Since we have the operation of a dihedral group, the orbits are either cycles or chains and the cycles have even order. A constraint for the sequence compatible to both structures appears only in the cycles where the choice of bases is not independent. It remains to be shown that there is a valid choice of bases for each cycle, which is obvious since these have even order. Therefore, it suffices to choose an alternating sequence of the pairing partners X and Y. Thus, there are at least two different choices for the first base in the orbit.

Remark. A generalization of the statement of theorem 5 to three different structures is false.

Reference for the definition of the intersection and the proof of the *intersection theorem*

Optimization of RNA molecules in silico

W.Fontana, P.Schuster, *A computer model of evolutionary optimization*. Biophysical Chemistry **26** (1987), 123-147

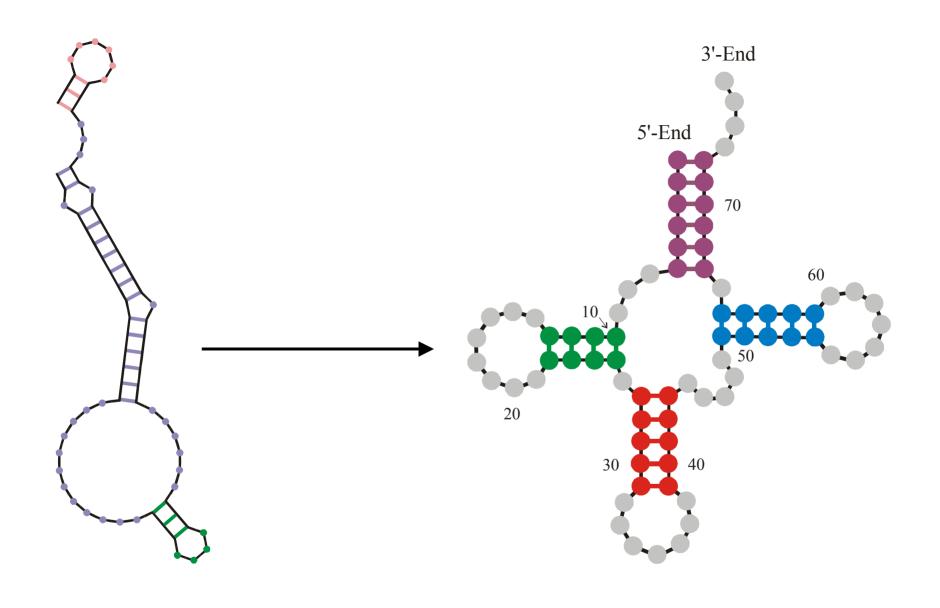
W.Fontana, W.Schnabl, P.Schuster, *Physical aspects of evolutionary optimization and adaptation*. Phys.Rev.A **40** (1989), 3301-3321

M.A.Huynen, W.Fontana, P.F.Stadler, *Smoothness within ruggedness. The role of neutrality in adaptation*. Proc.Natl.Acad.Sci.USA **93** (1996), 397-401

W.Fontana, P.Schuster, *Continuity in evolution. On the nature of transitions*. Science **280** (1998), 1451-1455

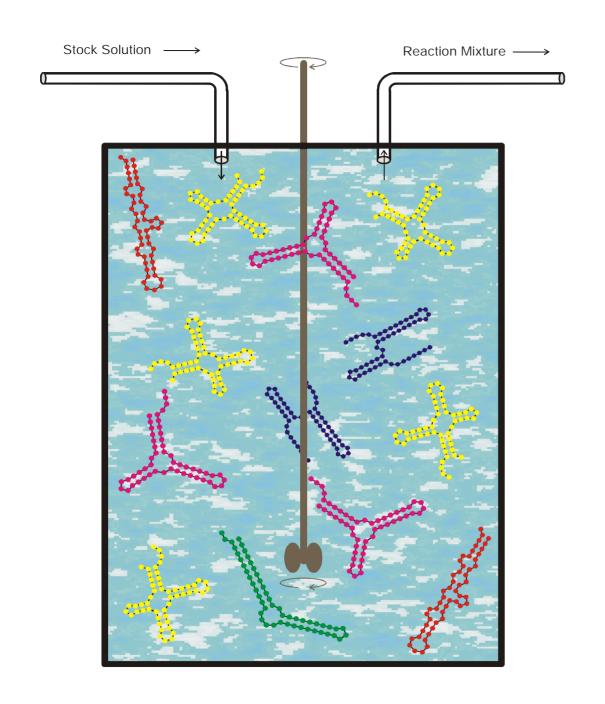
W.Fontana, P.Schuster, *Shaping space. The possible and the attainable in RNA genotype-phenotype mapping*. J.Theor.Biol. **194** (1998), 491-515

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Randomly chosen initial structure

Phenylalanyl-tRNA as target structure

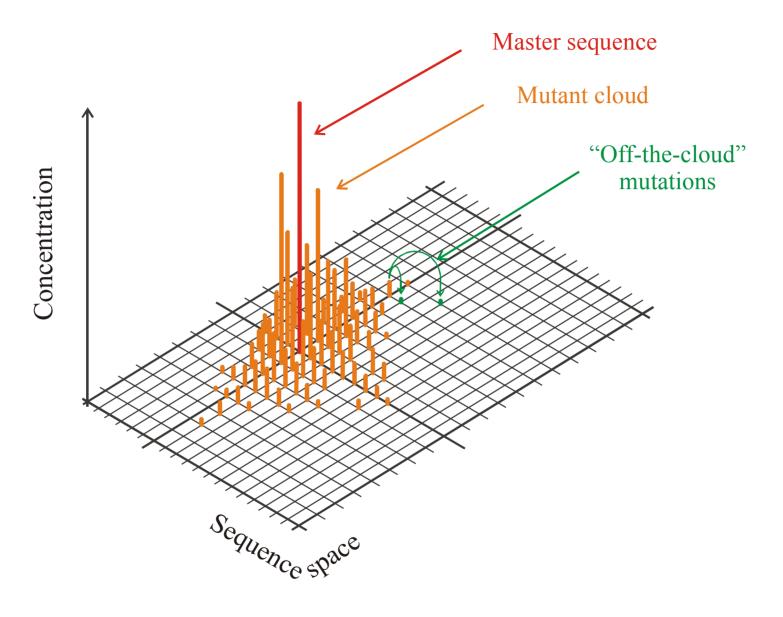


Fitness function:

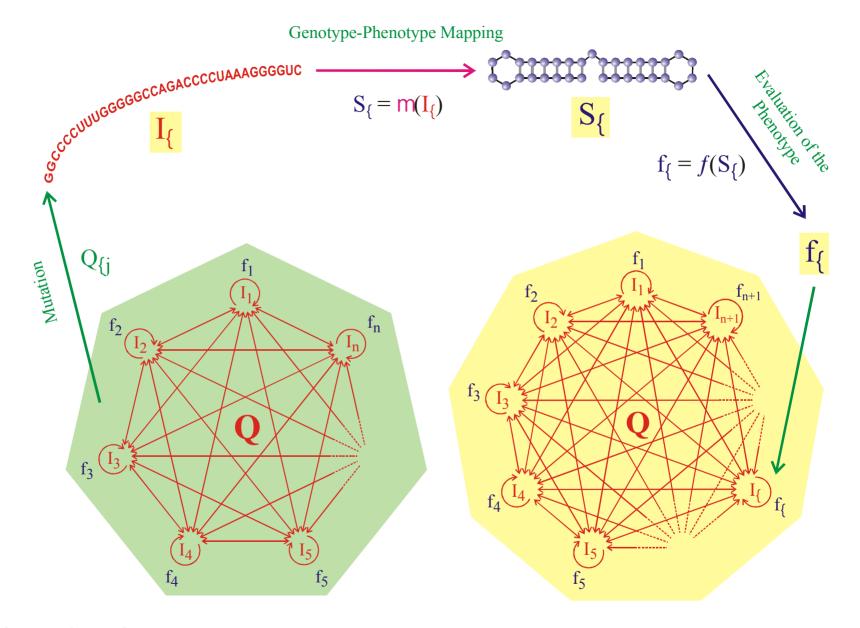
$$f_k = [/[U + 8d_S^{(k)}]$$

$$8d_{S}^{(k)} = d^{s}(I_{k}, I_{h})$$

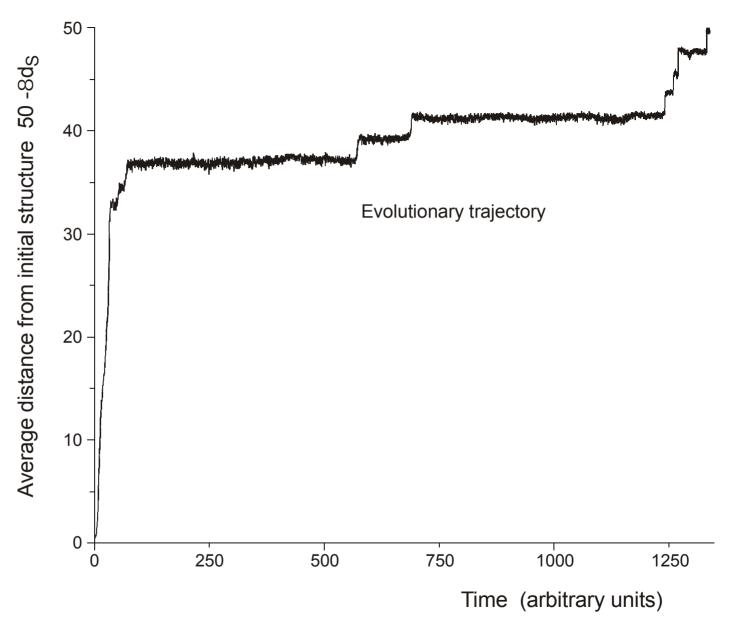
The flowreactor as a device for studies of evolution *in vitro* and *in silico*



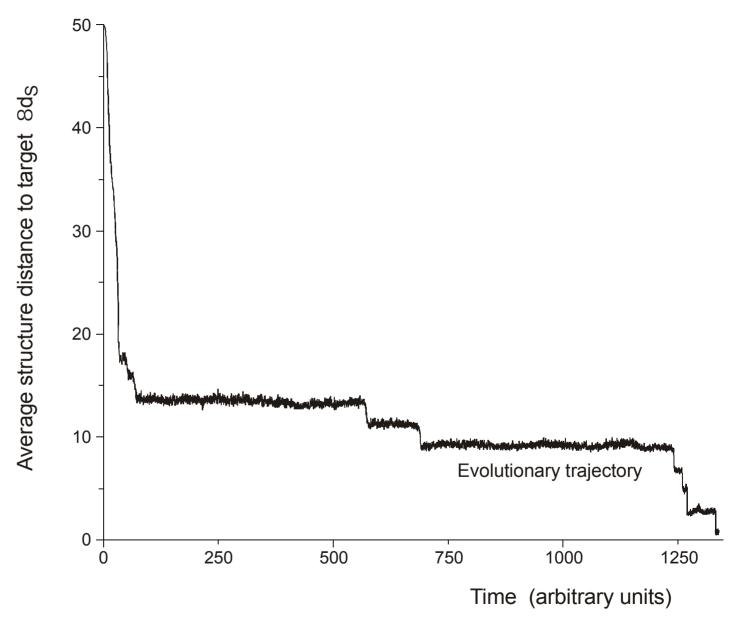
The molecular quasispecies in sequence space



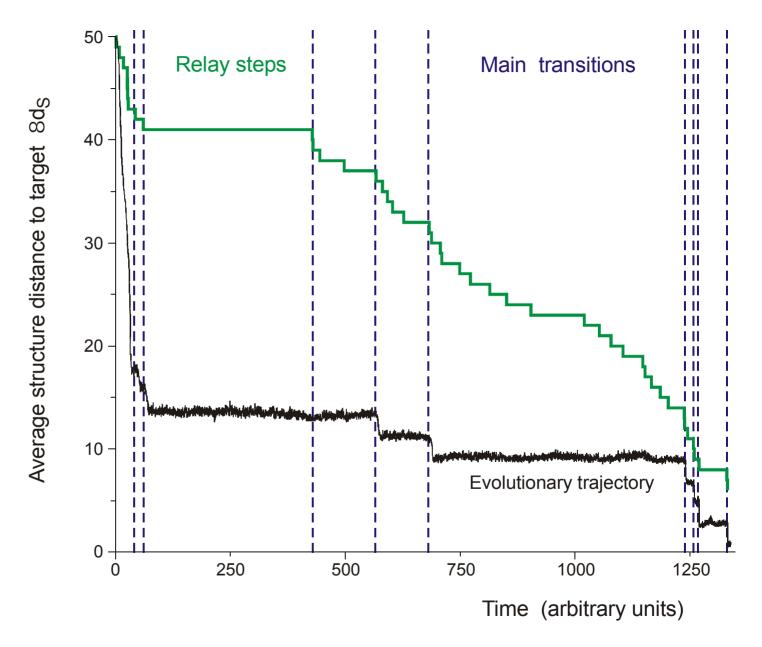
Evolutionary dynamics including molecular phenotypes



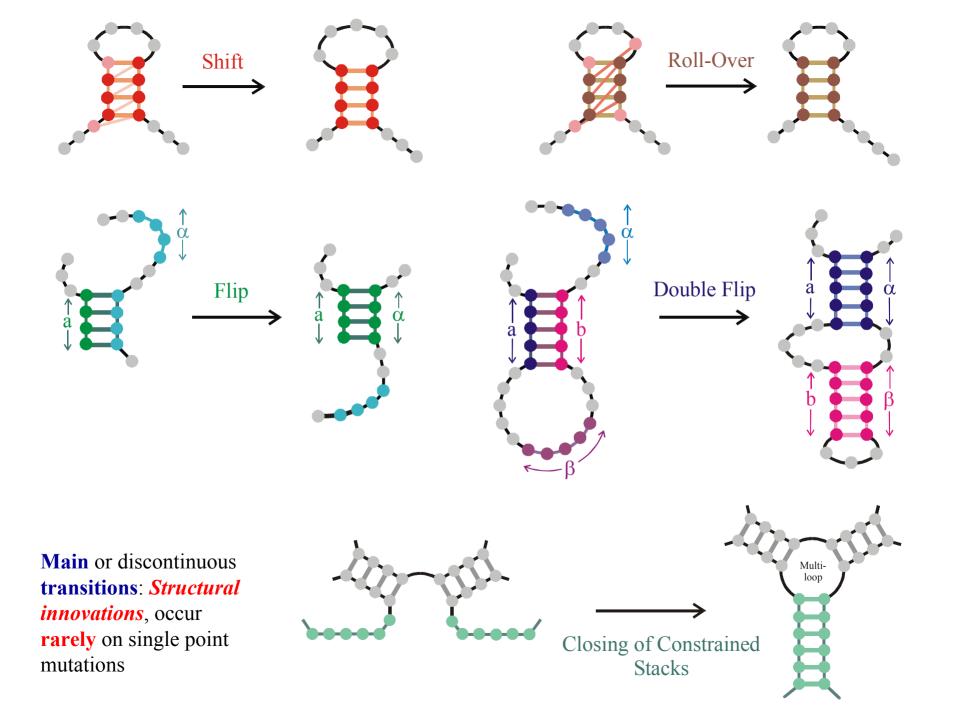
In silico optimization in the flow reactor: Trajectory (biologists' view)

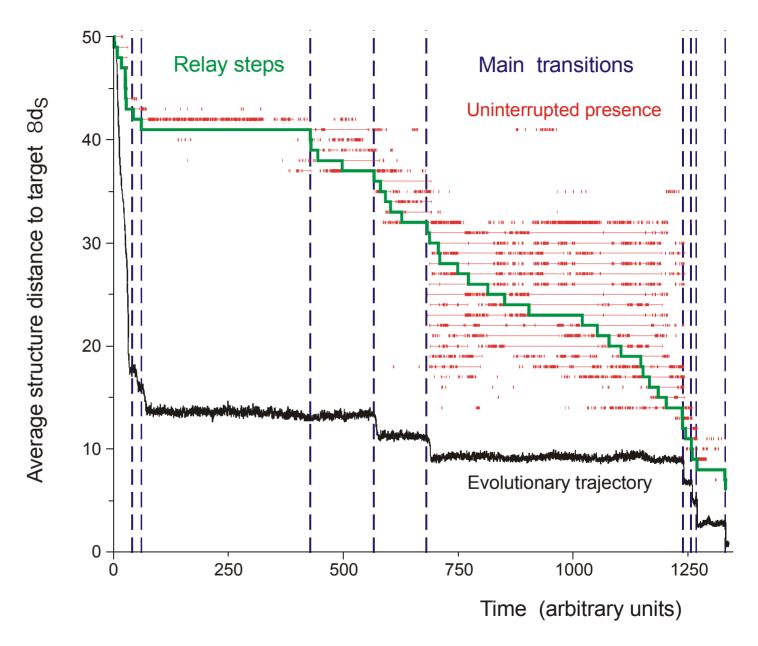


In silico optimization in the flow reactor: Trajectory (physicists' view)

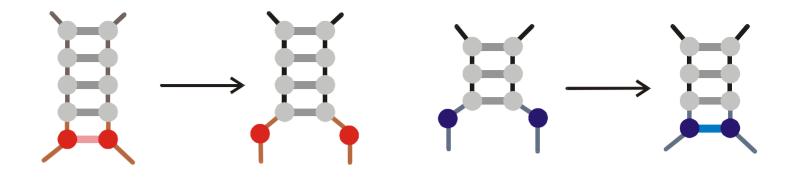


In silico optimization in the flow reactor: Main transitions



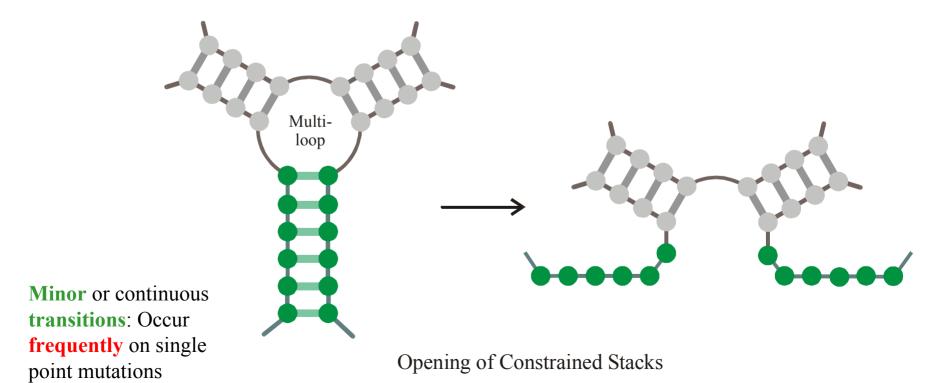


In silico optimization in the flow reactor



Shortening of Stacks

Elongation of Stacks

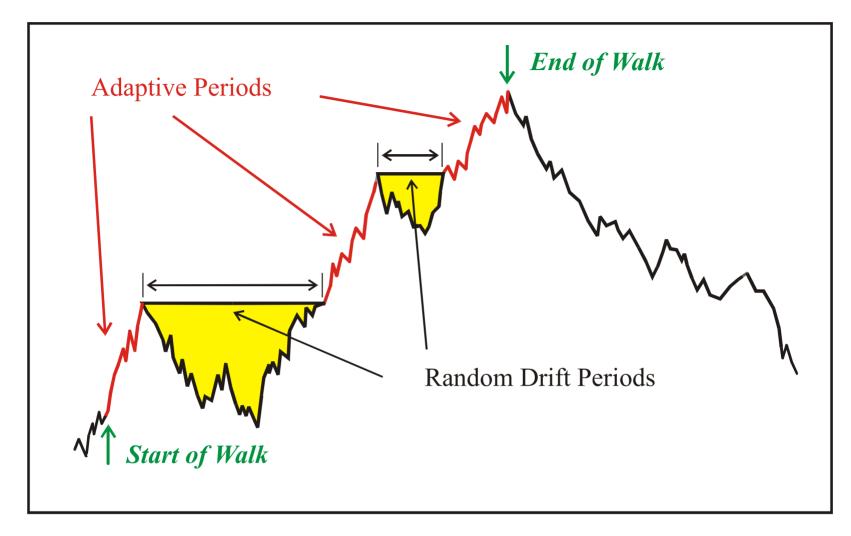


Statistics of evolutionary trajectories

Population size	Number of replications < n _{rep} >	Number of transitions < n _{tr} >	Number of main transitions < n _{dtr} >
1 000	$(5.5 \pm [6.9, 3.1]) \times 10^7$	92.7 ± [80.3,43.0]	8.8 ± [2.4,1.9]
2 000	$\textbf{(6.0} \pm \textbf{[11.1,3.9])} \times \textbf{10}^{\textbf{7}}$	$\textbf{55.7} \pm \textbf{[30.7,19.8]}$	8.9 ± [2.8,2.1]
3 000	(6.6 \pm [21.0,5.0]) $ imes$ 10 7	$\textbf{44.2} \pm \textbf{[25.9,16.3]}$	8.1 ± [2.3,1.8]
10 000	$\text{(1.2} \pm \text{[1.3,0.6])} \times \text{10}^{8}$	$35.9 \pm [10.3, 8.0]$	10.3 ± [2.6,2.1]
20 000	$(\textbf{1.5} \pm [\textbf{1.4,0.7]}) \times \textbf{10}^{\textbf{8}}$	$ 28.8 \pm \textbf{[5.8,4.8]} $	9.0 ± [2.8,2.2]
30 000	$\textbf{(2.2} \pm \textbf{[3.1,1.3])} \times \textbf{10}^{\textbf{8}}$	$ 29.8 \pm \mathbf{[7.3,5.9]} $	8.7 ± [2.4,1.9]
100 000	$\textbf{(3 \pm [2,1])} \times \textbf{10}^{\textbf{8}}$	24 ± [6,5]	9 ± 2

The number of **main transitions** or evolutionary innovations is constant.

"... Variations neither useful not injurious would not be affected by natural selection, and would be left either a fluctuating element, as perhaps we see in certain polymorphic species, or would ultimately become fixed, owing to the nature of the organism and the nature of the conditions. ... "



Genotype Space

Evolution in genotype space sketched as a non-descending walk in a fitness landscape

Evolutionary design of RNA molecules

D.B.Bartel, J.W.Szostak, **In vitro** *selection of RNA molecules that bind specific ligands*. Nature **346** (1990), 818-822

C.Tuerk, L.Gold, SELEX - Systematic evolution of ligands by exponential enrichment: RNA ligands to bacteriophage T4 DNA polymerase. Science 249 (1990), 505-510

D.P.Bartel, J.W.Szostak, *Isolation of new ribozymes from a large pool of random sequences*. Science **261** (1993), 1411-1418

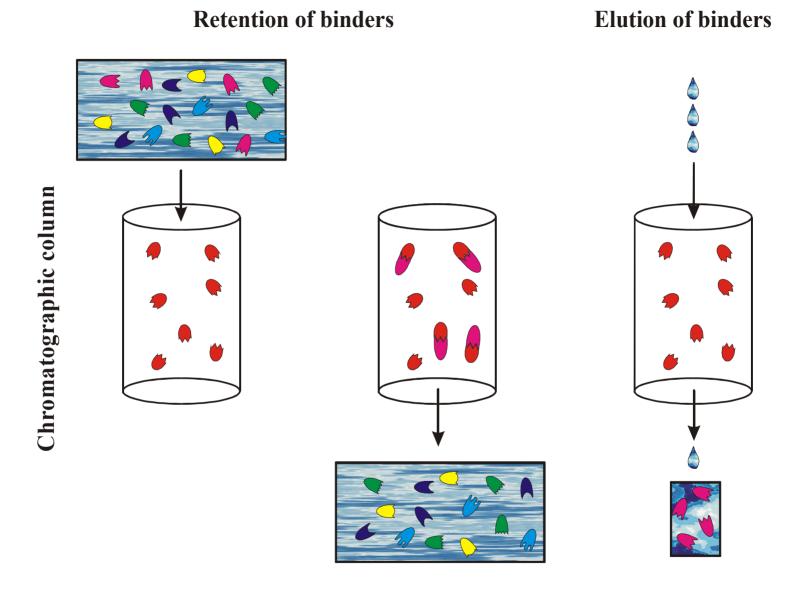
R.D.Jenison, S.C.Gill, A.Pardi, B.Poliski, *High-resolution molecular discrimination by RNA*. Science **263** (1994), 1425-1429

Y.Wang, R.R.Rando, *Specific binding of aminoglycoside antibiotics to RNA*. Chemistry & Biology **2** (1995), 281-290

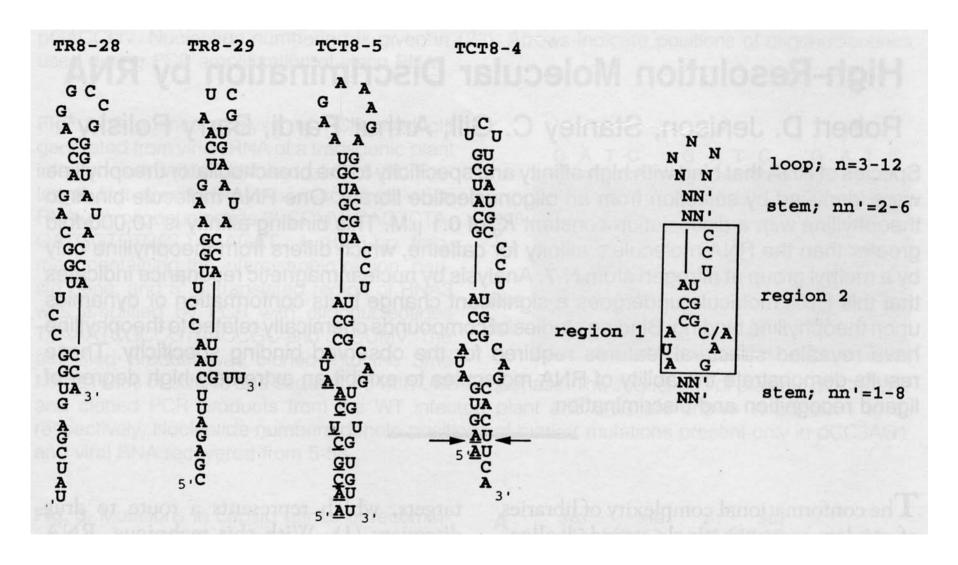
L.Jiang, A.K.Suri, R.Fiala, D.J.Patel, *Saccharide-RNA recognition in an aminoglycoside antibiotic-RNA aptamer complex*. Chemistry & Biology 4 (1997), 35-50

Amplification Diversification Genetic Diversity Selection Cycle Selection Desired Properties ??? no yes

Selection cycle used in applied molecular evolution to design molecules with predefined properties



The SELEX technique for the evolutionary design of *aptamers*



Secondary structures of aptamers binding theophyllin, caffeine, and related compounds

additional methyl group

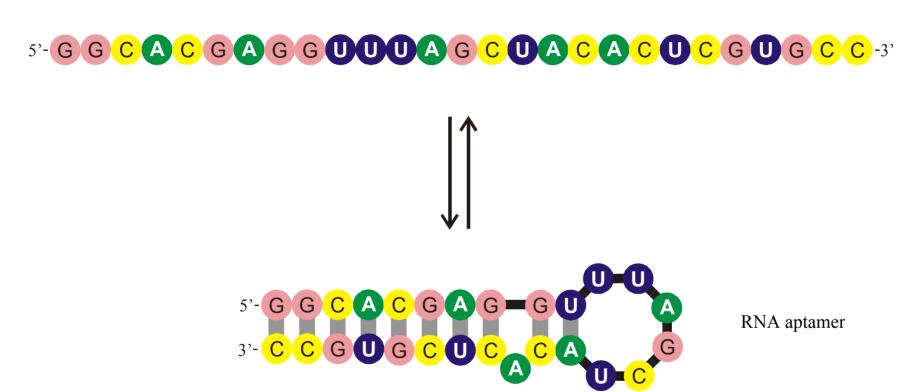
Dissociation constants and specificity of theophylline, caffeine, and related derivatives of uric acid for binding to a discriminating aptamer TCT8-4

Table 1. Competition binding analysis with TCT8-4 RNA. The chemical structures are shown for a series of derivatives used in competitive binding experiments with TCT8-4 RNA (Fig. 2) (20). The right column represents the affinity of the competitor relative to theophylline, $K_{\rm d}(c)/K_{\rm d}(t)$, where $K_{\rm d}(c)$ is the individual competitor dissociation constant and $K_{\rm d}(t)$ is the competitive dissociation constant of theophylline. Certain data (denoted by >) are minimum values that were limited by the solubility of the competitor. Each experiment was carried out in duplicate. The average error is shown.

Compound	Structure	<i>K</i> _d (c) (μM)	$K_{\rm d}(c)/K_{\rm d}(t)$
Theophylline	H ₃ C.N N N	0.32 ± 0.13	
CP-theophylline F	HOOC N N N N	0.93 ± 0.20	2.9
Xanthine	HN TH	8.5 ± 0.40	27
1-Methylxanthine	H ³ C·N	9.0 ± 0.30	28
3-Methylxanthine	HN TN OCH3	2.0 ± 0.7	6.3
7-Methylxanthine	HN CHS	> 500	>1500
3,7-Dimethylxanthine	HN CH ₃	> 500	> 1500
1,3-Dimethyluric acid	H ₃ C.N N N N N N N N N N N N N N N N N N N	> 1000	>3100
Hypoxanthine	HN	49 ± 10	153
Caffeine	H ₃ C.N CH ₃	3500 ± 1500	10,900

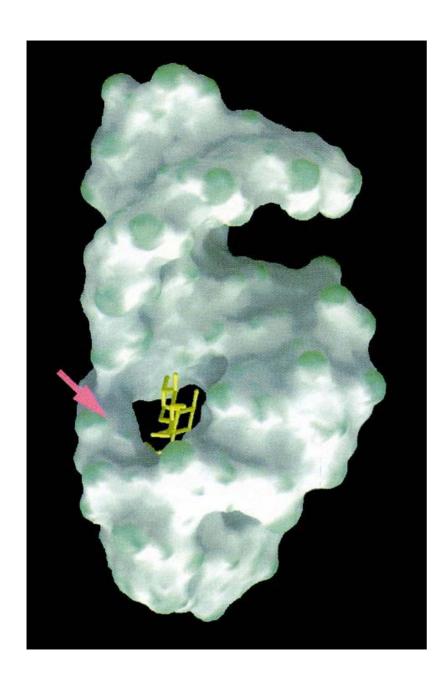
HO
$$\frac{4'' \ 6'' \ 5'' \ OH}{1}$$
 $\frac{1'' \ 6'' \ 5'' \ OH}{1}$ $\frac{1'' \ 6'' \ 5'' \ OH}{1}$ $\frac{1'' \ OH}{1}$ $\frac{1'' \ OH}{2}$ $\frac{1'' \ OH}{1}$ $\frac{1'' \ OH}{2}$ $\frac{1'' \ OH}{3'' \ NH_2}$

tobramycin



Formation of secondary structure of the tobramycin binding RNA aptamer

L. Jiang, A. K. Suri, R. Fiala, D. J. Patel, *Saccharide-RNA recognition in an aminoglycoside antibiotic-RNA aptamer complex.* Chemistry & Biology **4**:35-50 (1997)

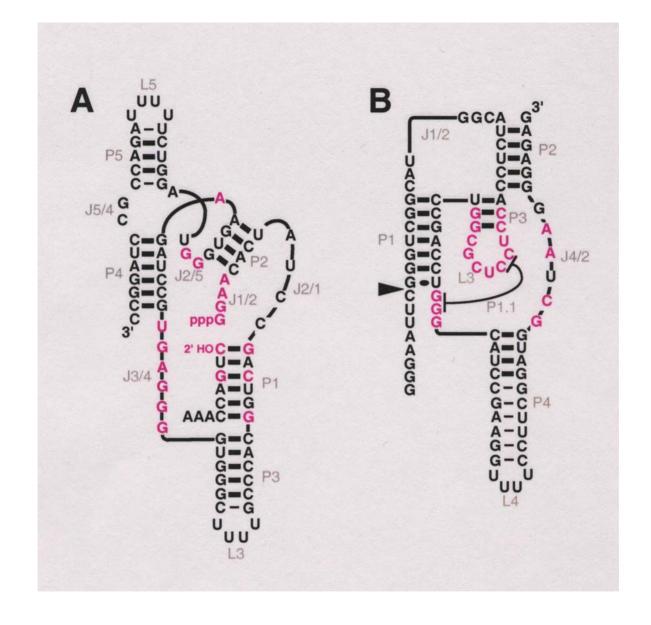


The three-dimensional structure of the tobramycin aptamer complex

L. Jiang, A. K. Suri, R. Fiala, D. J. Patel, Chemistry & Biology 4:35-50 (1997)

A ribozyme switch

E.A.Schultes, D.B.Bartel, *One sequence, two ribozymes: Implication for the emergence of new ribozyme folds*. Science **289** (2000), 448-452

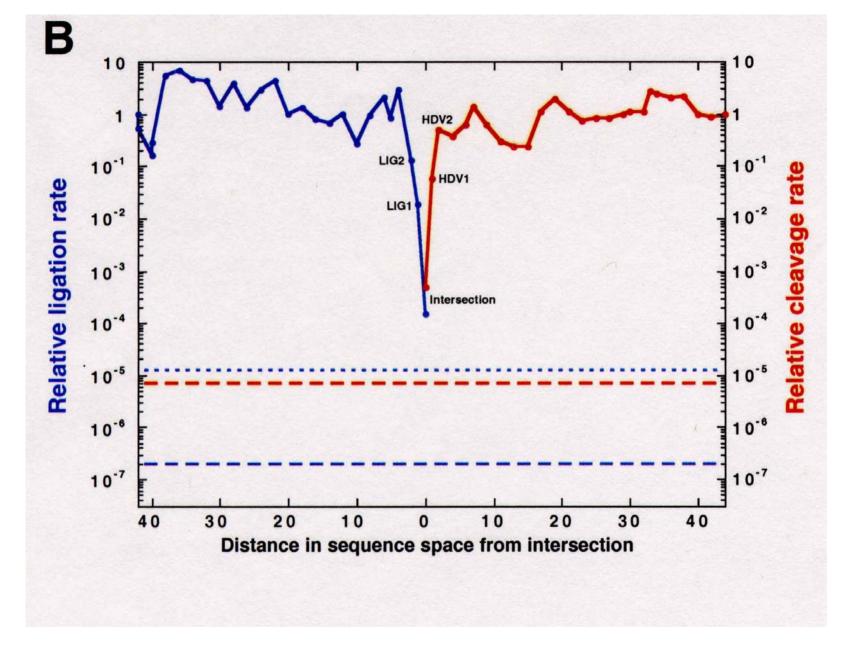


Two ribozymes of chain lengths n = 88 nucleotides: An artificial ligase (**A**) and a natural cleavage ribozyme of hepatitis-X-virus (**B**)

HDV1 LIG1 LIG1 HDV1 Ligase fold **HDV** fold

The sequence at the *intersection*:

An RNA molecules which is 88 nucleotides long and can form both structures



Two neutral walks through sequence space with conservation of structure and catalytic activity

From sequences to shapes and back: a case study in RNA secondary structures

PETER SCHUSTER^{1,2,3}, WALTER FONTANA³, PETER F. STADLER^{2,3} AND IVO L. HOFACKER²

³ Santa Fe Institute, Santa Fe, U.S.A.

SUMMARY

RNA folding is viewed here as a map assigning secondary structures to sequences. At fixed chain length the number of sequences far exceeds the number of structures. Frequencies of structures are highly non-uniform and follow a generalized form of Zipf's law: we find relatively few common and many rare ones. By using an algorithm for inverse folding, we show that sequences sharing the same structure are distributed randomly over sequence space. All common structures can be accessed from an arbitrary sequence by a number of mutations much smaller than the chain length. The sequence space is percolated by extensive neutral networks connecting nearest neighbours folding into identical structures. Implications for evolutionary adaptation and for applied molecular evolution are evident: finding a particular structure by mutation and selection is much simpler than expected and, even if catalytic activity should turn out to be sparse in the space of RNA structures, it can hardly be missed by evolutionary processes.

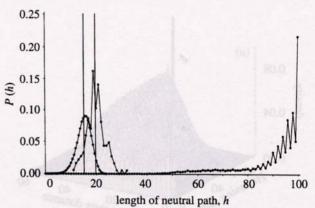


Figure 4. Neutral paths. A neutral path is defined by a series of nearest neighbour sequences that fold into identical structures. Two classes of nearest neighbours are admitted: neighbours of Hamming distance 1, which are obtained by single base exchanges in unpaired stretches of the structure. and neighbours of Hamming distance 2, resulting from base pair exchanges in stacks. Two probability densities of Hamming distances are shown that were obtained by searching for neutral paths in sequence space: (i) an upper bound for the closest approach of trial and target sequences (open circles) obtained as endpoints of neutral paths approaching the target from a random trial sequence (185 targets and 100 trials for each were used); (ii) a lower bound for the closest approach of trial and target sequences (open diamonds) derived from secondary structure statistics (Fontana et al. 1993a; see this paper, §4); and (iii) longest distances between the reference and the endpoints of monotonously diverging neutral paths (filled circles) (500 reference sequences were used).

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Ulrike Göbel, Institut für Molekulare Biotechnologie, Jena, GE Walter Grüner, Stefan Kopp, Jaqueline Weber