



Happy birthday Manuel !

Good health, enjoy science

and

ad multos annos !

Present Day Biology Seen in the Looking Glass of the Physics of Complex Systems

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„A Week of Science“ at Instituto Pluridisciplinar, UCM

In honor of Manuel G. Velarde

Madrid, 15.09.2011

Web-Page for further information:

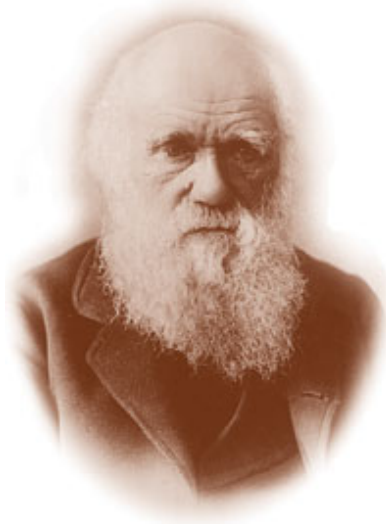
<http://www.tbi.univie.ac.at/~pks>

Nothing in biology makes sense
except in the light of evolution.

Theodosius Dobzhansky. 1973.
Am.Biol.Teach. **35**:125-129.

1. Darwin and evolutionary optimization
2. Evolution as an exercise in chemical kinetics
3. Sequences and structures in biopolymers
4. Evolution on simple model landscapes
5. Evolution on realistic landscapes
6. Neutrality in evolution
7. Perspectives

1. **Darwin and evolutionary optimization**
2. Evolution as an exercise in chemical kinetics
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Three necessary conditions for Darwinian evolution are:

1. **Multiplication,**
2. **Variation,** and
3. **Selection.**

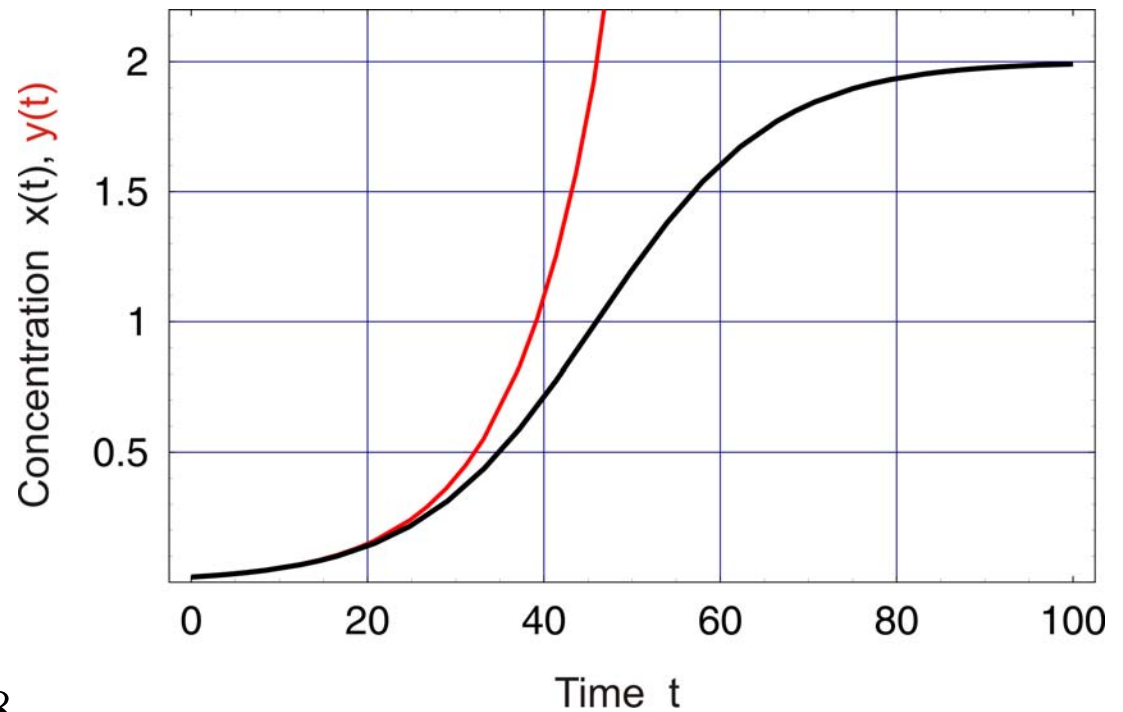
Biologists distinguish the **genotype** - the genetic information - and the **phenotype** - the organisms and all its properties. The **genotype** is unfolded in development and yields the **phenotype**.

Variation operates on the **genotype** - through mutation and recombination - whereas the **phenotype** is the target of **selection**. Without human intervention **natural selection** is based on the number of fertile progeny in forthcoming generations that is called **fitness**.



Pierre-François Verhulst,
1804-1849

$$\frac{dx}{dt} = r x \left(1 - \frac{x}{C} \right), \quad x(t) = \frac{x(0) C}{x(0) + (C - x(0)) e^{-rt}}$$



The logistic equation, 1828

$$\frac{dx}{dt} = r x \left(1 - \frac{x}{C}\right) \implies \frac{dx}{dt} = r x - \frac{x}{C} r x$$

$$\Phi(t) \equiv r x, C = 1: \frac{dx}{dt} = x(r - \Phi)$$

Population: $X_1, X_2, \dots, X_N; [X_i] = x_i$

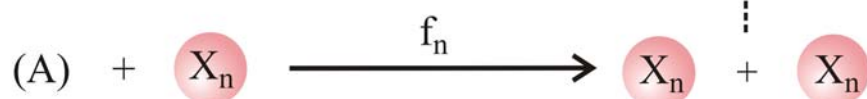
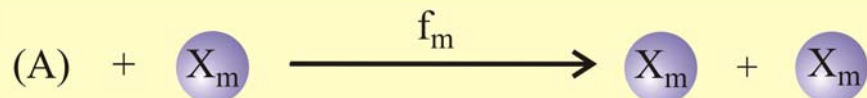
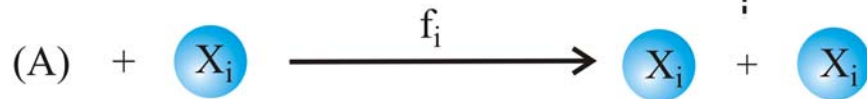
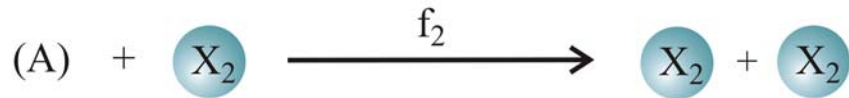
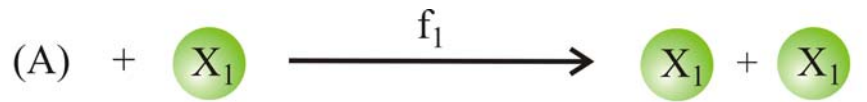
$$\text{Stationary population: } \sum_{i=1}^N x_i = C = 1$$

$$\frac{dx_j}{dt} = x_j \left(f_j - \sum_{i=1}^n f_i x_i \right) = x_j (f_j - \Phi); \quad \Phi = \sum_{i=1}^n f_i x_i$$

Darwin

$$\frac{d\Phi}{dt} = 2 \left(\langle f^2 \rangle - \langle \bar{f} \rangle^2 \right) = 2 \text{ var } \{f\} \geq 0$$

Generalization of the logistic equation to n variables yields selection



$$x_j(t) = N_j(t) / \sum_{i=1}^n N_i(t)$$

$$f_m = \max \{f_j; j=1,2,\dots,n\}$$

$$x_m(t) \rightarrow 1 \text{ for } t \rightarrow \infty$$

Reproduction of organisms or replication of molecules as the basis of selection

Selection equation: $[X_i] = x_i \geq 0, f_i \geq 0$

$$\frac{dx_i}{dt} = x_i (f_i - \phi), \quad i=1,2,\dots,n; \quad \sum_{i=1}^n x_i = 1; \quad \phi = \sum_{j=1}^n f_j x_j = \bar{f}$$

mean fitness or dilution flux, $\phi(t)$, is a **non-decreasing function** of time,

$$\frac{d\phi}{dt} = \sum_{i=1}^n f_i \frac{dx_i}{dt} = \overline{f^2} - (\bar{f})^2 = \text{var}\{f\} \geq 0$$

solutions are obtained by integrating factor transformation

$$x_i(t) = \frac{x_i(0) \cdot \exp(f_i t)}{\sum_{j=1}^n x_j(0) \cdot \exp(f_j t)}; \quad i = 1, 2, \dots, n$$

The mean reproduction rate or mean fitness, $\phi(t)$, is optimized in populations.

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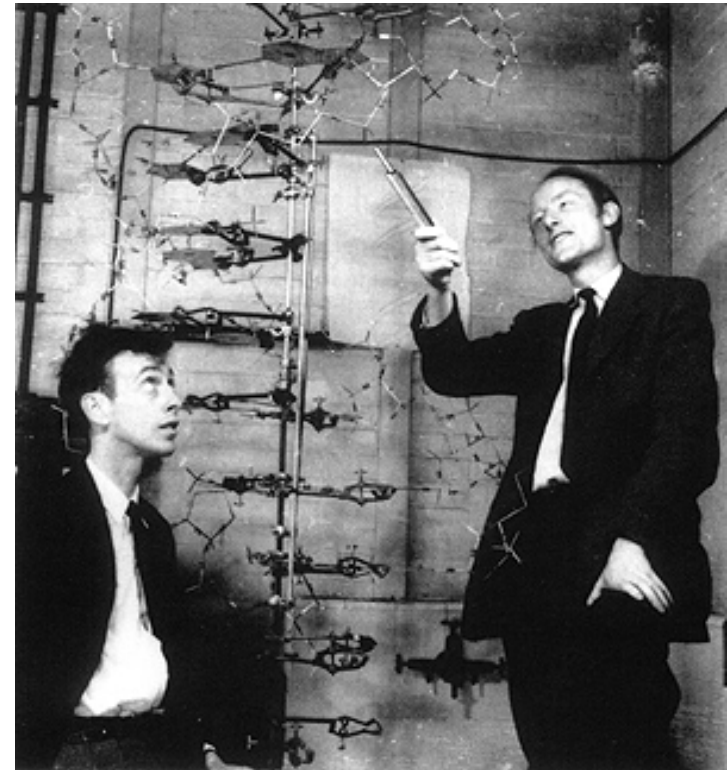
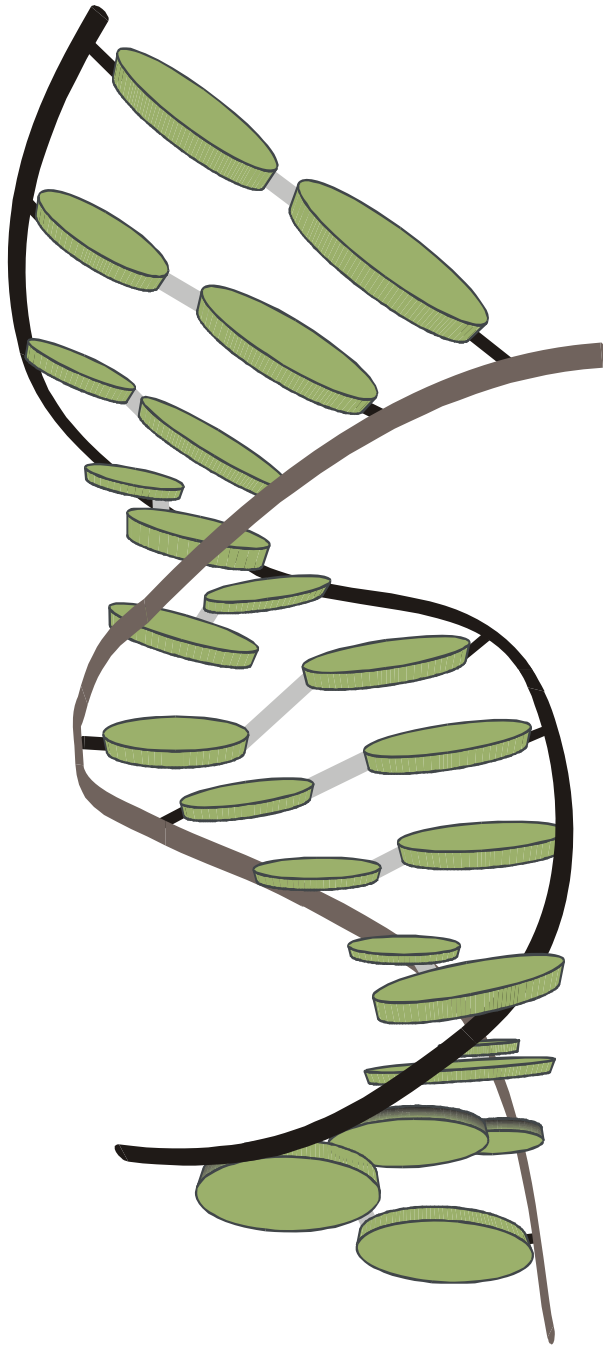
Charles Darwin, 1809-1882

Three necessary conditions for Darwinian evolution are:

1. **Multiplication,**
2. **Variation,** and
3. **Selection.**

All three conditions are fulfilled not only by cellular organisms but also by **nucleic acid molecules** - DNA or RNA - **in** suitable **cell-free experimental assays**:

Darwinian evolution in the test tube

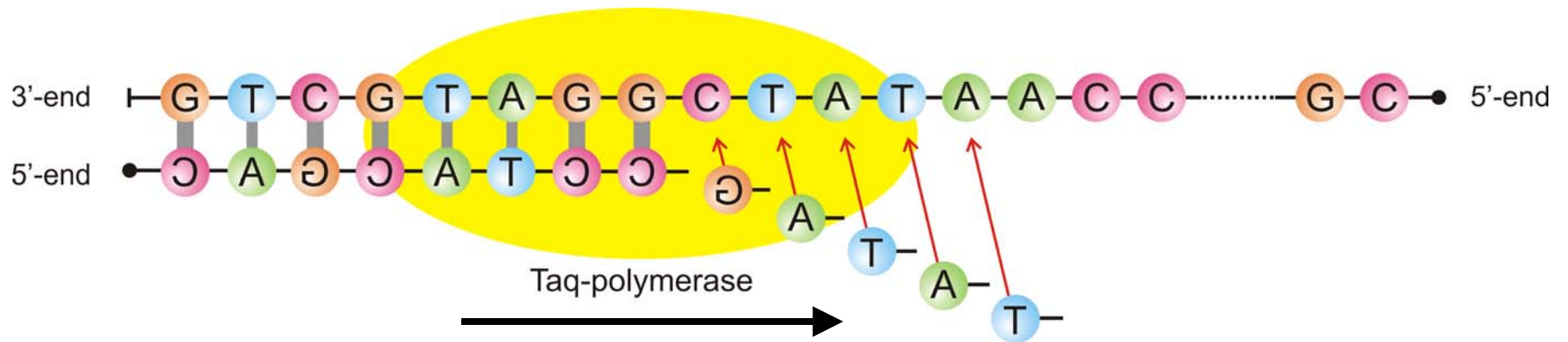


James D. Watson, 1928-, and Francis H.C. Crick, 1916-2004

Nobel prize 1962

1953 – 2003 fifty years double helix

The three-dimensional structure of a short double helical stack of B-DNA



Accuracy of replication: $Q = q_1 \cdot q_2 \cdot q_3 \cdot \dots \cdot q_n$

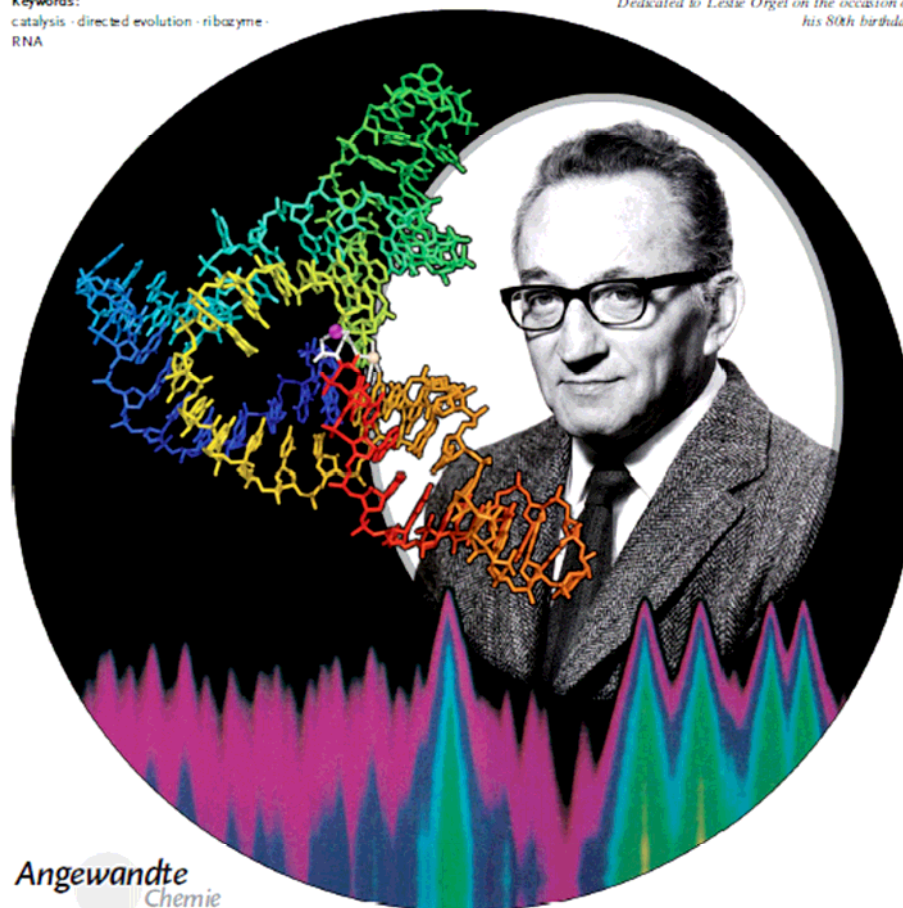
Template induced nucleic acid synthesis proceeds from 5'-end to 3'-end

Forty Years of In Vitro Evolution**

Gerald F. Joyce*

Keywords:
catalysis · directed evolution · ribozyme ·
RNA

Dedicated to Leslie Orgel on the occasion of
his 80th birthday

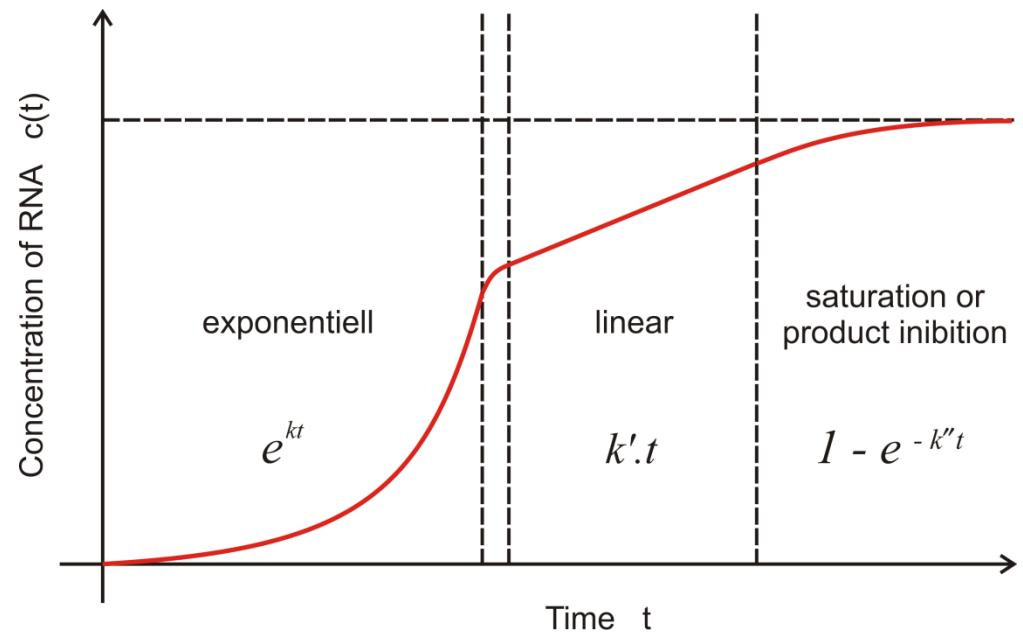
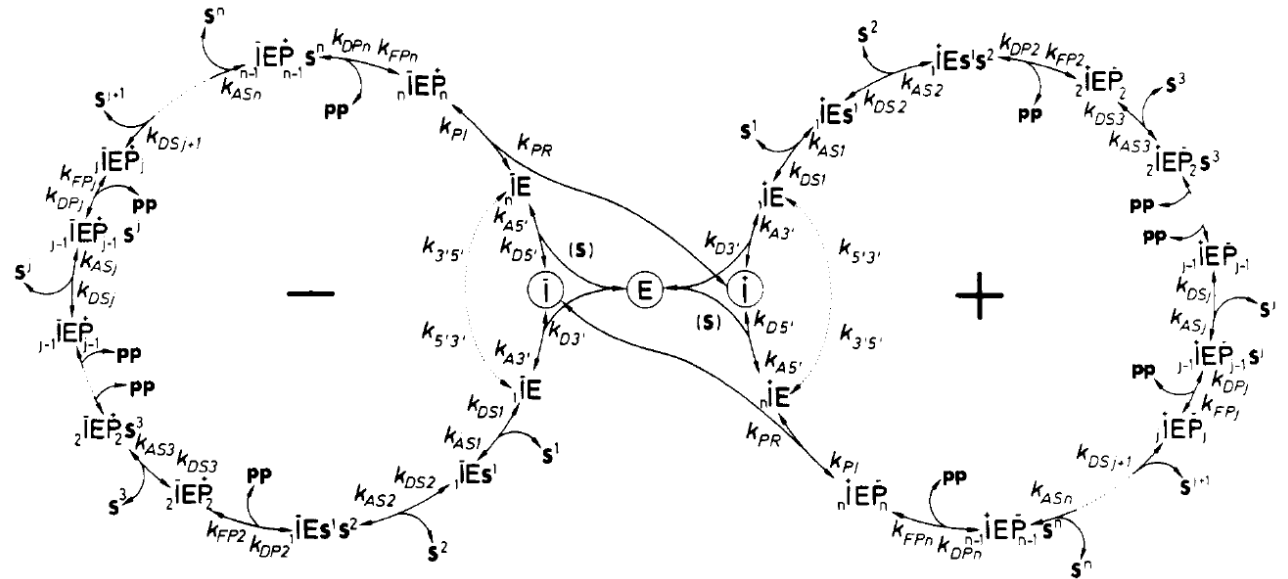


Evolution in the test tube:

G.F. Joyce, *Angew. Chem. Int. Ed.*
46 (2007), 6420-6436

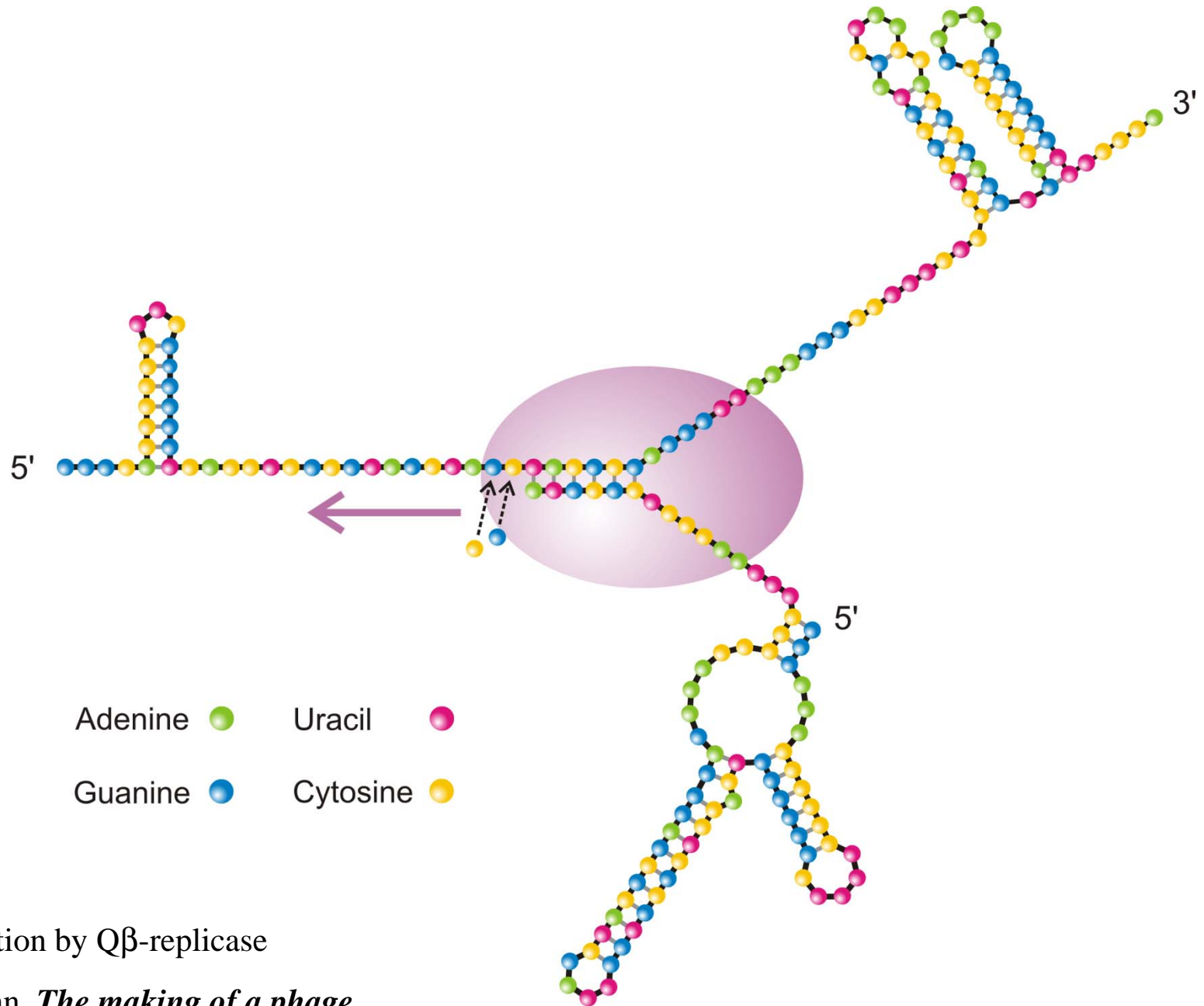


Christof K. Biebricher,
1941-2009



Kinetics of RNA replication

C.K. Biebricher, M. Eigen, W.C. Gardiner, Jr.
Biochemistry **22**:2544-2559, 1983



RNA replication by Q β -replicase

C. Weissmann, *The making of a phage.*

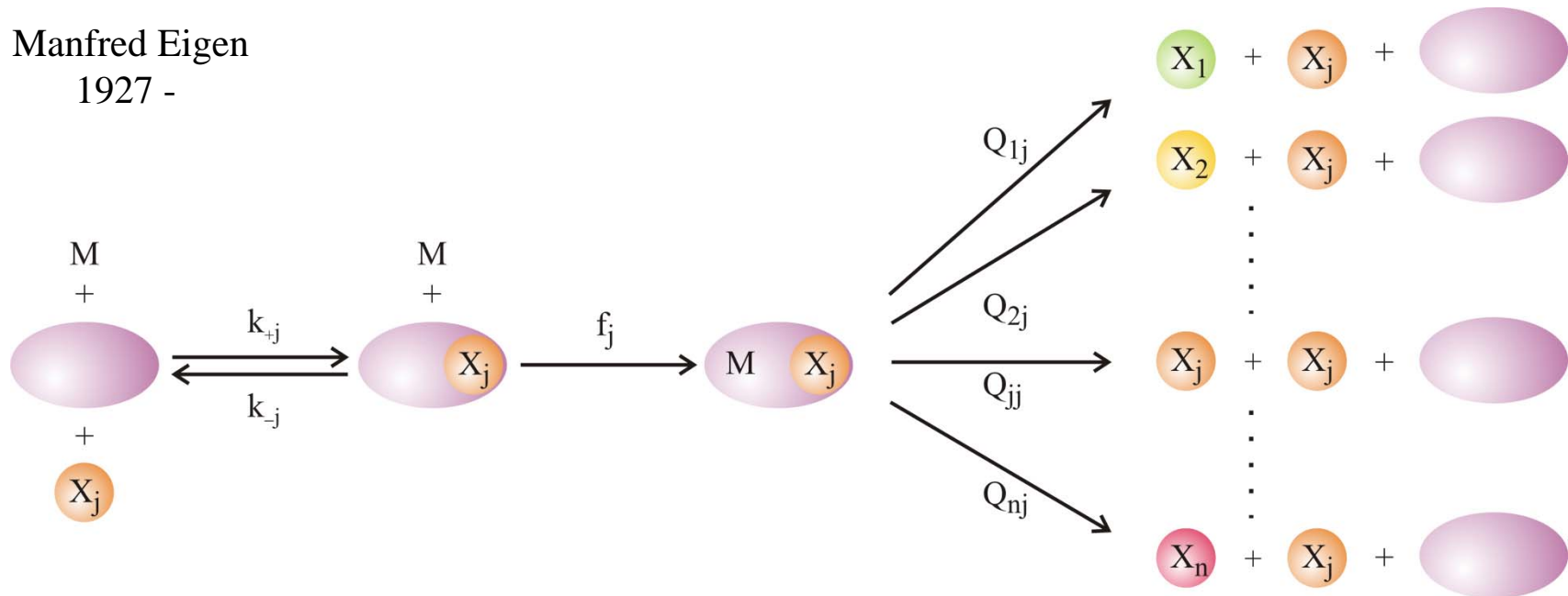
FEBS Letters **40** (1974), S10-S18



Manfred Eigen
1927 -

$$\frac{dx_j}{dt} = \sum_{i=1}^n W_{ji} x_i - x_j \Phi; \quad j = 1, 2, \dots, n$$

$$\Phi = \sum_{i=1}^n f_i x_i / \sum_{i=1}^n x_i$$



Mutation and (correct) replication as parallel chemical reactions

M. Eigen. 1971. *Naturwissenschaften* 58:465,

M. Eigen & P. Schuster. 1977. *Naturwissenschaften* 64:541, 65:7 und 65:341

Mutation-selection equation: $[I_i] = x_i \geq 0, f_i > 0, Q_{ij} \geq 0$

$$\frac{dx_i}{dt} = \sum_{j=1}^n f_j Q_{ji} x_j - x_i \phi, \quad i=1,2,\dots,n; \quad \sum_{i=1}^n x_i = 1; \quad \phi = \sum_{j=1}^n f_j x_j = \bar{f}$$

Solutions are obtained after integrating factor transformation by means of an eigenvalue problem

$$x_i(t) = \frac{\sum_{k=0}^{n-1} \ell_{ik} \cdot c_k(0) \cdot \exp(\lambda_k t)}{\sum_{j=1}^n \sum_{k=0}^{n-1} \ell_{jk} \cdot c_k(0) \cdot \exp(\lambda_k t)}; \quad i=1,2,\dots,n; \quad c_k(0) = \sum_{i=1}^n h_{ki} x_i(0)$$

$$W \div \{f_i Q_{ij}; i, j=1,2,\dots,n\}; \quad L = \{\ell_{ij}; i, j=1,2,\dots,n\}; \quad L^{-1} = H = \{h_{ij}; i, j=1,2,\dots,n\}$$

$$L^{-1} \cdot W \cdot L = \Lambda = \{\lambda_k; k=0,1,\dots,n-1\}$$

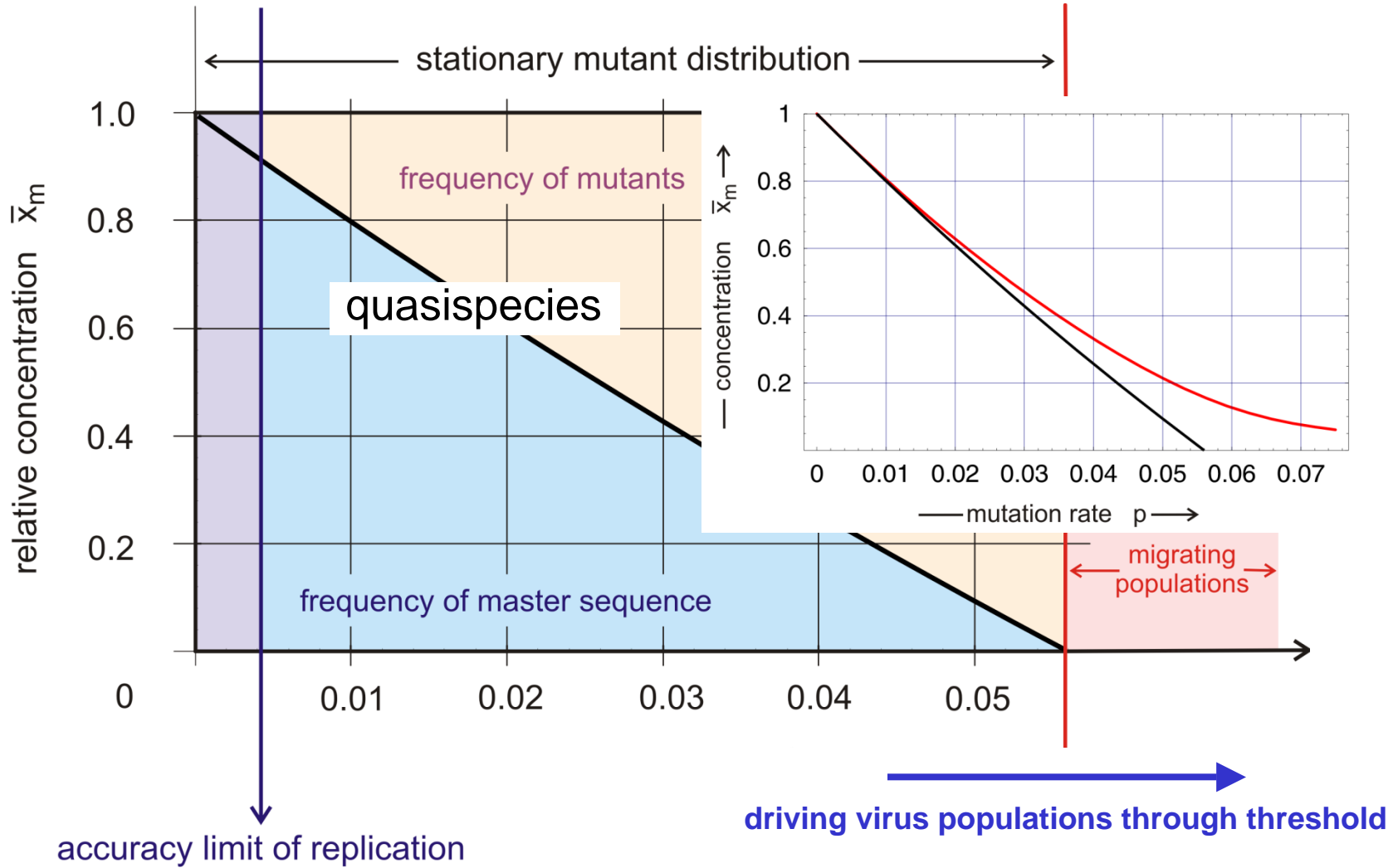
$$\frac{dx_m^{(0)}}{dt} = x_m^{(0)} (Q_{mm} f_m - \phi(t)) = 0 \quad \text{and} \quad \phi(t) = Q_{mm} f_m$$

$$\bar{x}_m^{(0)} = \frac{Q_{mm} - \sigma_m^{-1}}{1 - \sigma_m^{-1}} = \frac{1}{\sigma_m - 1} (\sigma_m (1-p)^n - 1)$$

$$\bar{x}_m^{(0)} = 0 \quad \Rightarrow \quad (1-p)^n = \sigma_m^{-1} \quad \text{and} \quad p_{cr} \approx 1 - (\sigma_m)^{-1/n}$$

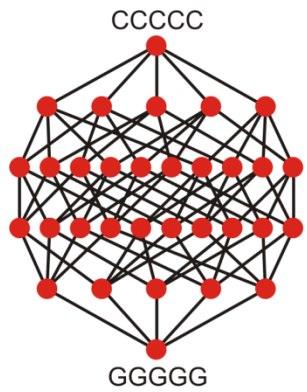
$$\sigma_m = \frac{f_m}{\bar{f}_{-m}} \quad \text{and} \quad \bar{f}_{-m} = \frac{1}{(1-x_m)} \sum_{i=1, i \neq m}^N x_i f_i$$

The ,no-mutational-backflow‘ or zeroth order approximation

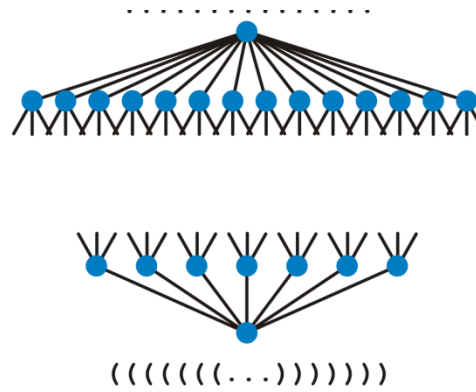


The error threshold in replication and mutation

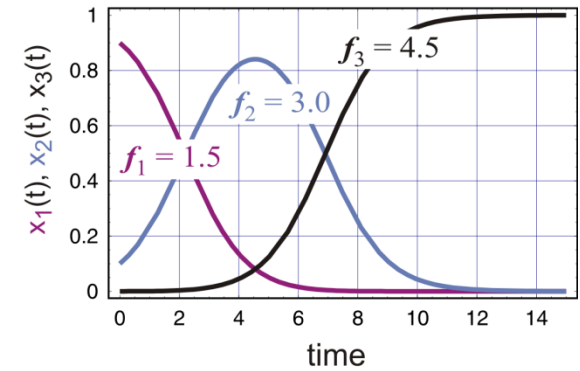
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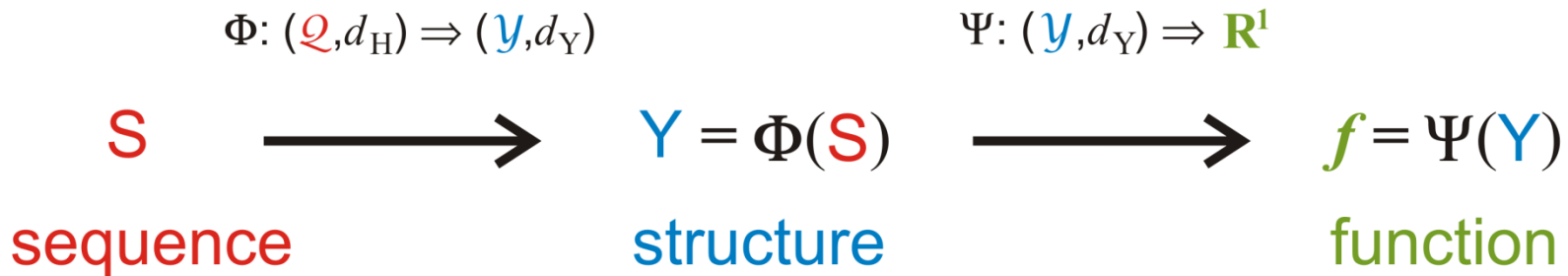
sequence space



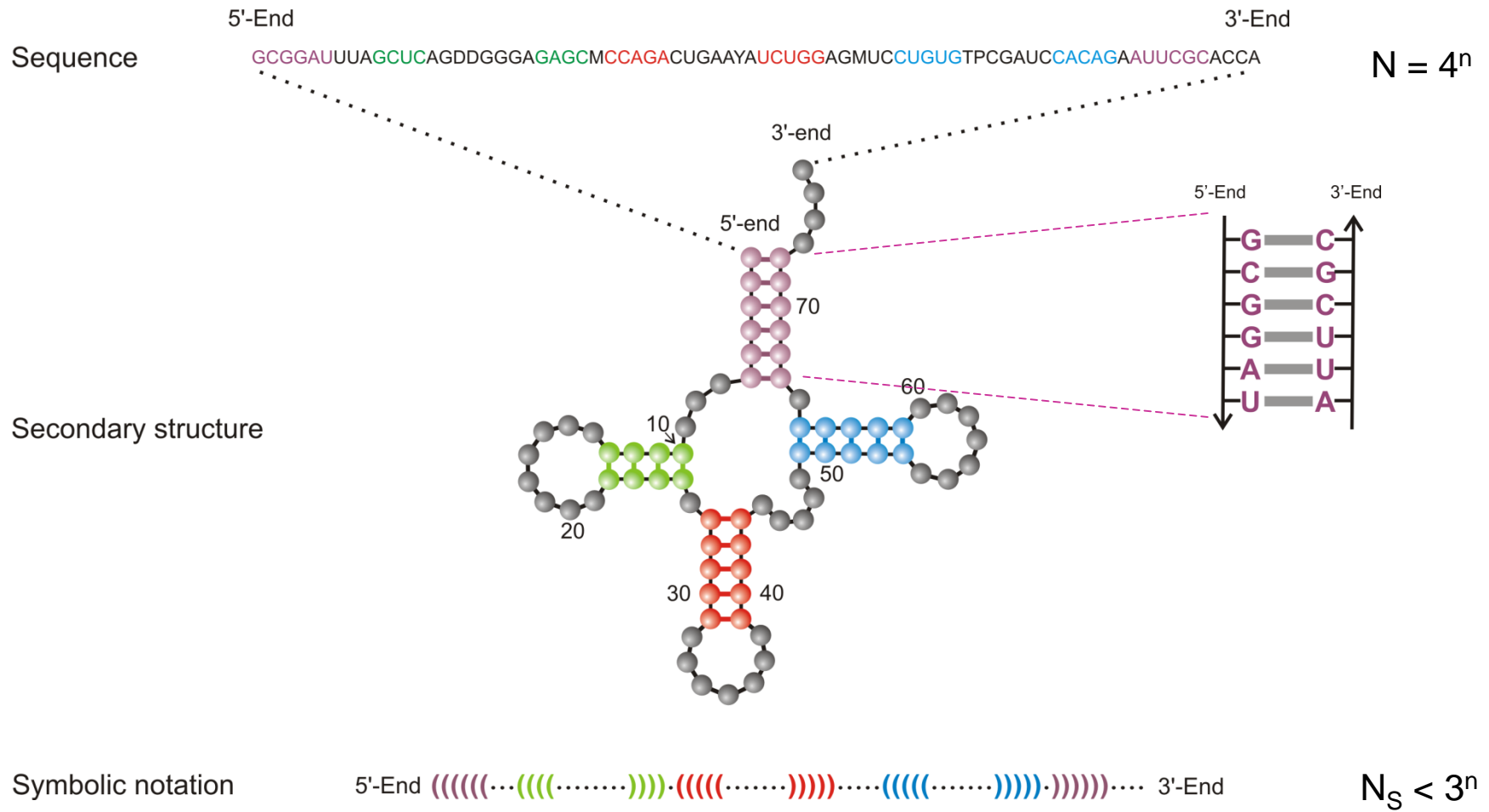
shape space



parameter space



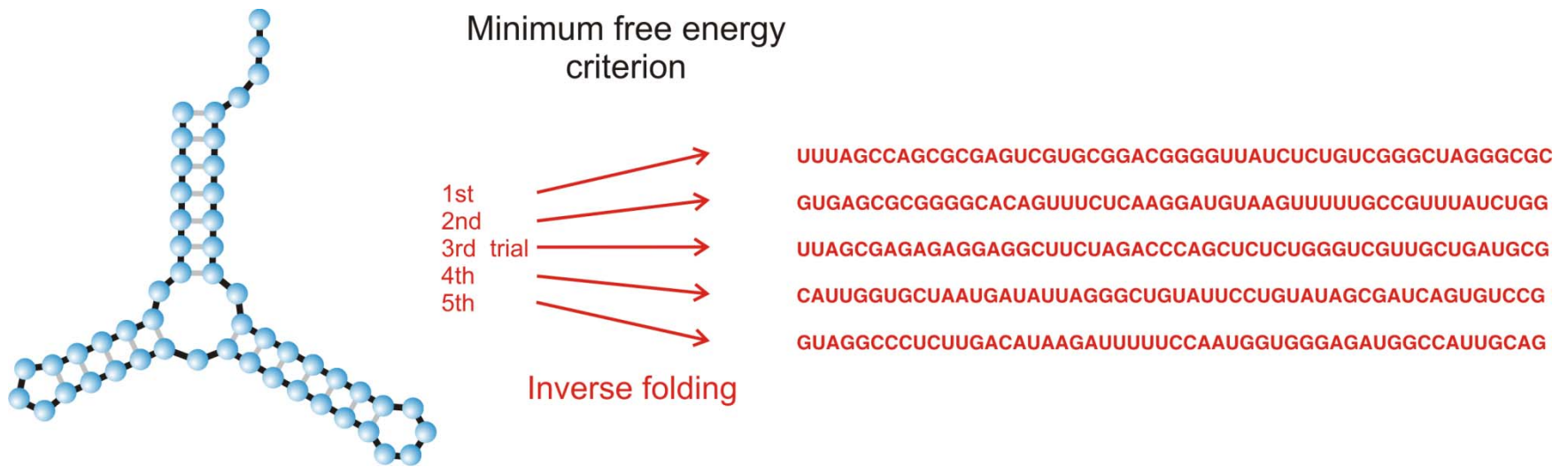
The paradigm of structural biology



Criterion: Minimum free energy (mfe)

Rules: $_ (_) _ \in \{AU, CG, GC, GU, UA, UG\}$

A symbolic notation of RNA secondary structure that is equivalent to the conventional graphs



The **inverse folding algorithm** searches for sequences that form a given RNA secondary structure under the minimum free energy criterion.

space of genotypes: $\mathcal{Q} = \{S_1, S_2, S_3, \dots, S_N\}$

space of phenotypes: $\mathcal{Y} = \{Y_1, Y_2, Y_3, \dots, Y_M\}$

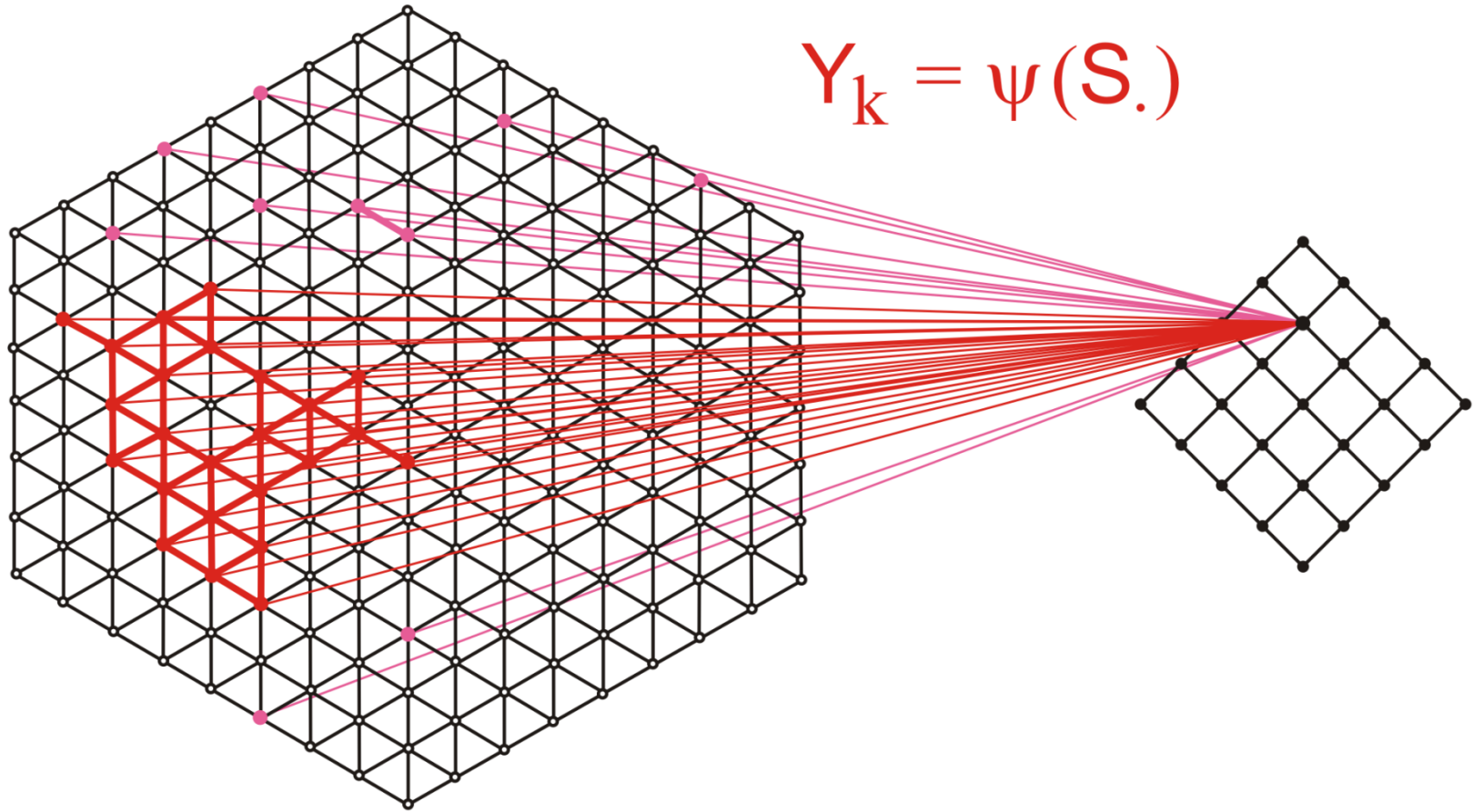
$$N \gg M$$

$$\Phi(S_j) = Y_k$$

$$G_k = \Phi^{-1}(Y_k) \equiv \{S_j \mid \Phi(S_j) = Y_k\}$$

Inversion of genotype-phenotype mapping

neutral network



$$Y_k = \psi(S.)$$

sequence space

shape space

Neutral networks in sequence space



$$G_k = \psi^{-1}(Y_k) \equiv \{ S_j \mid \psi(S_j) = Y_k \}$$

$$\lambda_j = 12 / 27 = 0.444, \quad \bar{\lambda}_k = \frac{\sum_{j \in G_k} \lambda_j(k)}{|G_k|}$$

Connectivity threshold: $\lambda_{cr} = 1 - \kappa^{-1/(\kappa-1)}$

$\bar{\lambda}_k > \lambda_{cr}$ network G_k is **connected**

$\bar{\lambda}_k < \lambda_{cr}$ network G_k is **not connected**

The parameter κ is the size of the alphabet underlying the strings in sequence space

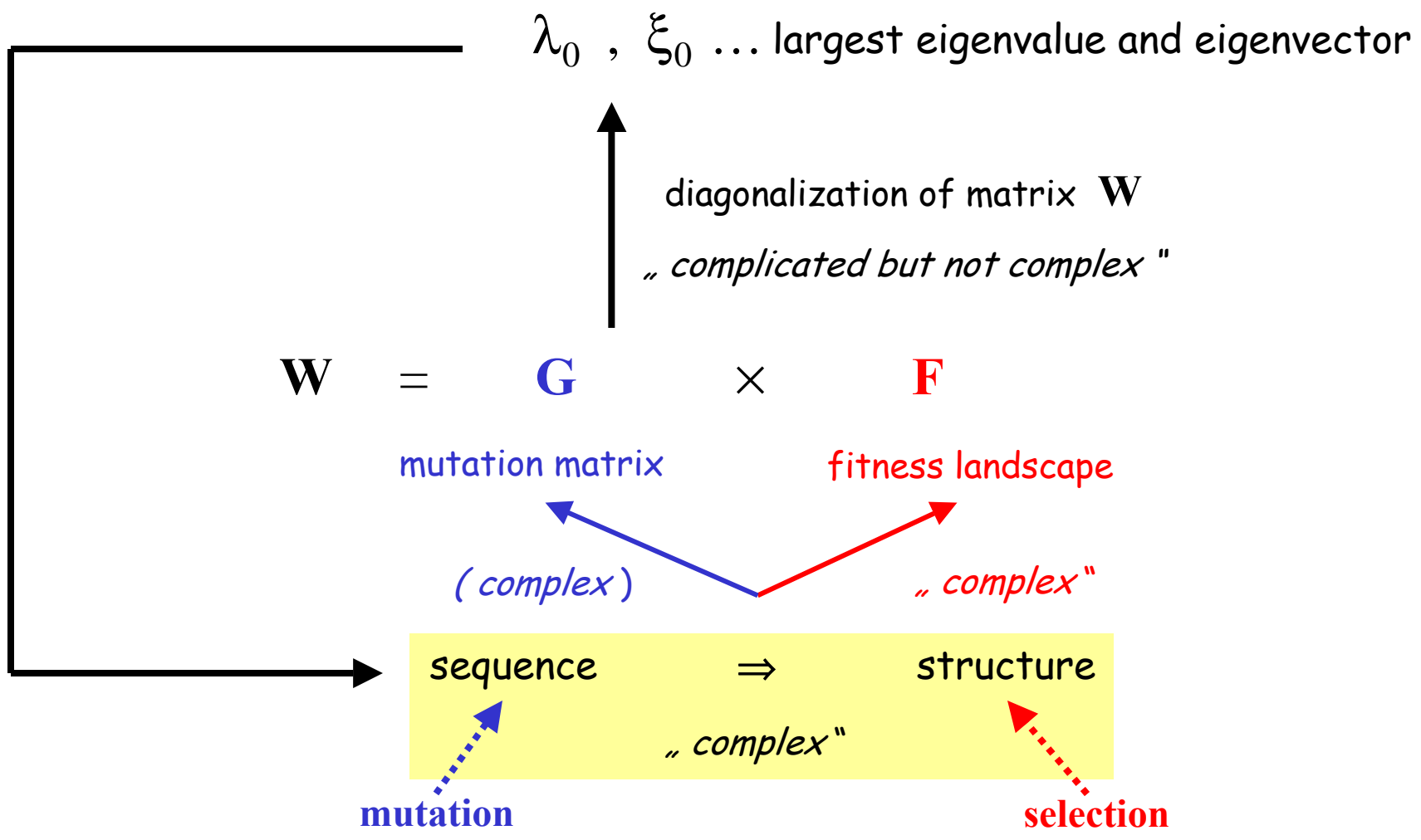
κ	λ_{cr}	
2	0.5	GC, DU
3	0.423	AUG
4	0.370	AUGC

Degree of neutrality of neutral networks and the connectivity threshold

Realistic fitness landscapes

1. Ruggedness: nearby lying genotypes may unfold into very different phenotypes

2. Neutrality: many different genotypes give rise to phenotypes with identical selection behavior



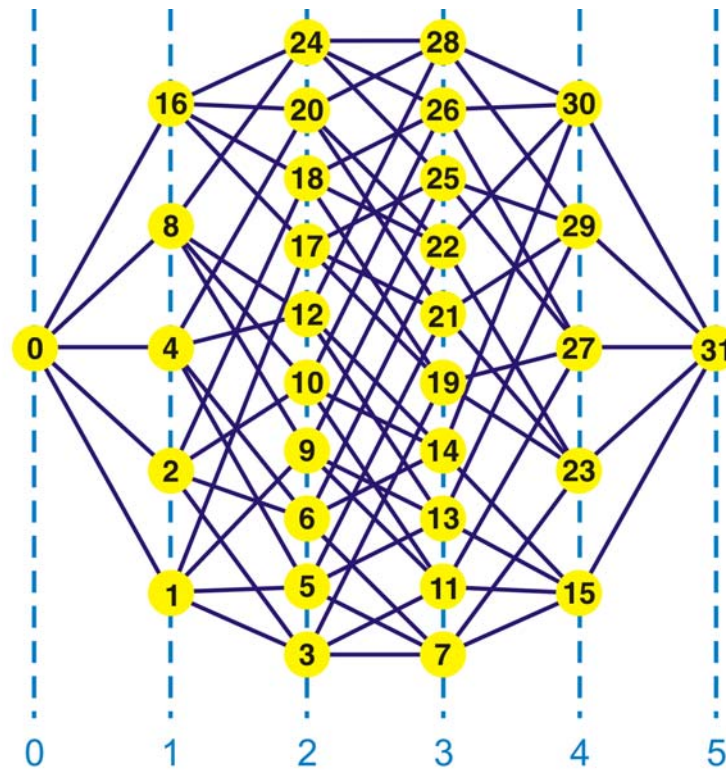
Complexity in molecular evolution

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Make things as simple as possible,
but not simpler !

Albert Einstein

Albert Einstein's razor, precise reference is unknown.



Binary sequences are encoded by their decimal equivalents:

C = 0 and **G** = 1, for example,

"0" ≡ 00000 = **CCCCC**,

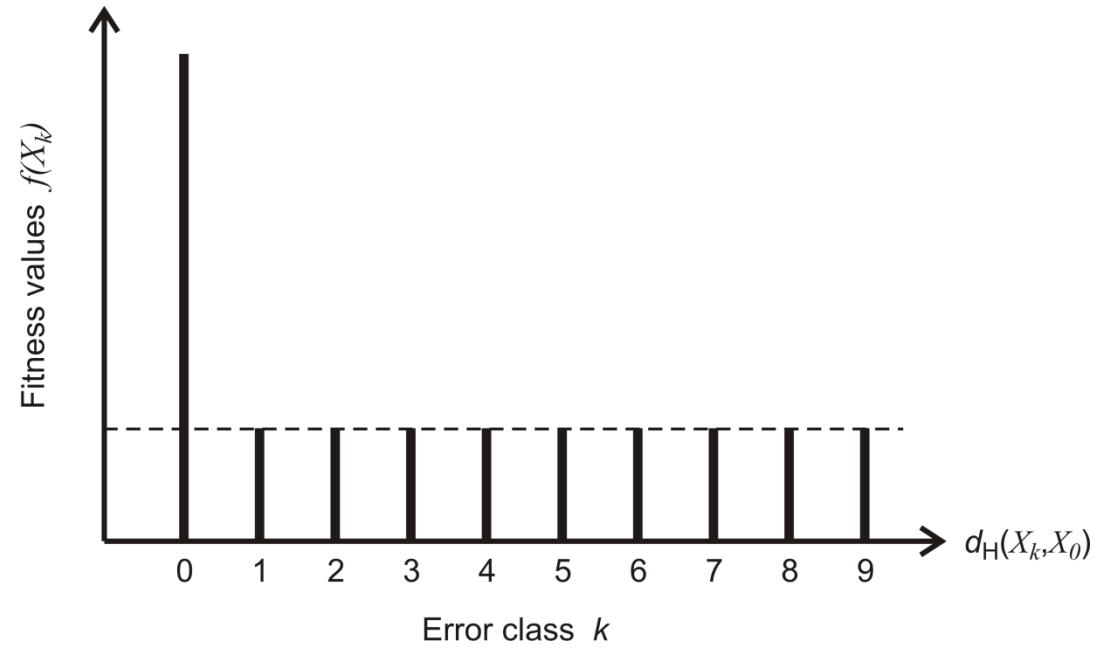
"14" ≡ 01110 = **CGGGC**,

"29" ≡ 11101 = **GGGCG**, etc.

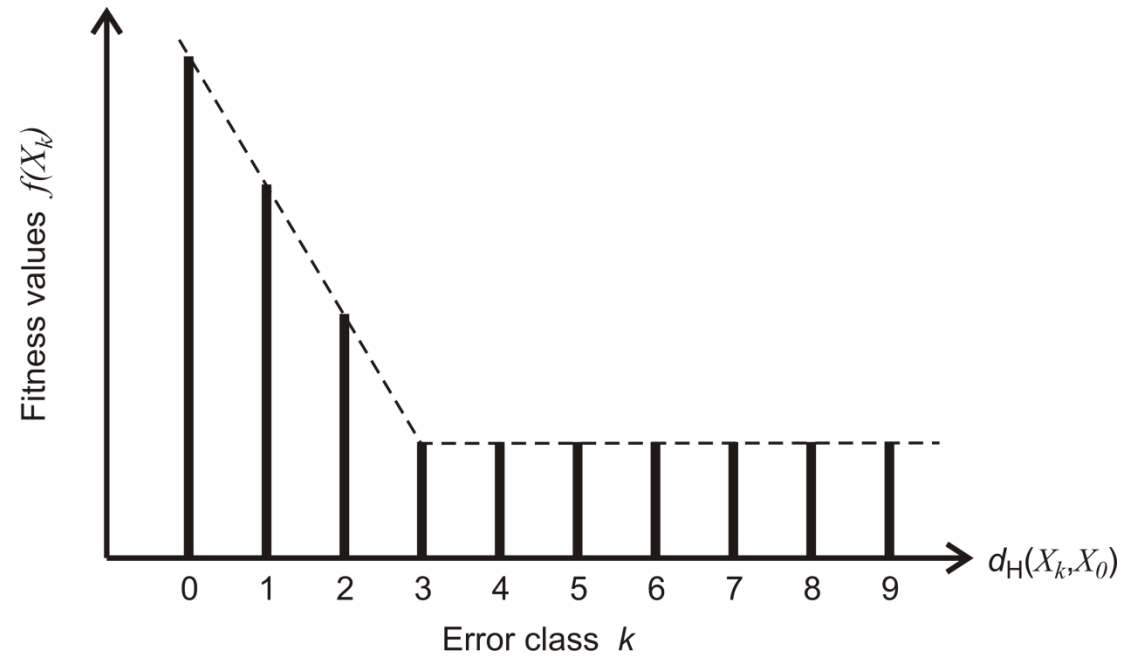
Concentrations of entire error classes: $[\Gamma_k] = y_k(p), k = 0, 1, \dots, n$

$$y_k(p) = \sum_{i=1, d_H(X_i, X_k)=k}^N x_i(p), \quad |\Gamma_k| = \binom{n}{k}$$

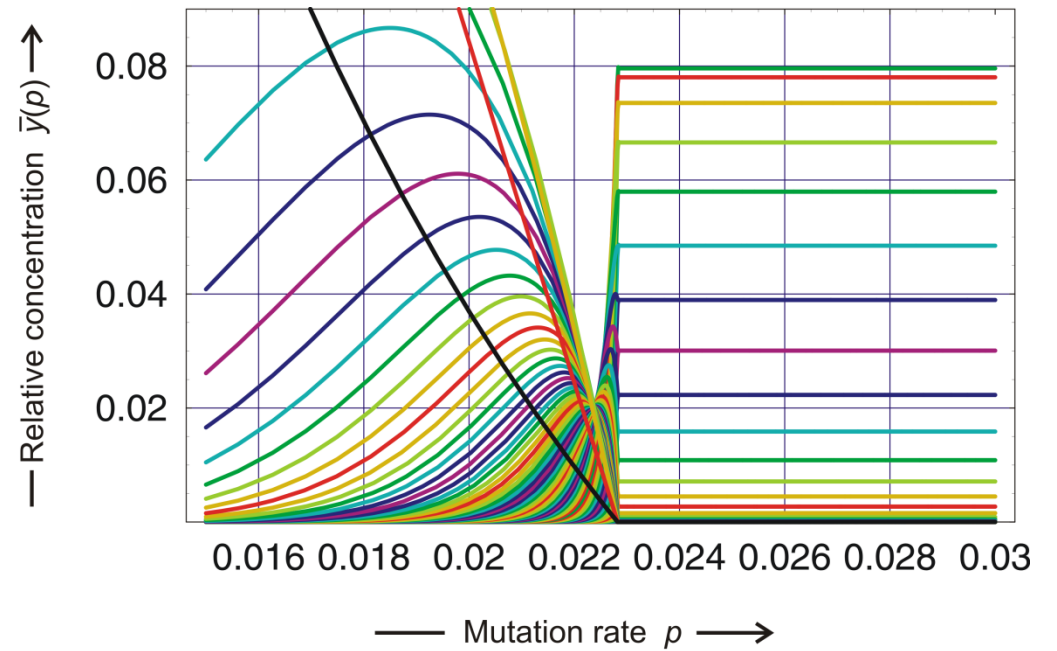
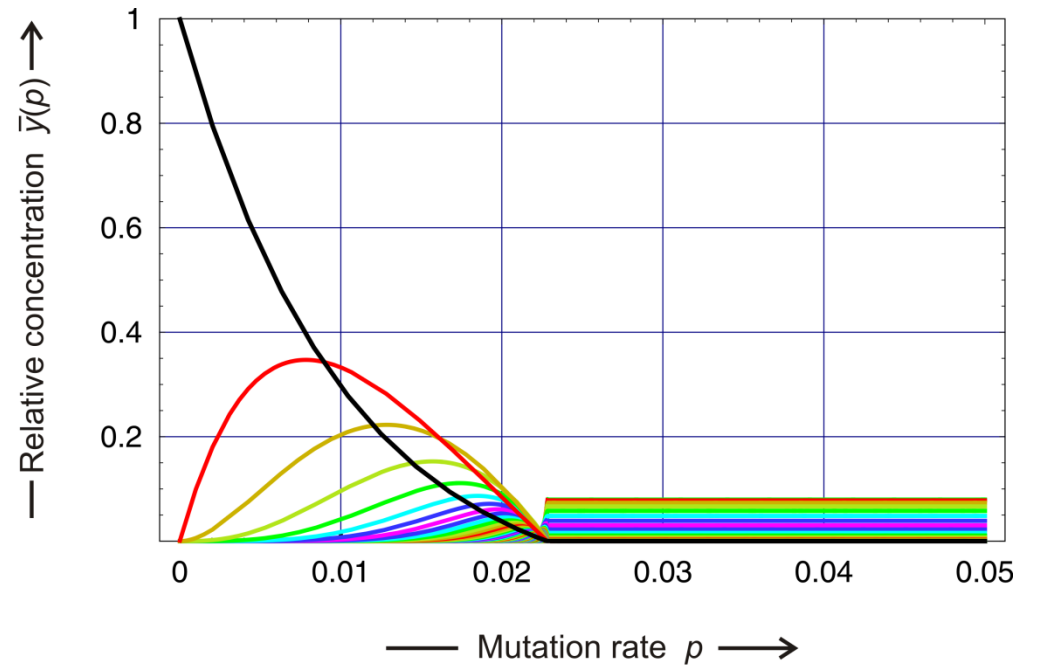
single peak landscape



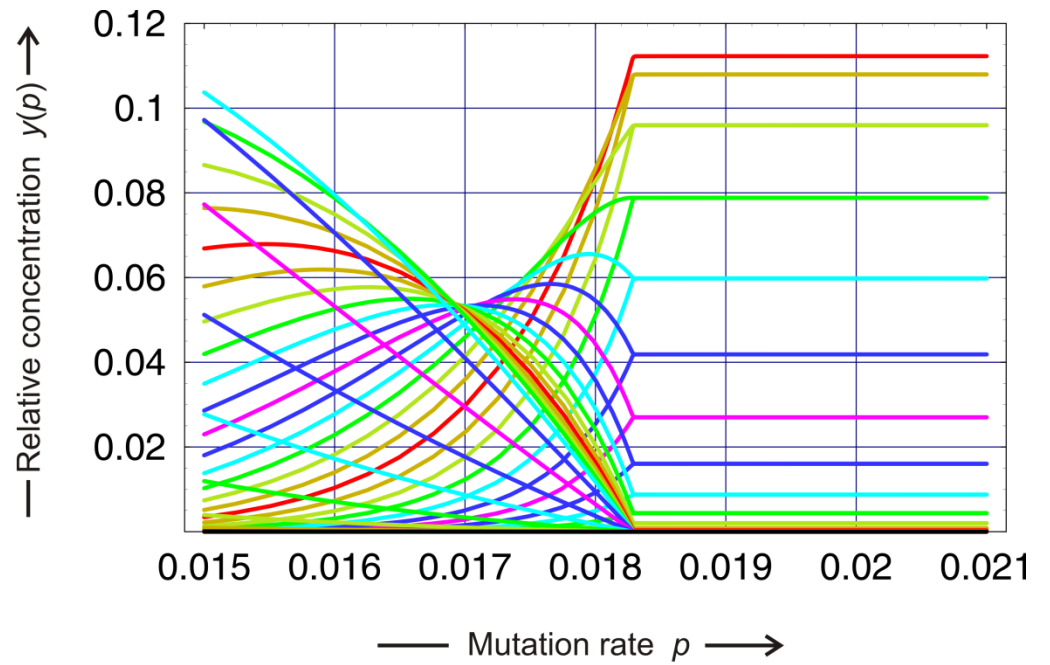
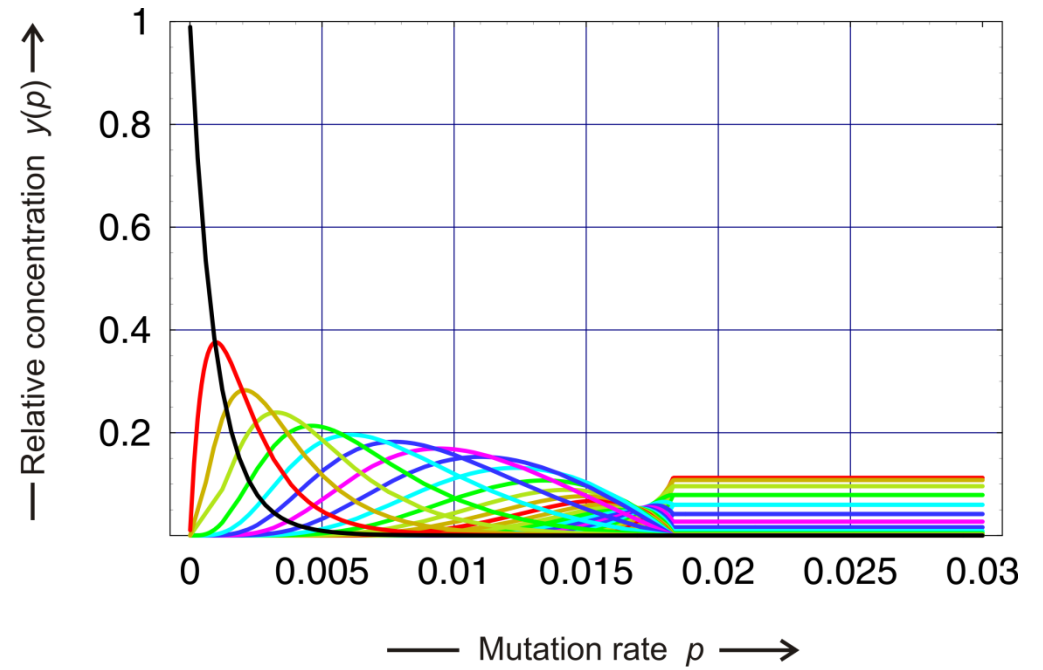
step linear landscape



Model fitness landscapes I

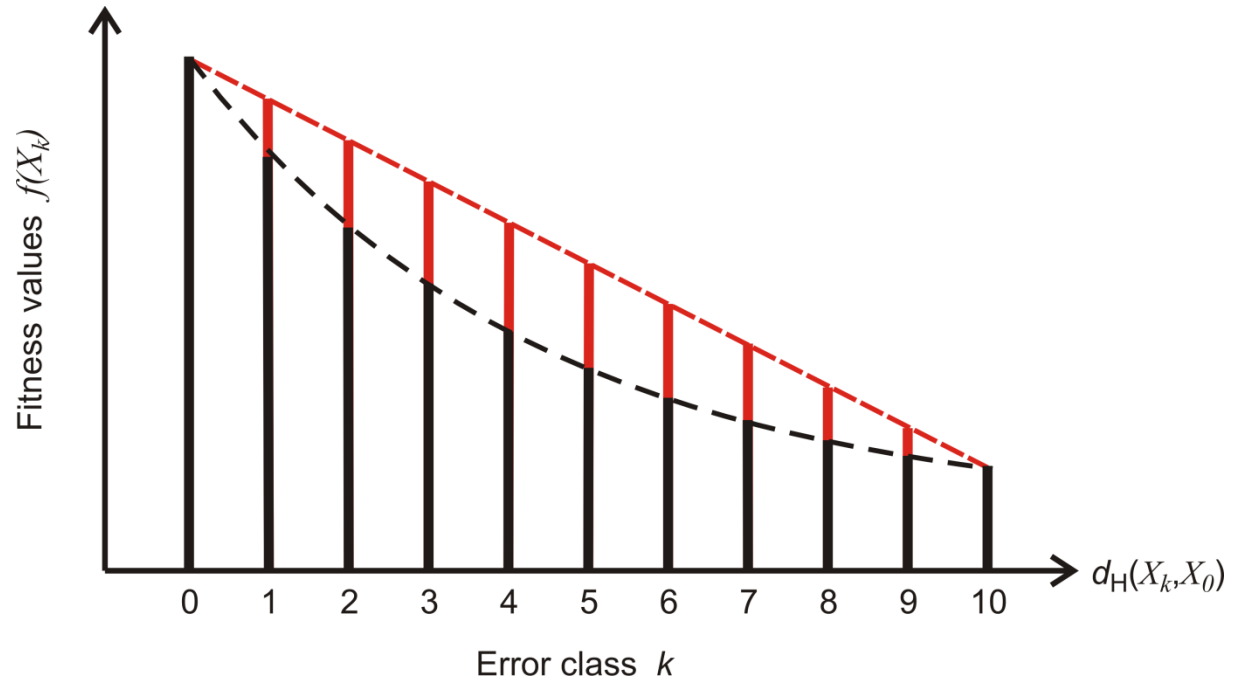


Error threshold on the single peak landscape

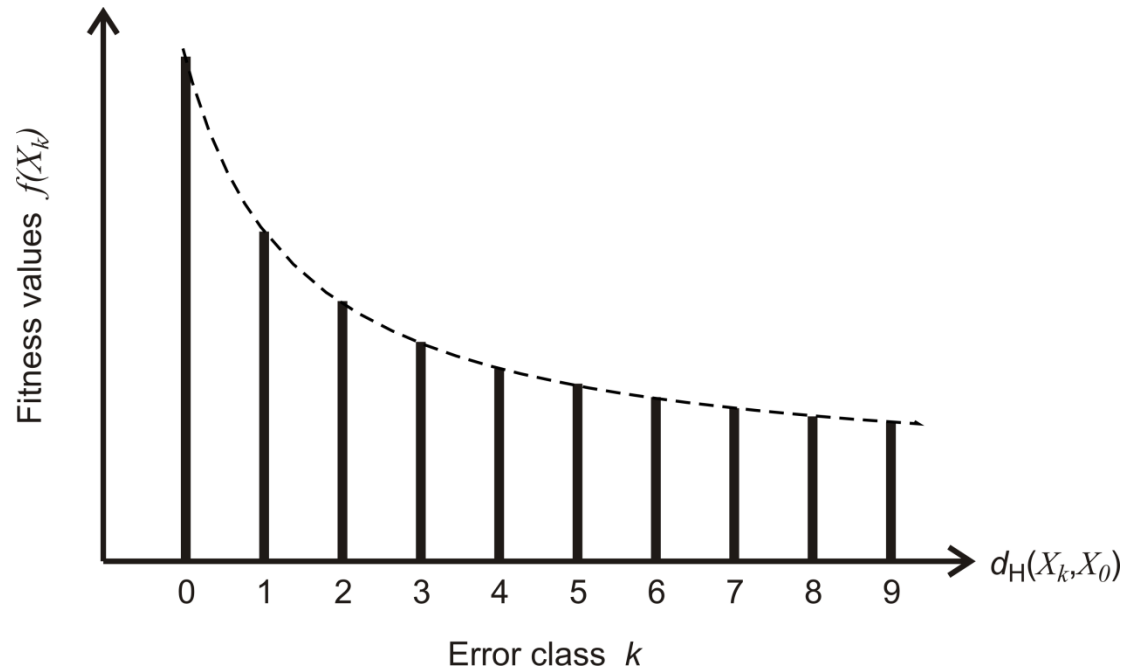


Error threshold on the step linear landscape

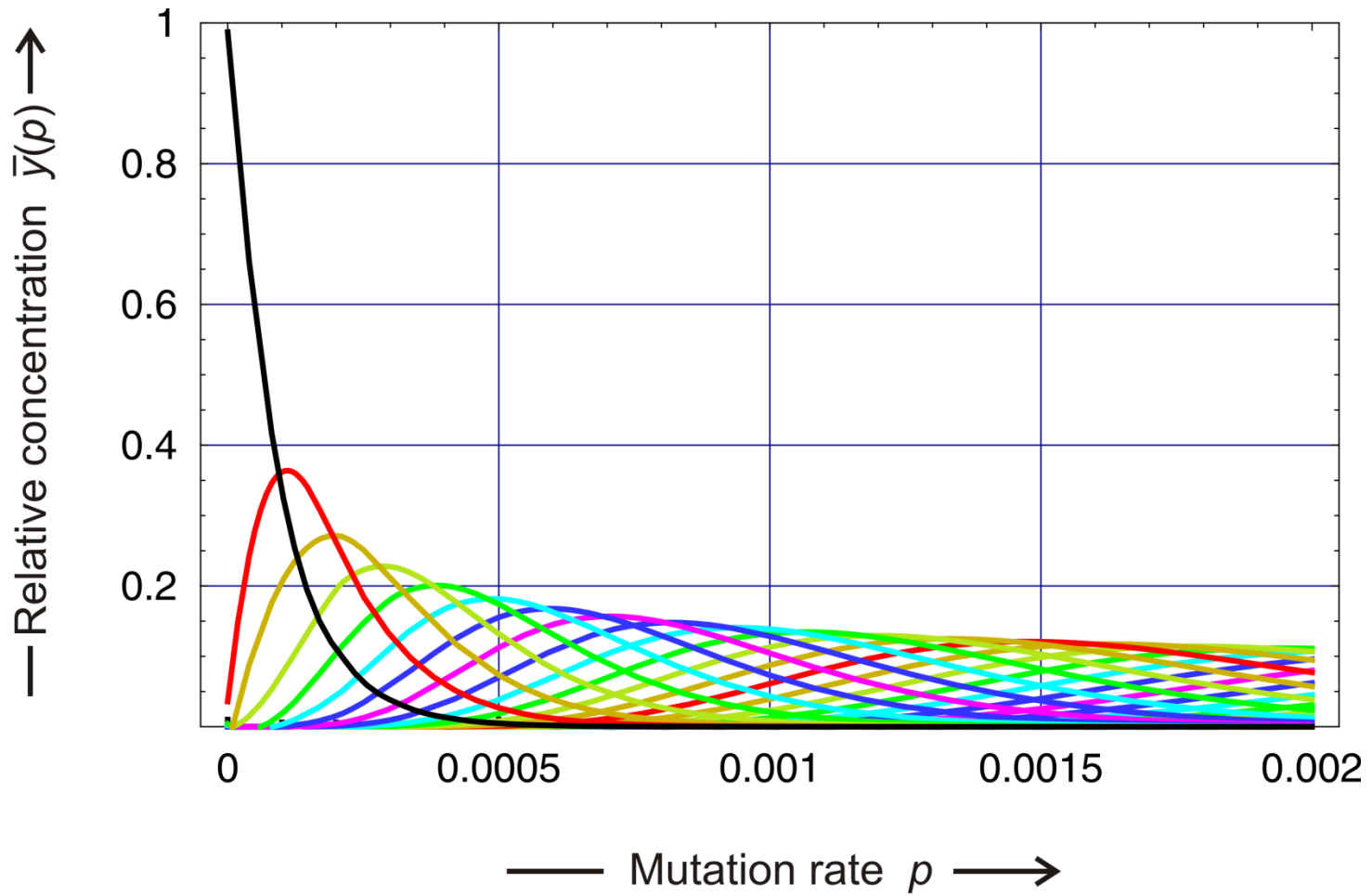
linear and
multiplicative



hyperbolic



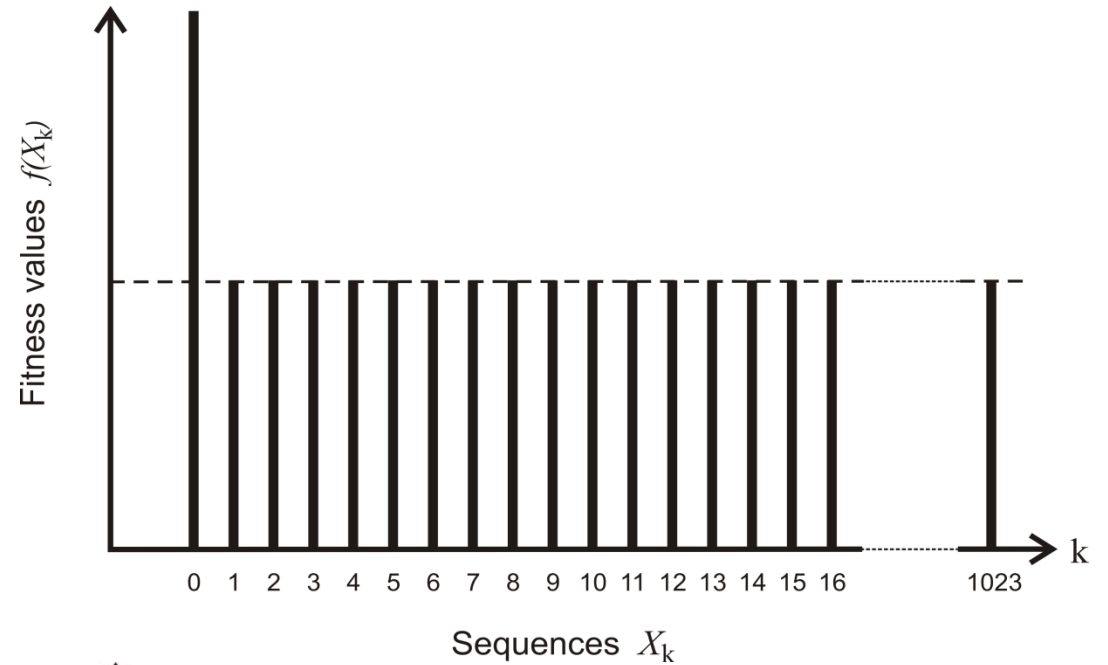
Model fitness landscapes II



The linear fitness landscape shows no error threshold

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single peak landscape



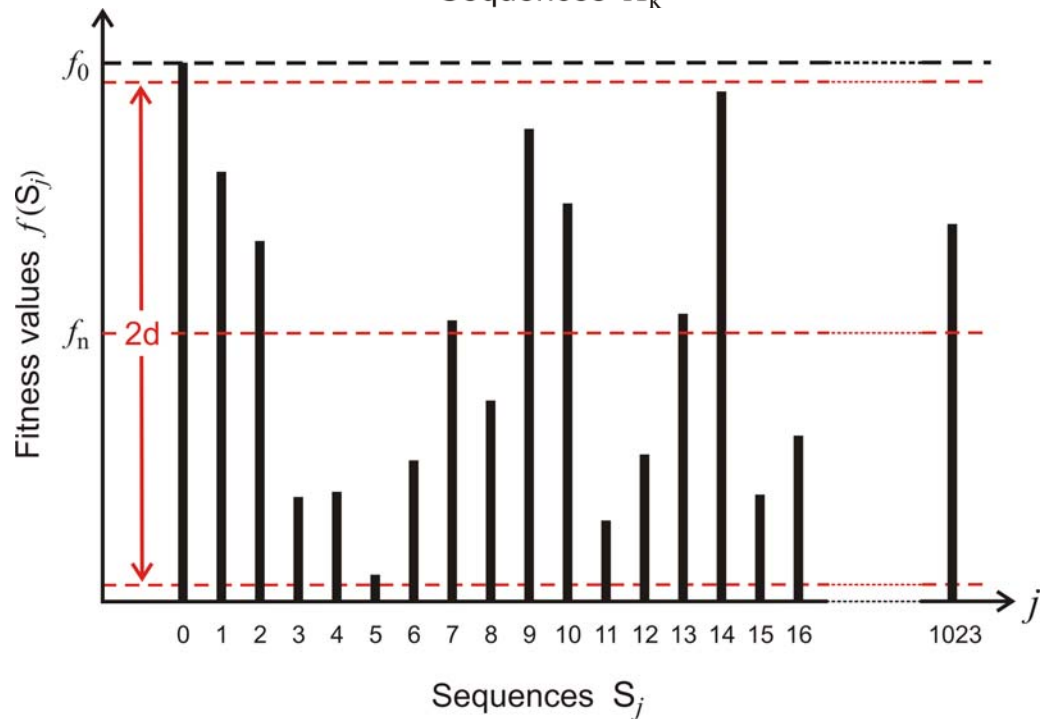
$$f(S_j) = f_n + 2d(f_0 - f_n) \left(\eta_j^{(s)} - 0.5 \right)$$

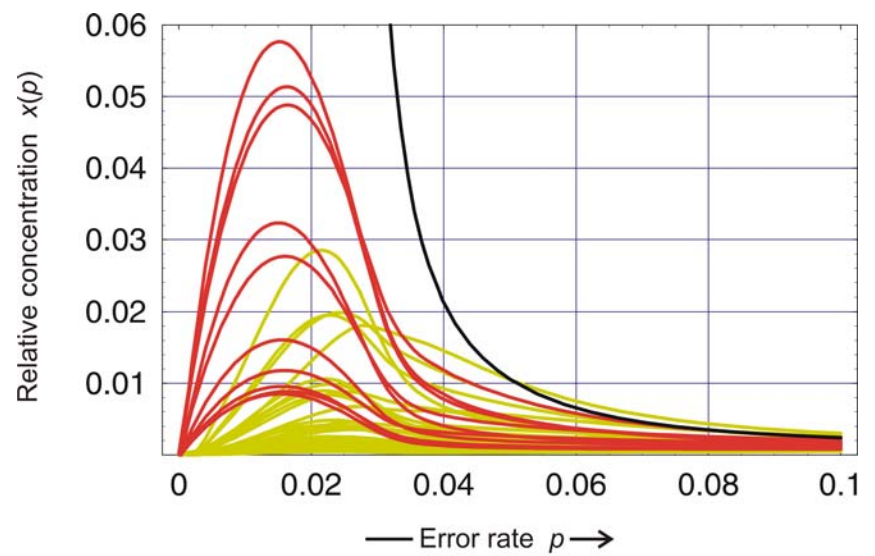
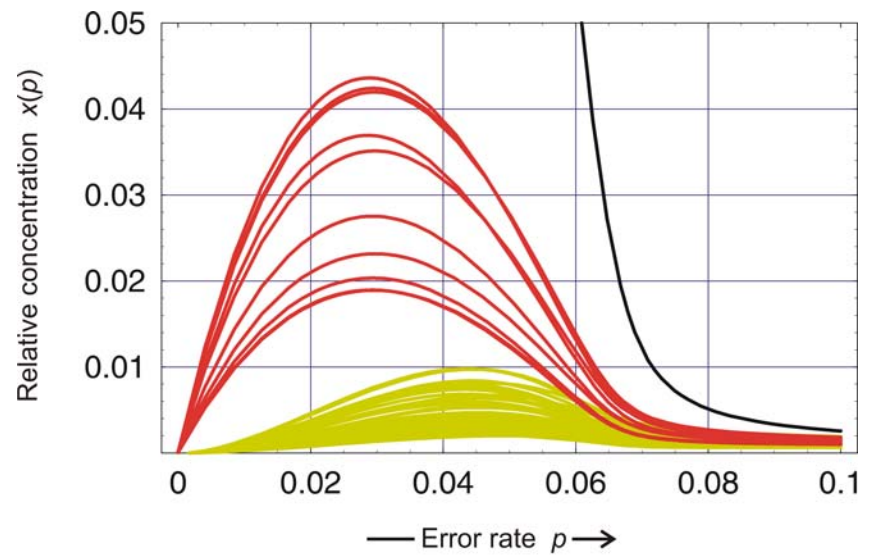
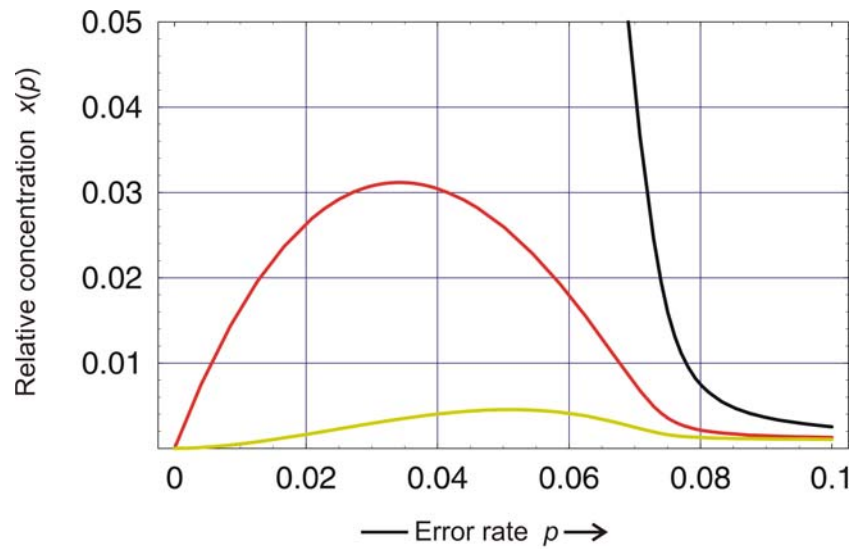
$$j = 1, 2, \dots, N; j \neq m,$$

η ... random number; s ... seeds

„realistic“ landscape

Rugged fitness landscapes
over individual binary sequences
with $n = 10$

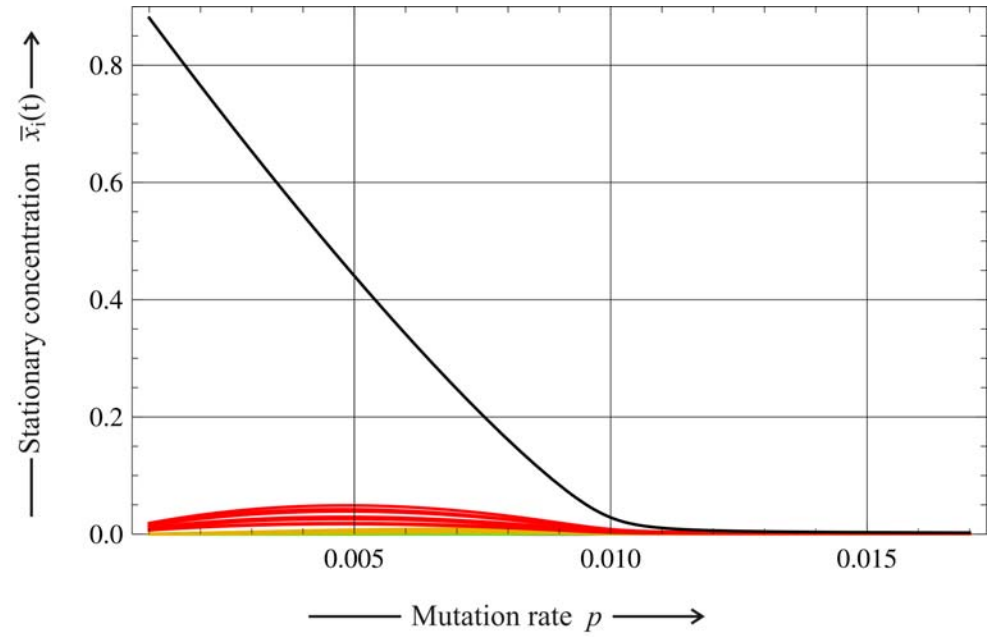




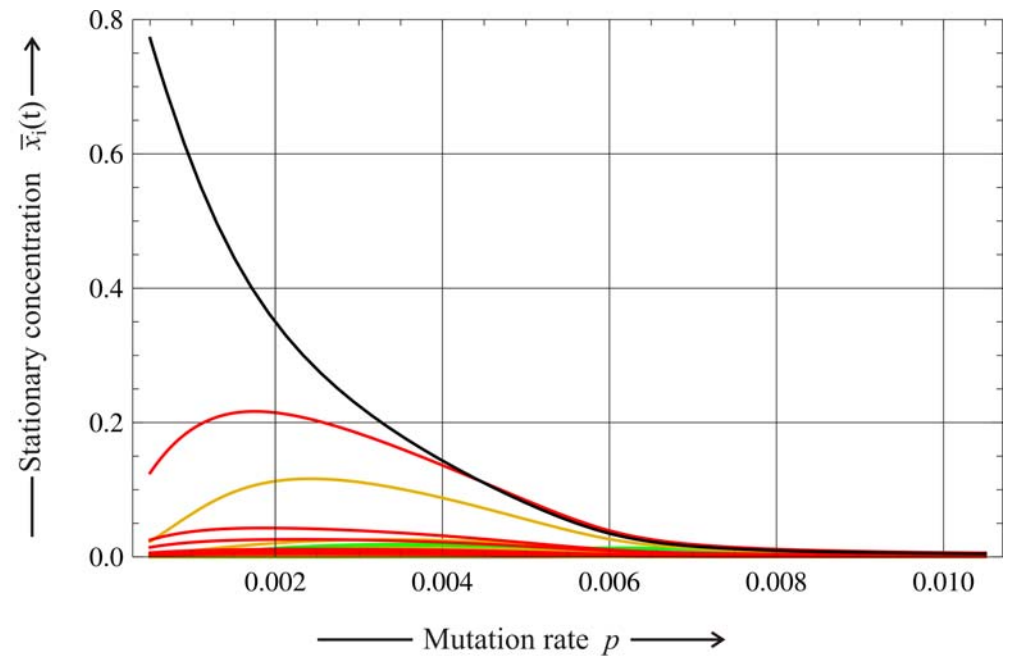
Error threshold: Individual sequences

$n = 10$, $\sigma = 2$, $s = 491$ and $d = 0, 0.5, 0.9375$

$d = 0.5$



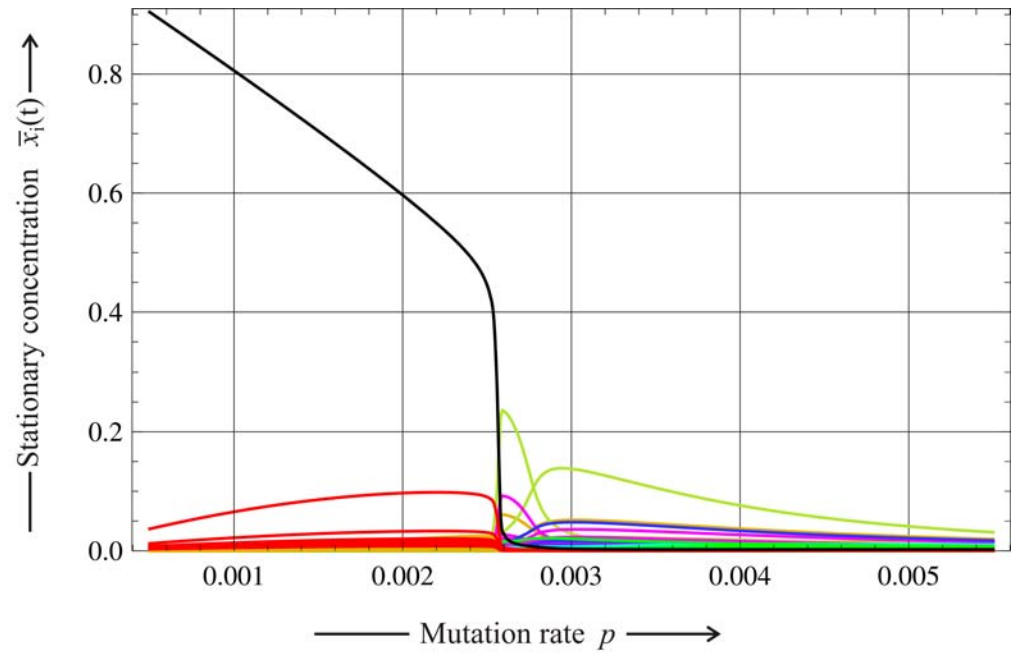
$d = 1.0$



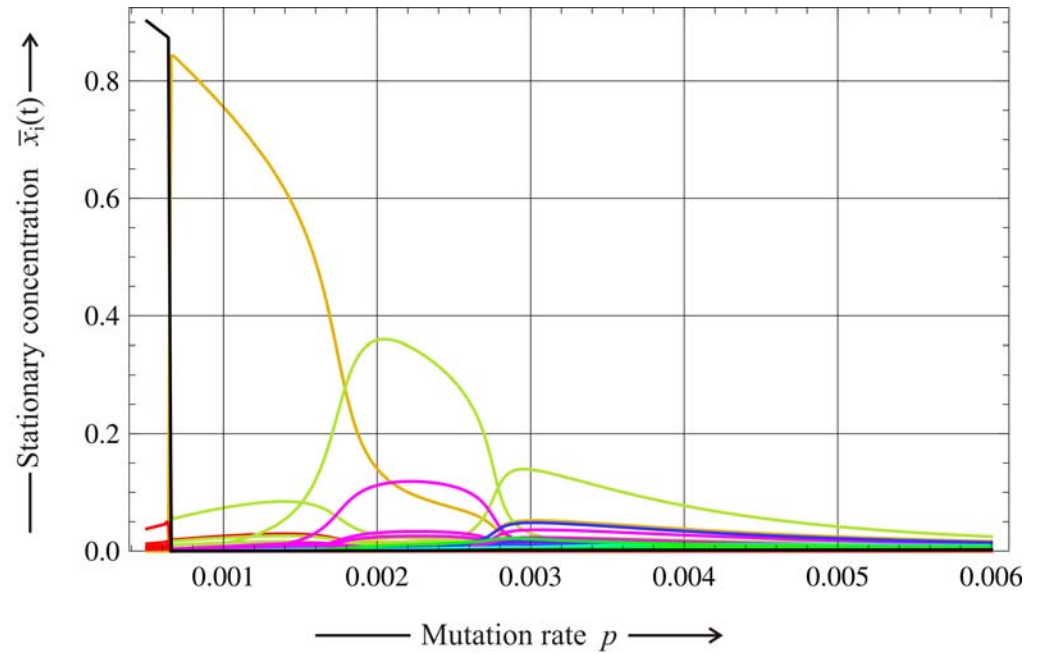
Case I: Strong quasispecies

$n = 10, f_0 = 1.1, f_n = 1.0, s = 919$

$d = 0.995$



$d = 1.0$



Case III: multiple transitions

$n = 10, f_0 = 1.1, f_n = 1.0, s = 637$

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Motoo Kimuras population genetics of neutral evolution.

Evolutionary rate at the molecular level.
Nature **217**: 624-626, 1955.

The Neutral Theory of Molecular Evolution.
Cambridge University Press. Cambridge,
UK, 1983.

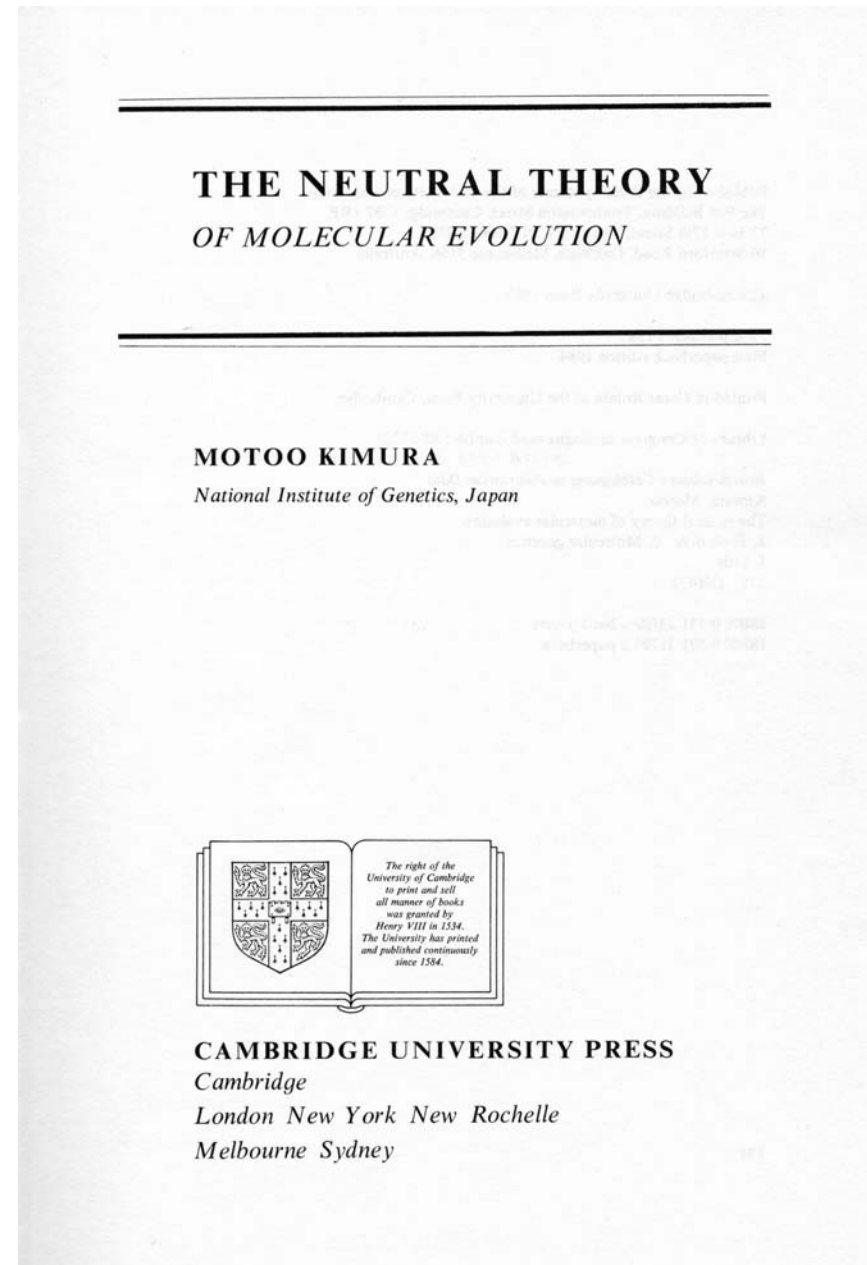
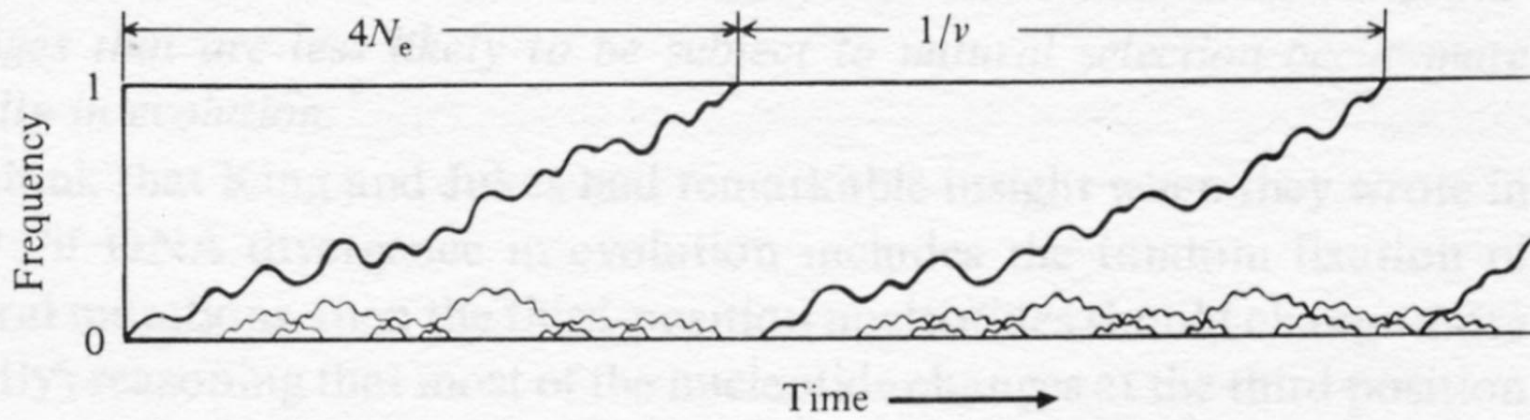


Fig. 3.1. Behavior of mutant genes following their appearance in a finite population. Courses of change in the frequencies of mutants destined to fixation are depicted by thick paths. N_e stands for the effective population size and v is the mutation rate.



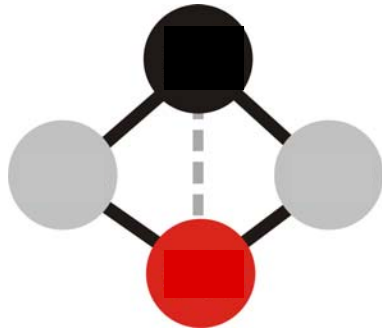
Motoo Kimura

Is the Kimura scenario correct for frequent mutations?



$$d_H = 1$$

$$\lim_{p \rightarrow 0} x_1(p) = x_2(p) = 0.5$$



$$d_H = 2$$

$$\lim_{p \rightarrow 0} x_1(p) = a$$

$$\lim_{p \rightarrow 0} x_2(p) = 1 - a$$

$$d_H \geq 3$$

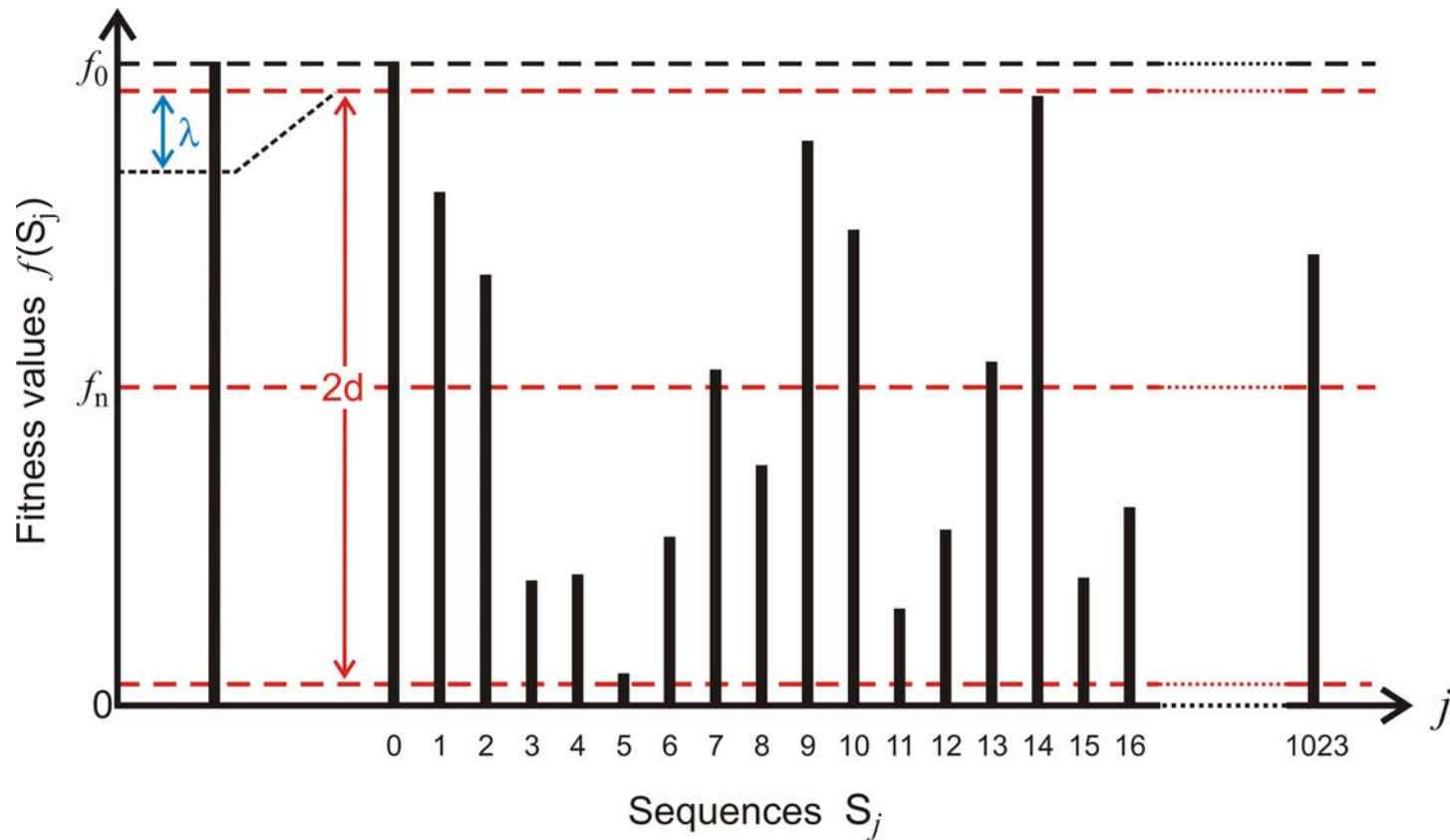
$$\lim_{p \rightarrow 0} x_1(p) = 1, \lim_{p \rightarrow 0} x_2(p) = 0 \text{ or}$$

$$\lim_{p \rightarrow 0} x_1(p) = 0, \lim_{p \rightarrow 0} x_2(p) = 1$$

Pairs of neutral sequences in replication networks

Random fixation in the sense of Motoo Kimura

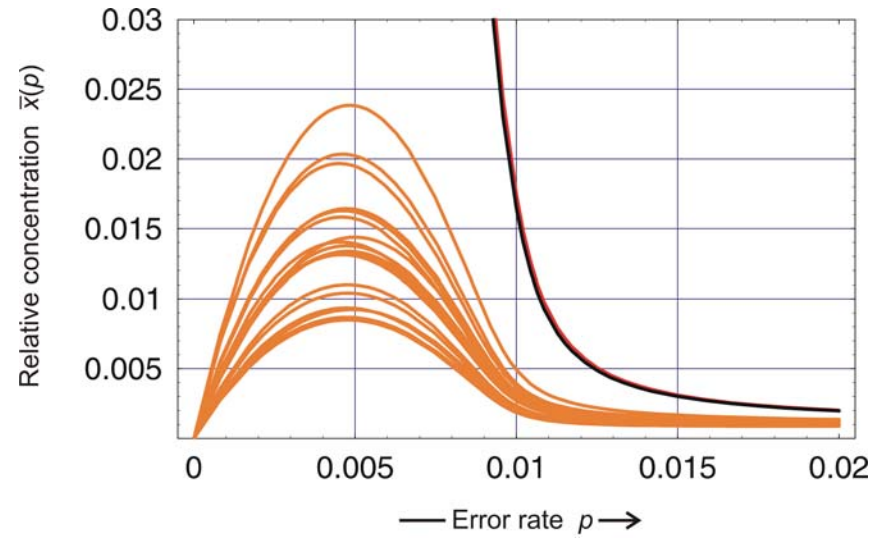
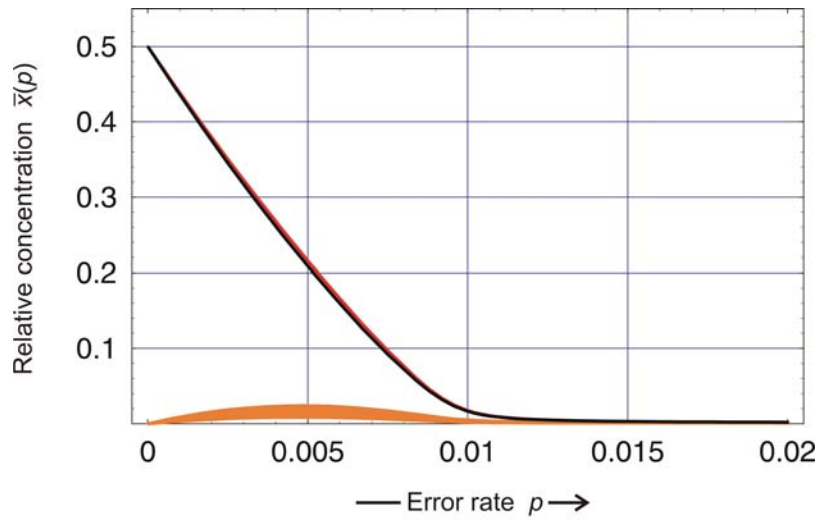
P. Schuster, J. Swetina. 1988. Bull. Math. Biol. 50:635-650



$$f(S_j) = \begin{cases} f_0 & \text{if } \eta_j(s) \geq 1 - \lambda, \\ f_n + \frac{2d}{1-\lambda} (f_0 - f_n) (\eta_j^{(s)} - 0.5) & \text{if } \eta_j(s) \leq 1 - \lambda, \end{cases}$$

$j = 1, 2, \dots, N; j \neq m; \eta \dots$ random number; $s \dots$ seeds

A fitness landscape including neutrality

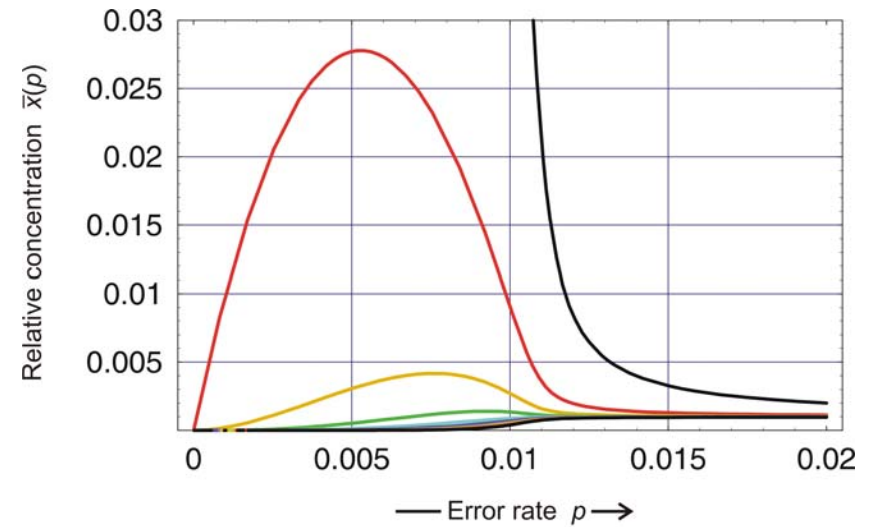


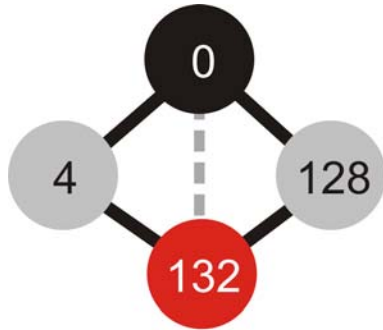
Neutral network

$\lambda = 0.01, s = 367$

Neutral network: Individual sequences

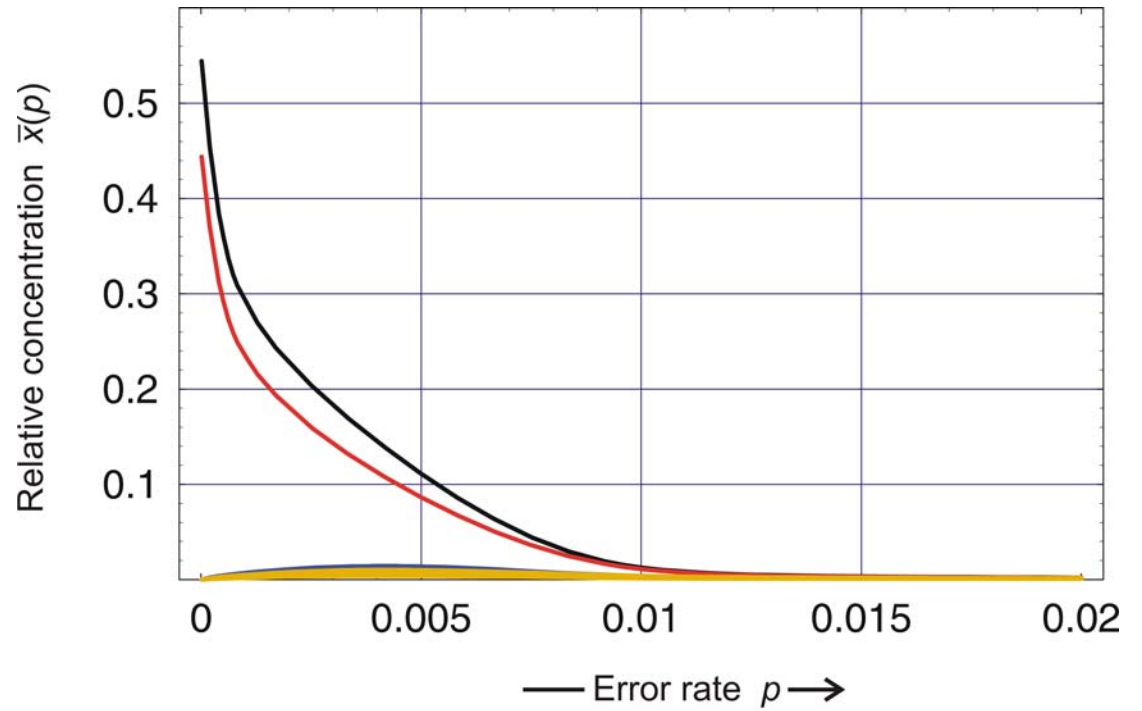
$n = 10, \sigma = 1.1, d = 0.5$





Neutral network

$\lambda = 0.01, s = 877$



Neutral network: Individual sequences

$n = 10, \sigma = 1.1, d = 0.5$

..... ACAUGCGAA
 AUAUACGAA
 ACAUGCGCA
 GCAUACGAA
 ACAUGC UAA
 ACAUGC GAG
 ACACGCGAA
 ACGUACGAA
 ACAUAGGAA
 ACAUACGAA

master sequence 1



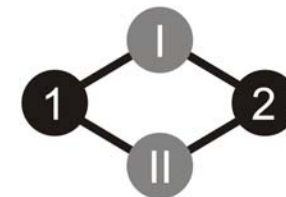
master sequence 2

..... ACAU^G_ACGAA

consensus sequence

..... ACAGUCAGAA
 ACAGUC CGAA
 AUAAUCCGAA
 ACAGUCAGCA
 GCAGUCAGAA
 ACAGUCAUAA
 ACAGUCAGAG
 ACAACCCGAA
 ACGGUCAGAA
 ACAGUGAGAA
 ACAAUUCAGAA
 ACAAUCCGAA

master sequence 1
intermediate I

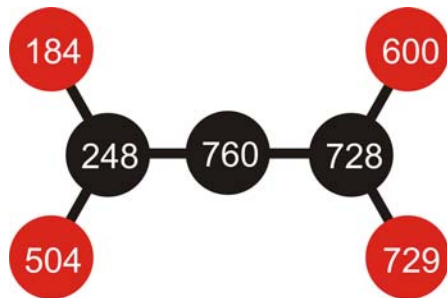
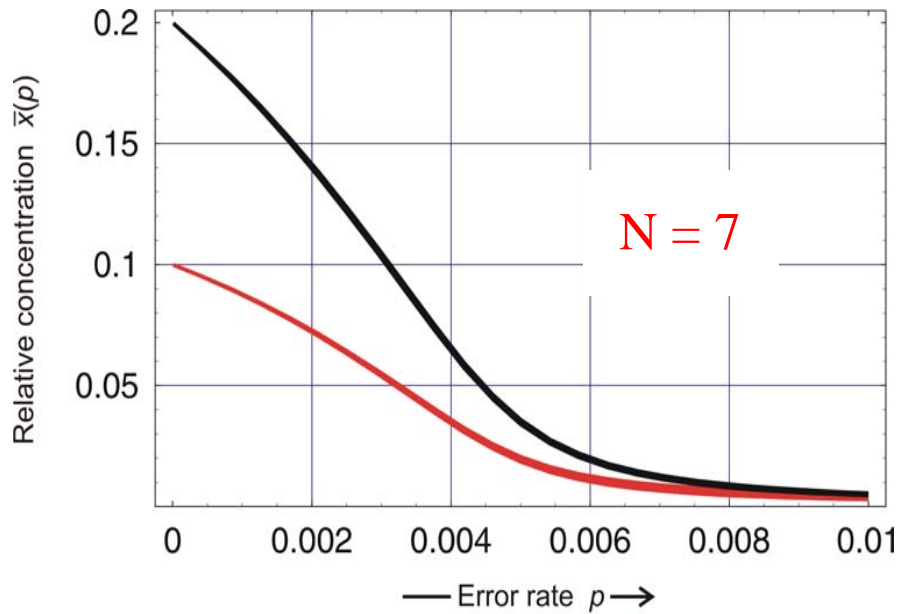


intermediate II
master sequence 2

..... ACA^G_AUC^A_CGAA

consensus sequence

Consensus sequence of a
 quasispecies with strongly
 coupled sequences of
 Hamming distance
 $d_H(X_i, X_j) = 1$ and 2.



Neutral network

$$\lambda = 0.10, s = 229$$

Selection-mutation matrix W

$$W = \begin{pmatrix} f & O(\varepsilon^2) & \varepsilon & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) \\ O(\varepsilon^2) & f & \varepsilon & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) \\ \varepsilon & \varepsilon & f & \varepsilon & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) \\ O(\varepsilon^2) & O(\varepsilon^2) & \varepsilon & f & \varepsilon & O(\varepsilon^2) & O(\varepsilon^2) \\ O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & \varepsilon & f & \varepsilon & \varepsilon \\ O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & \varepsilon & f & O(\varepsilon^2) \\ O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & O(\varepsilon^2) & \varepsilon & O(\varepsilon^2) & f \end{pmatrix}$$

Adjacency matrix A

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Eigenvalues of W and A

$$\begin{aligned} \lambda_0 &= f + 2\varepsilon, & \lambda_0 &= 2, \\ \lambda_1 &= f + \sqrt{2}\varepsilon, & \lambda_1 &= \sqrt{2}, \\ \lambda_{2,3,4} &= f, & \lambda_{2,3,4} &= 0, \\ \lambda_5 &= f - \sqrt{2}\varepsilon, & \lambda_5 &= -\sqrt{2}, \\ \lambda_6 &= f - 2\varepsilon, & \lambda_6 &= -2. \end{aligned}$$

Largest eigenvector of W and A

$$\xi_0 = (0.1, 0.1, 0.2, 0.2, 0.2, 0.1, 0.1).$$

Computation of sequences in the core of a neutral network

1. Darwin and evolutionary optimization
2. Evolution as an exercise in chemical kinetics
3. Sequences and structures in biopolymers
4. Evolution on simple model landscapes
5. Evolution on realistic landscapes
6. Neutrality in evolution
7. **Perspectives**

- (i) Fitness landscapes for the evolution of molecules are obtainable by standard techniques of physics and chemistry.
- (ii) Fitness landscapes for evolution of viroids and viruses under controlled conditions are accessible in principle.
- (iii) Systems biology can be carried out for especially small bacteria and an extension to bacteria of normal size is to be expected for the near future.
- (iv) The computational approach for selection on known fitness landscapes - ODEs or stochastic processes - is standard.
- (v) The efficient description of migration and splitting of populations in sequence space requires new mathematical techniques.

Consideration of multistep and nonlinear replication mechanisms as well as accounting for epigenetic phenomena is readily possible within the molecular approach.

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Acknowledgement of support

Fonds zur Förderung der wissenschaftlichen Forschung (FWF)
Projects No. 09942, 10578, 11065, 13093
13887, and 14898

Wiener Wissenschafts-, Forschungs- und Technologiefonds (WWTF)
Project No. Mat05

Jubiläumsfonds der Österreichischen Nationalbank
Project No. Nat-7813

European Commission: Contracts No. 98-0189, 12835 (NEST)

Austrian Genome Research Program – GEN-AU

Siemens AG, Austria

Universität Wien and the Santa Fe Institute



Universität Wien

Thank you for your attention !

Web-Page for further information:

<http://www.tbi.univie.ac.at/~pks>

