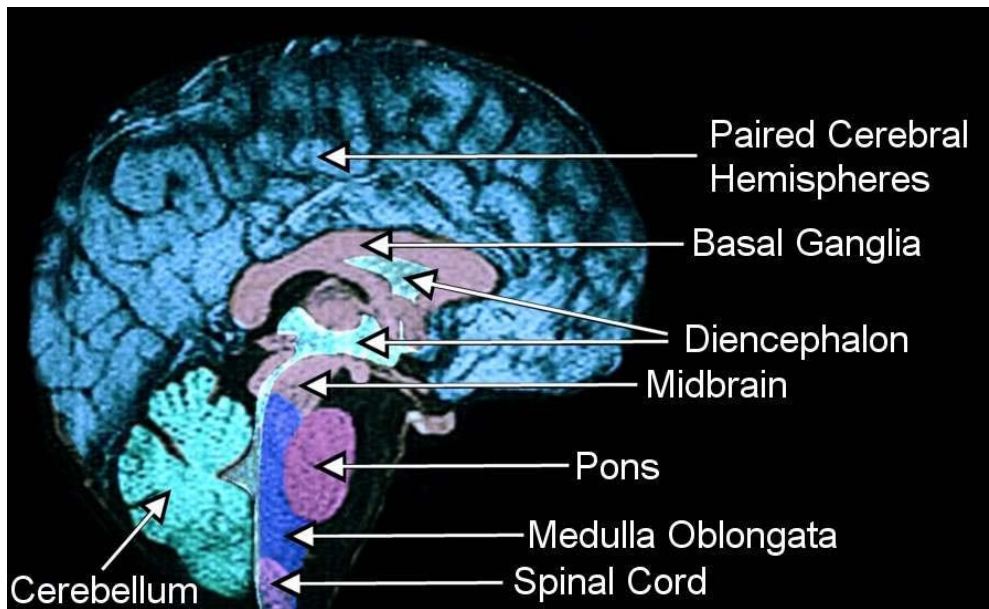
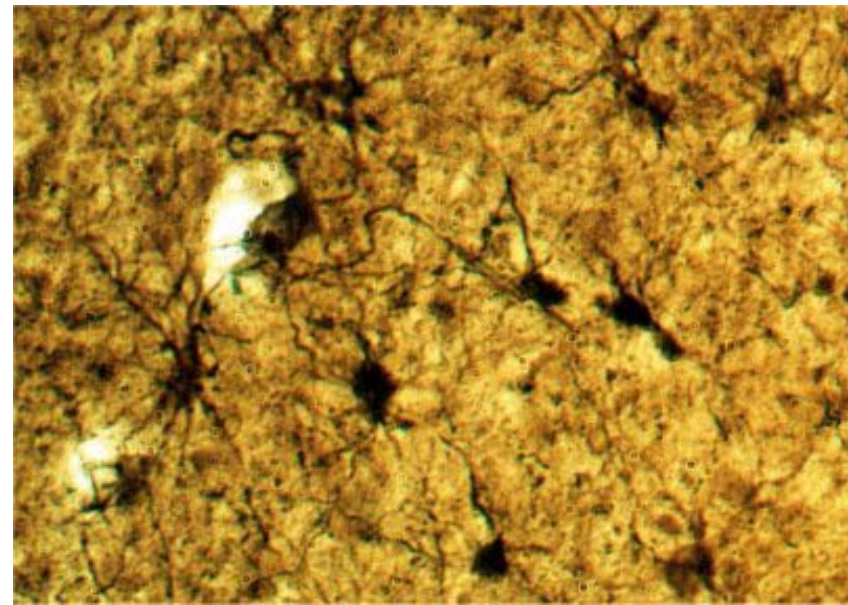






# Neurobiology

Neural networks, collective properties, nonlinear dynamics, signalling, ...



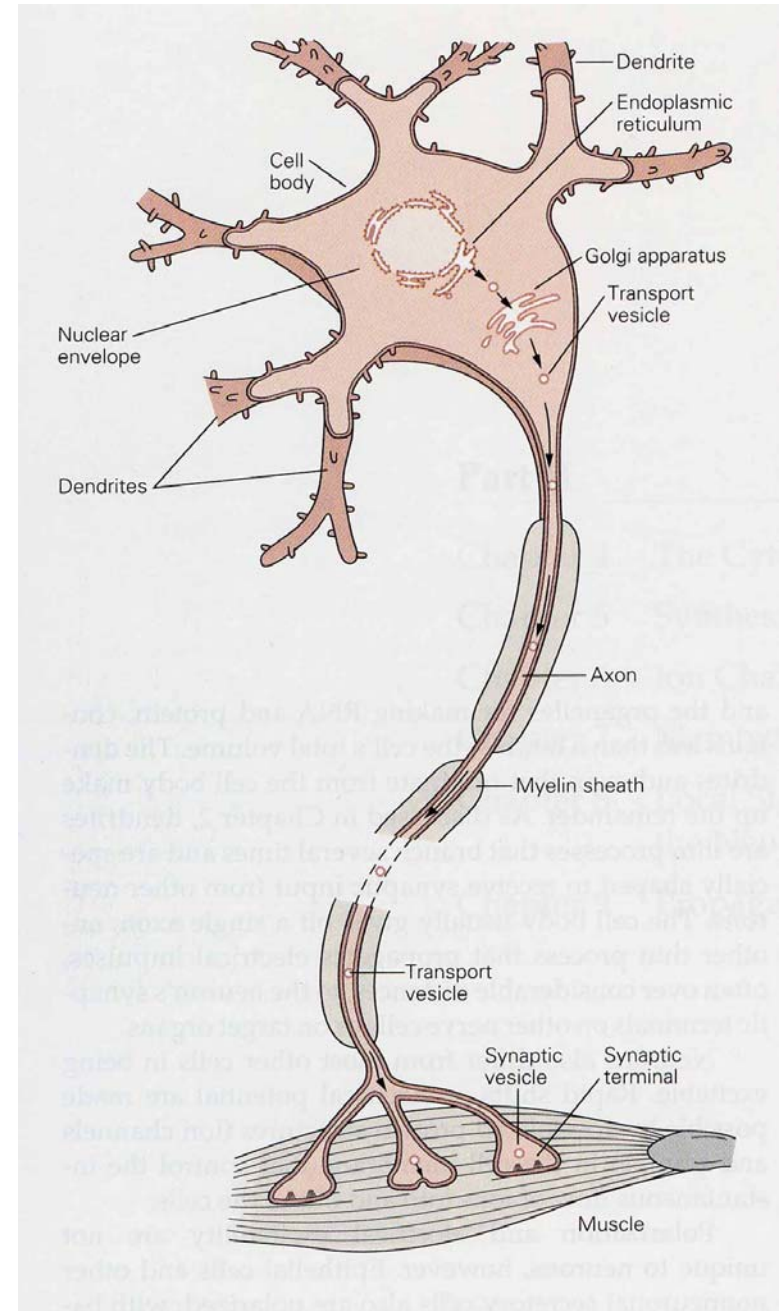
The human brain

$10^{11}$  neurons connected by  $10^{13}$  to  $10^{14}$  synapses

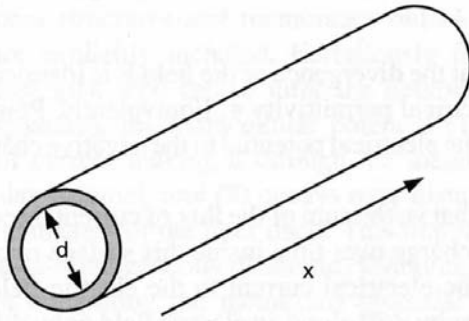


# Neurobiology

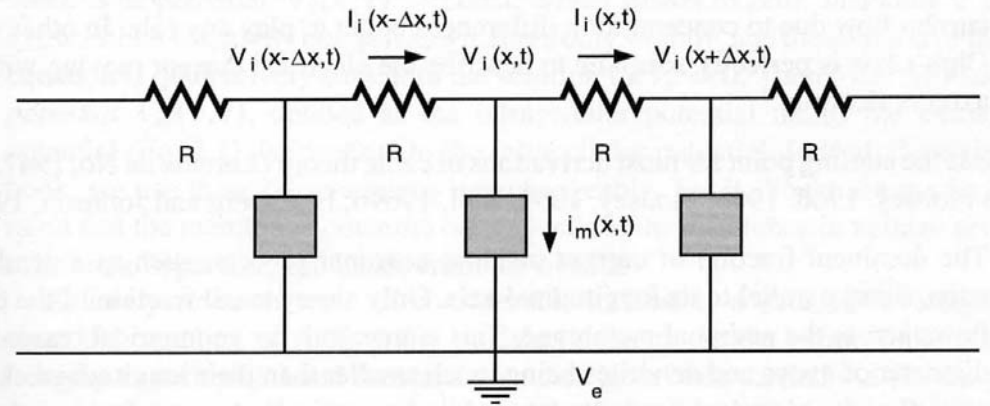
Neural networks, collective properties, nonlinear dynamics, signalling, ...



A single neuron signaling to a muscle fiber

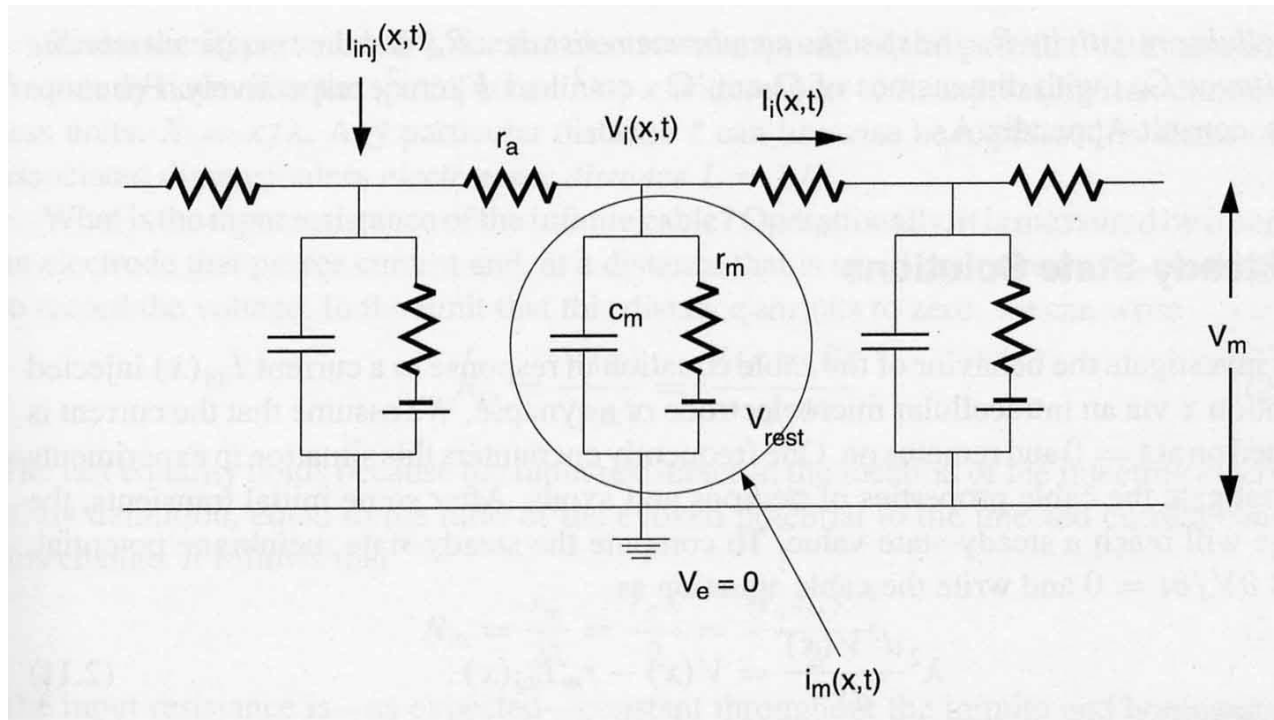


**A**

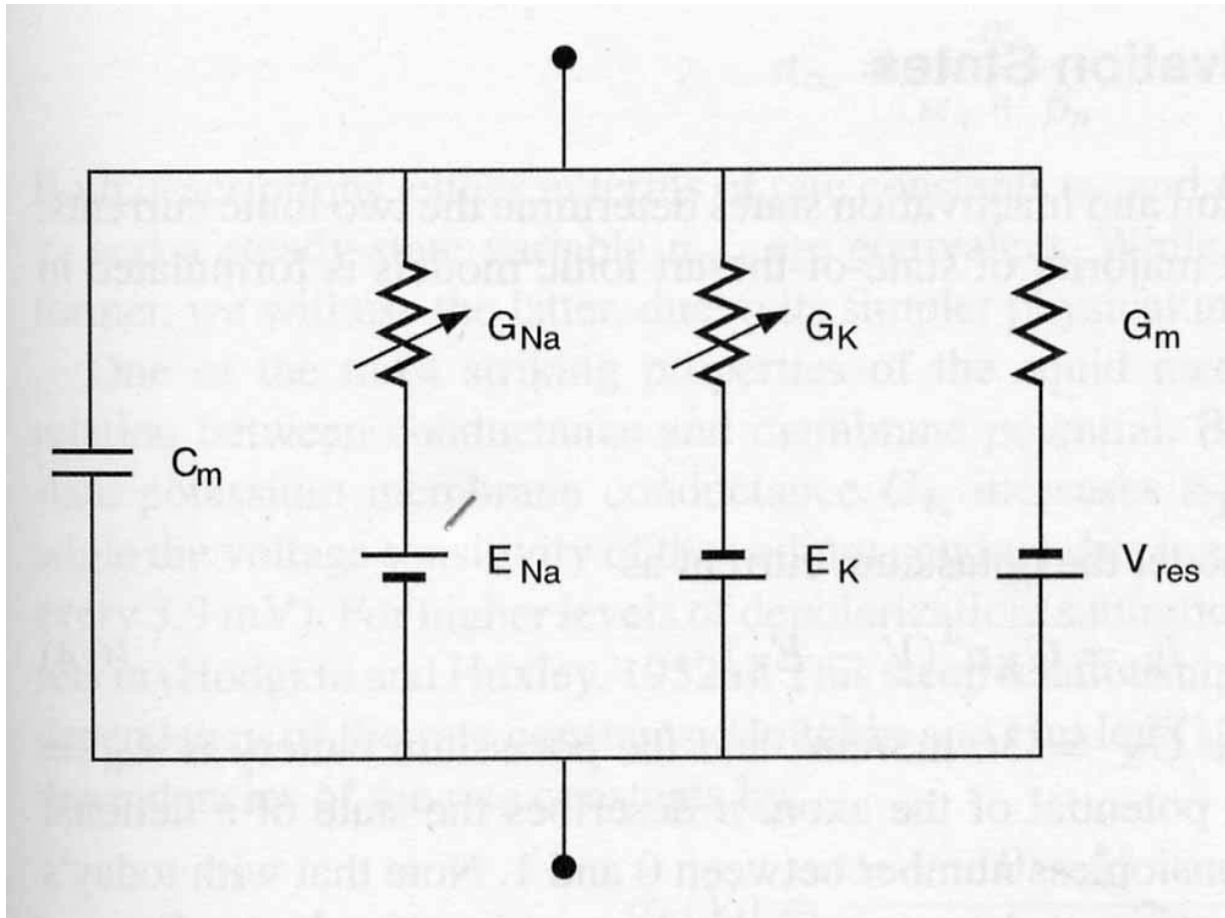


**B**

**Fig. 2.2 ELECTRICAL STRUCTURE OF A CABLE** (A) Idealized cylindrical axon or dendrite at the heart of one-dimensional cable theory. Almost all of the current inside the cylinder is longitudinal due to geometrical (the radius is much smaller than the length of the cable) and electrical factors (the membrane covering the axon or dendrite possesses a very high resistivity compared to the intracellular cytoplasm). As a consequence, the radial and angular components of the current can be neglected, and the problem of determining the potential in these structures can be reduced from three spatial dimensions to a single one. On the basis of the bidomain approximation, gradients in the extracellular potentials are neglected and the cable problem is expressed in terms of the transmembrane potential  $V_m(x, t) = V_i(x, t) - V_e$ . (B) Equivalent electrical structure of an arbitrary neuronal process. The intracellular cytoplasm is modeled by the purely ohmic resistance  $R$ . This tacitly assumes that movement of carriers is exclusively due to drift along the voltage gradient and not to diffusion. Here and in the following the extracellular resistance is assumed to be negligible and  $V_e$  is set to zero. The current per unit length across the membrane, whether it is passive or contains voltage-dependent elements, is described by  $i_m$  and the system is characterized by the second-order differential equation, Eq. 2.5.



**Fig. 2.3 A SINGLE PASSIVE CABLE** Equivalent lumped electrical circuit of an elongated neuronal fiber with passive membrane. The intracellular cytoplasm is described by an ohmic resistance per unit length  $r_a$  and the membrane by a capacitance  $c_m$  in parallel with a passive membrane resistance  $r_m$  and a battery  $V_{rest}$ . The latter two components are frequently referred to as *leak resistance* and *leak battery*. An external current  $I_{inj}(x, t)$  is injected into the cable. The associated linear cable equation (Eq. 2.7) describes the dynamics of the electrical potential  $V_m = V_i - V_e$  along the cable.



**Fig. 6.2 ELECTRICAL CIRCUIT FOR A PATCH OF SQUID AXON**  
 Hodgkin and Huxley modeled the membrane of the squid axon using four parallel branches: two passive ones (membrane capacitance  $C_m$  and the leak conductance  $G_m = 1/R_m$ ) and two time- and voltage-dependent ones representing the sodium and potassium conductances.

# Neurobiology

Neural networks, collective properties, nonlinear dynamics, signalling, ...

$$\frac{dV}{dt} = \frac{1}{C_M} \left[ I - g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) - g_l (V - V_l) \right]$$

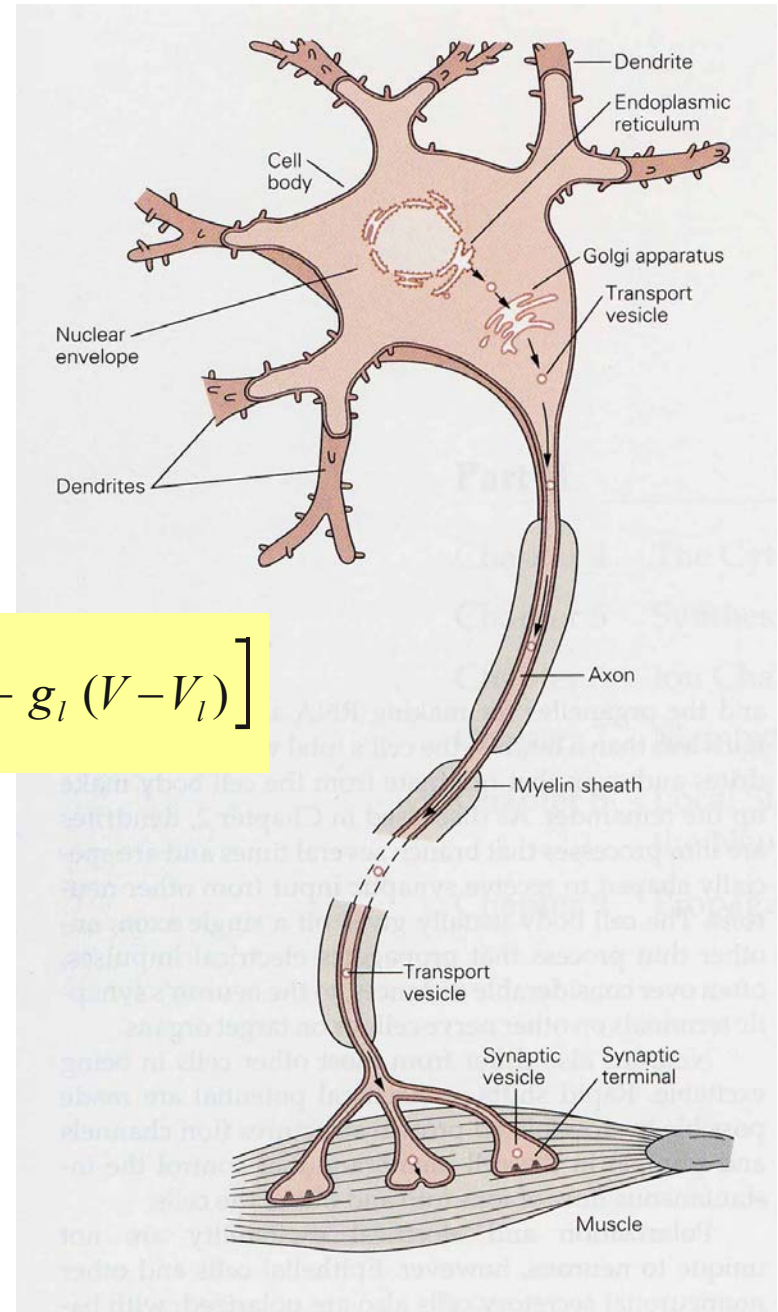
$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h$$

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n$$

Hogdkin-Huxley OD equations

A single neuron signaling to a muscle fiber





$$\frac{dV}{dt} = \frac{1}{C_M} \left[ I - \bar{g}_{Na} m^3 h (V - V_{Na}) - \bar{g}_K n^4 (V - V_K) - \bar{g}_l (V - V_l) \right]$$

$$\frac{dm}{dt} = \alpha_m (1 - m) - \beta_m m$$

$$\frac{dh}{dt} = \alpha_h (1 - h) - \beta_h h$$

$$\frac{dn}{dt} = \alpha_n (1 - n) - \beta_n n$$

Hogdkin-Huxley OD equations



Hhsim.Ink

Simulation of space independent Hodgkin-Huxley equations:

Voltage clamp and constant current

$$\frac{1}{R} \frac{\partial^2 V}{\partial x^2} = C \frac{\partial V}{\partial t} + \left[ g_{Na} m^3 h (V - V_{Na}) + g_K n^4 (V - V_K) + g_l (V - V_l) \right] 2\pi r L$$

$$\frac{\partial m}{\partial t} = \alpha_m (1 - m) - \beta_m m$$

$$\frac{\partial h}{\partial t} = \alpha_h (1 - h) - \beta_h h$$

$$\frac{\partial n}{\partial t} = \alpha_n (1 - n) - \beta_n n$$

Hodgkin-Huxley PDEquations

Travelling pulse solution:  $V(x,t) = V(l)$  with  
 $l = x + e t$

Hodgkin-Huxley equations describing pulse propagation along nerve fibers

$$\frac{1}{R} \frac{d^2 V}{d\xi^2} = C_M \theta \frac{dV}{d\xi} + \left[ \bar{g}_{Na} m^3 h (V - V_{Na}) + \bar{g}_K n^4 (V - V_K) + \bar{g}_l (V - V_l) \right] 2\pi r L$$

$$\theta \frac{dm}{d\xi} = \alpha_m (1 - m) - \beta_m m$$

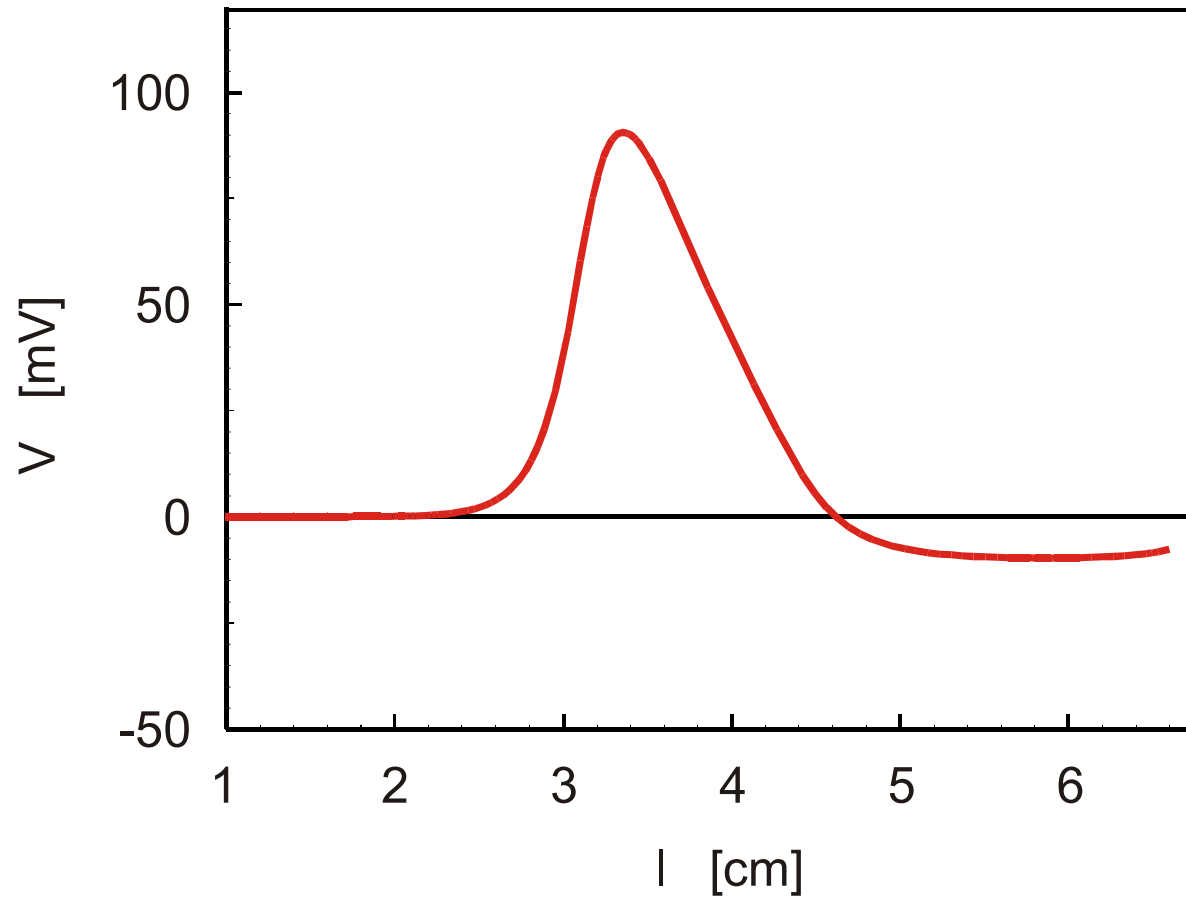
$$\theta \frac{dh}{d\xi} = \alpha_h (1 - h) - \beta_h h$$

$$\theta \frac{dn}{d\xi} = \alpha_n (1 - n) - \beta_n n$$

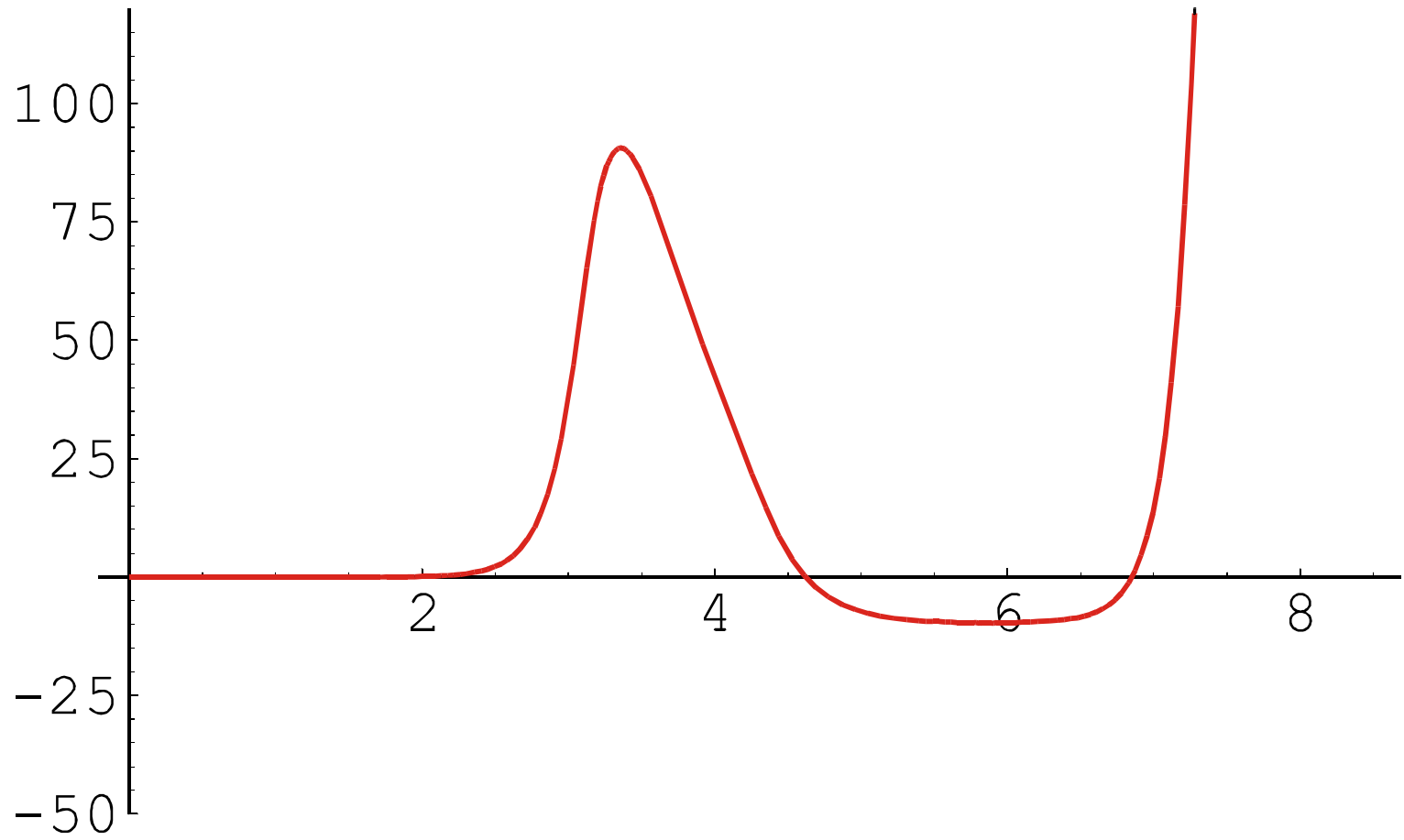
Hodgkin-Huxley PDEquations

Travelling pulse solution:  $V(x,t) = V(l)$  with  
 $l = x + e t$

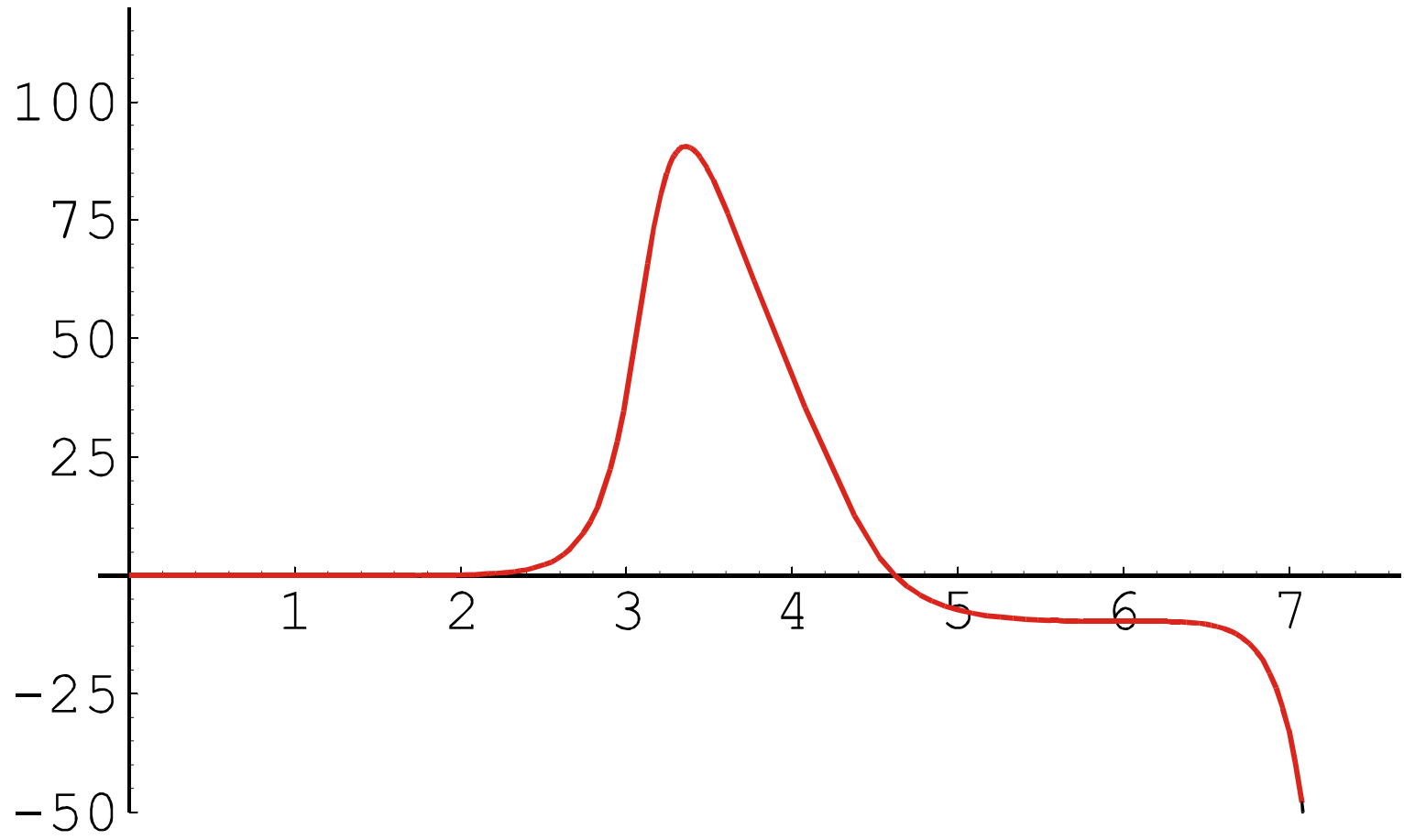
Hodgkin-Huxley equations describing pulse propagation along nerve fibers



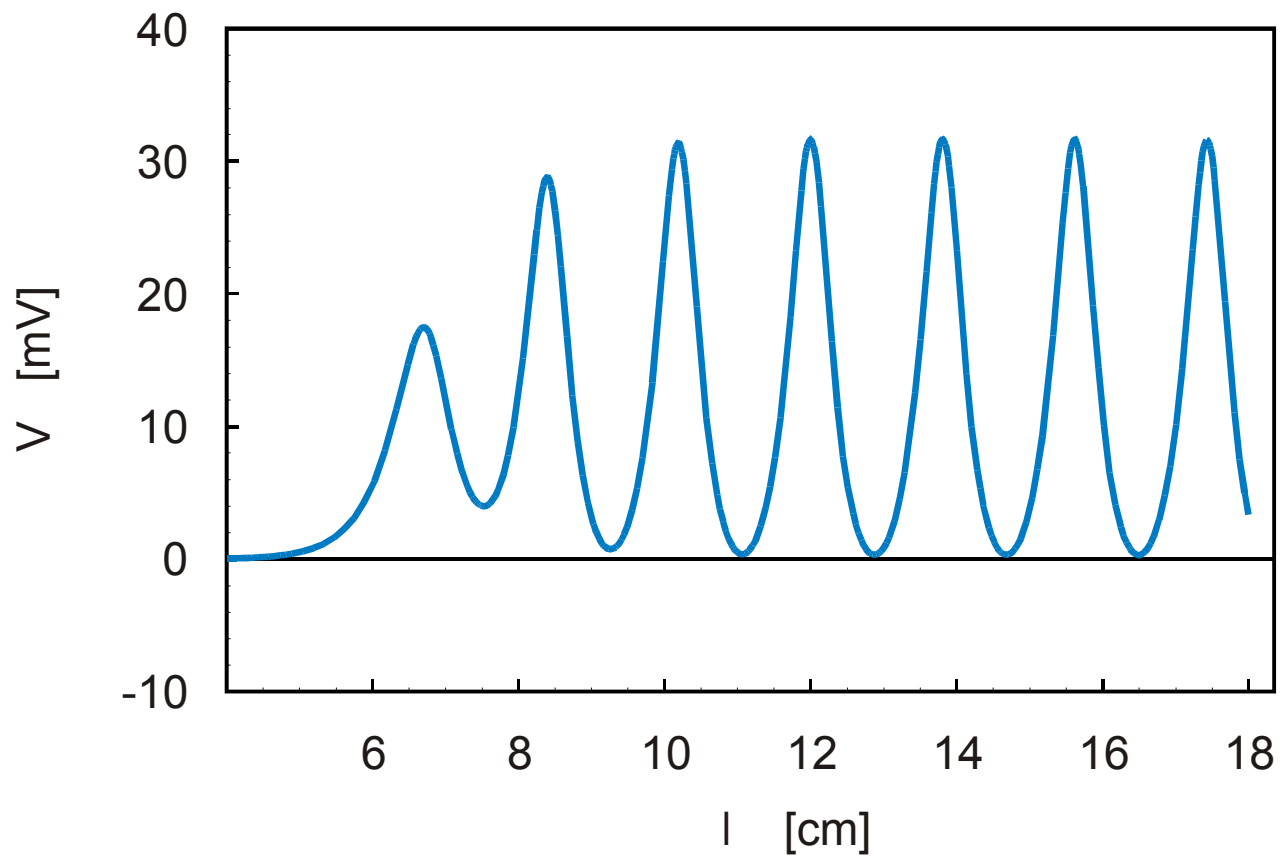
$T = 18.5 \text{ C}; \theta = 1873.33 \text{ cm / sec}$



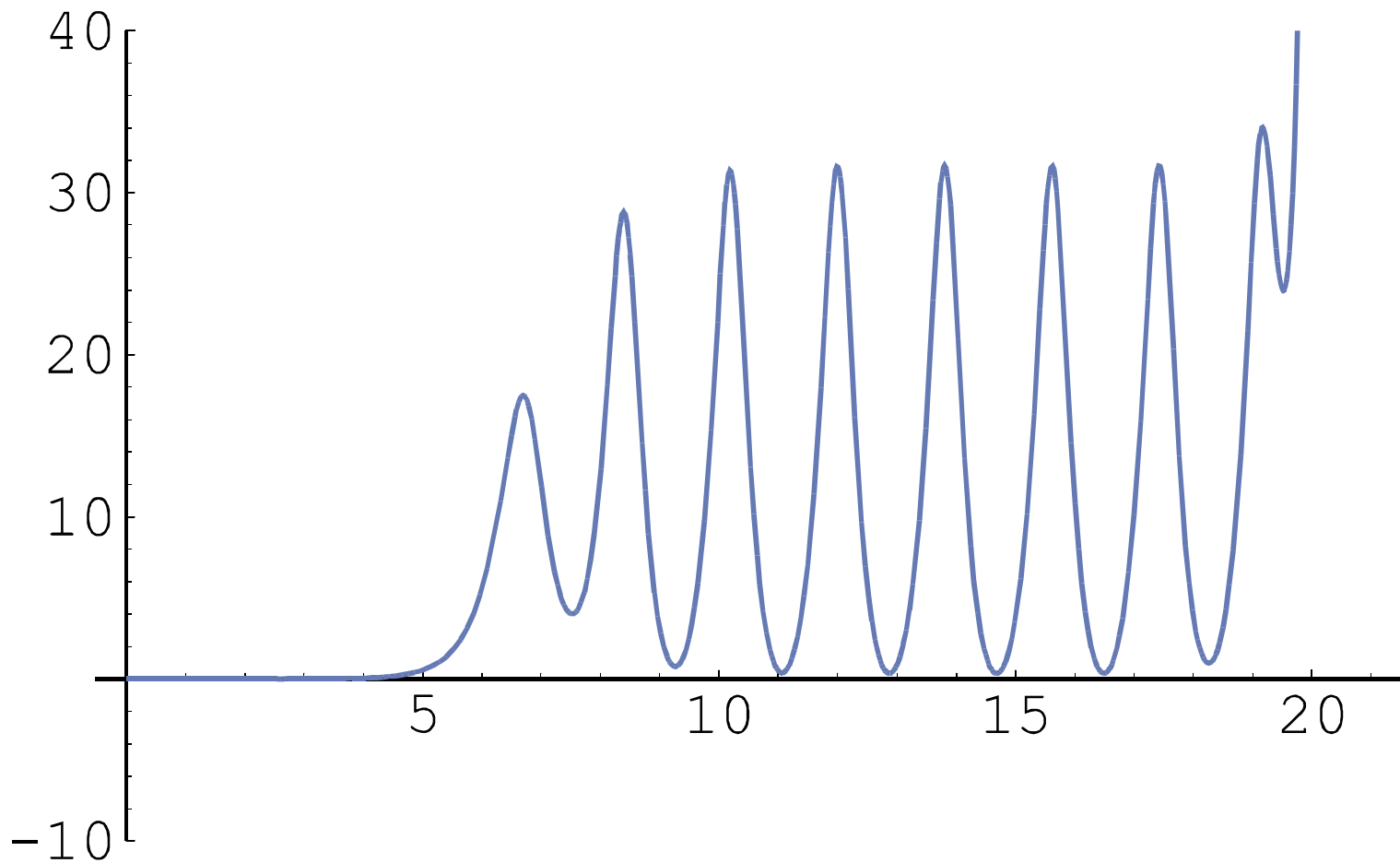
$T = 18.5 \text{ C}; \theta = 1873.3324514717698 \text{ cm / sec}$



$T = 18.5 \text{ C}; \theta = 1873.3324514717697 \text{ cm / sec}$

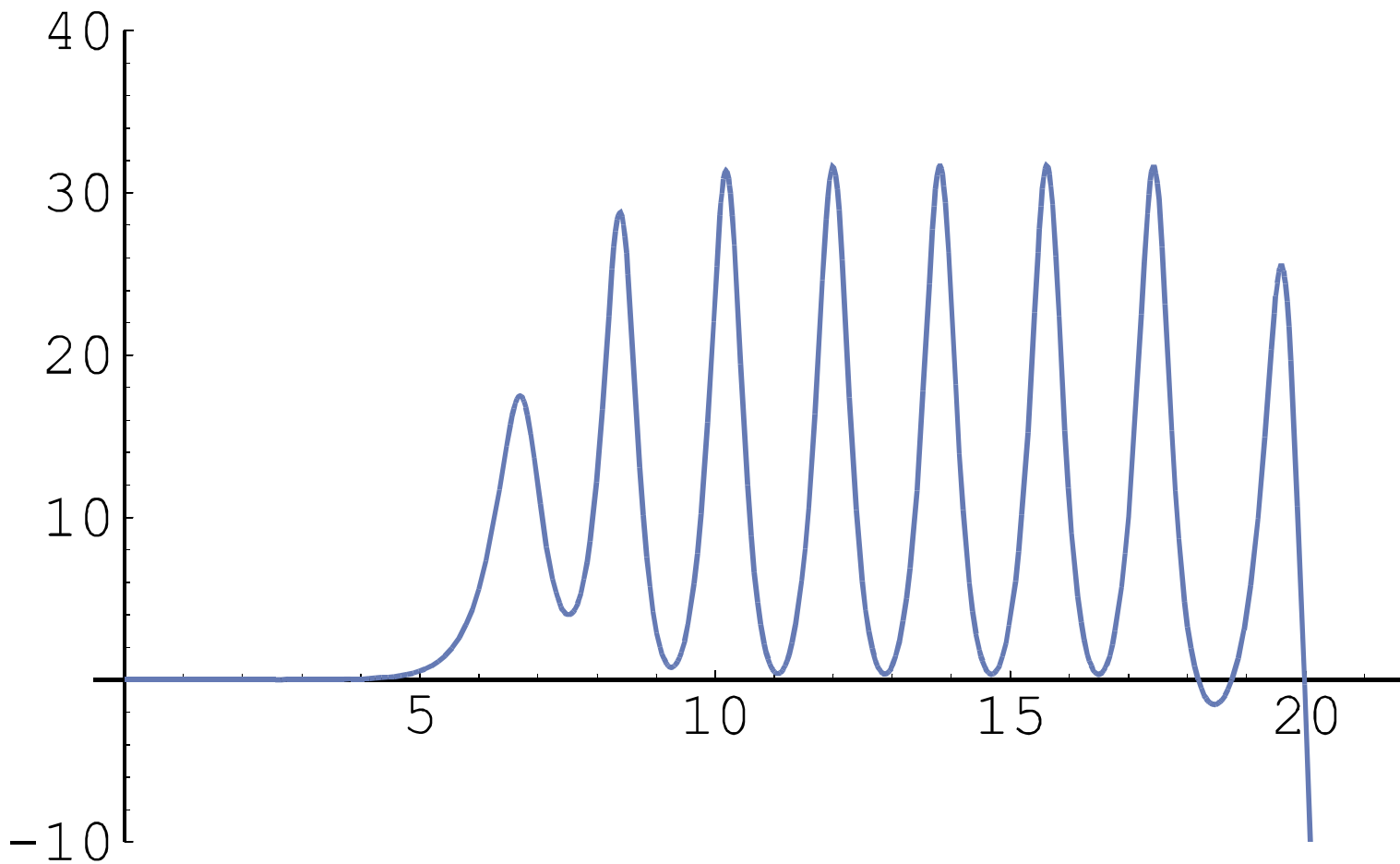


$T = 18.5 \text{ C}; \theta = 544.070 \text{ cm / sec}$

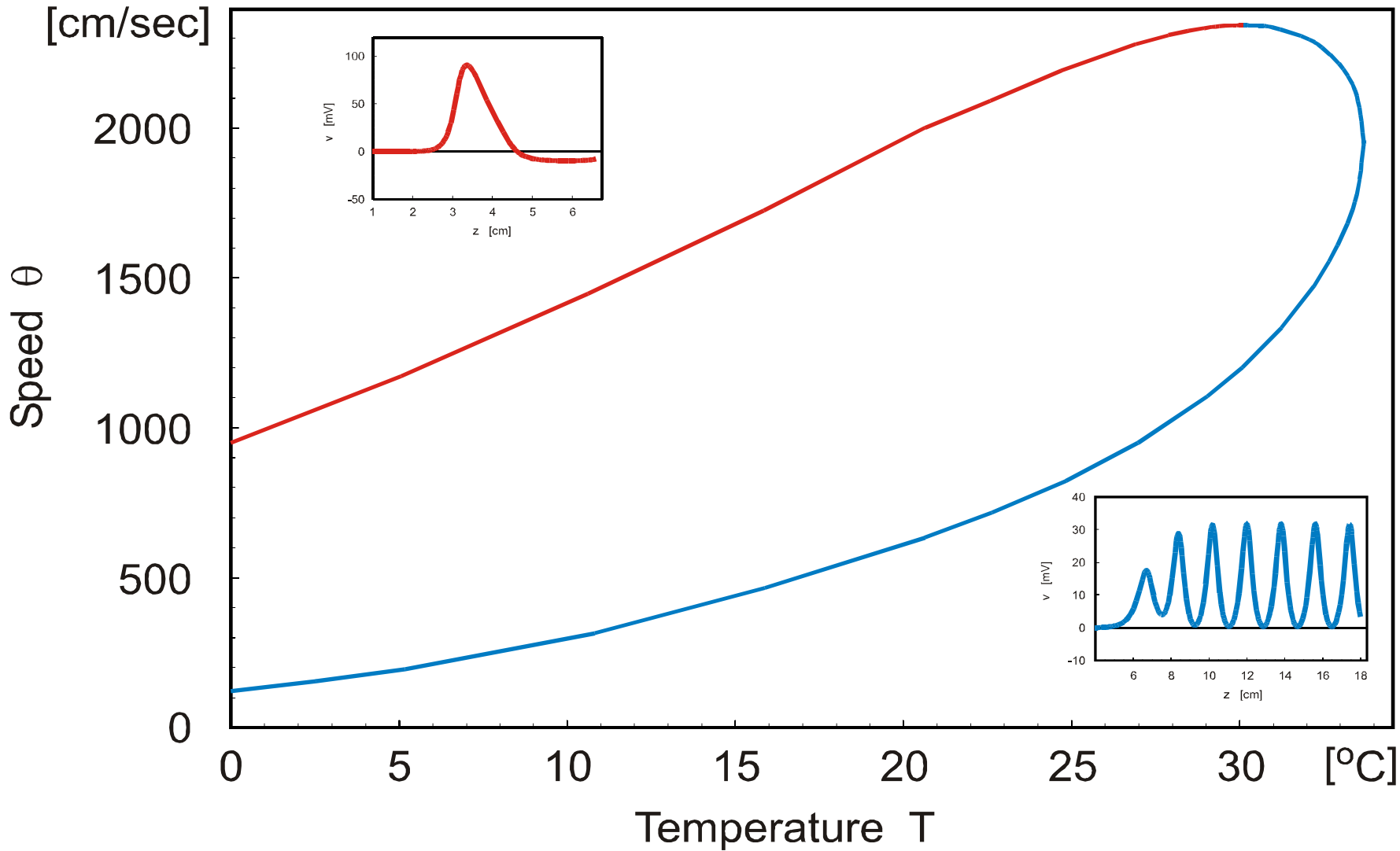


$T = 18.5 \text{ C}; \theta = 554.070286919319 \text{ cm/sec}$





$T = 18.5 \text{ C}; \theta = 554.070286919320 \text{ cm/sec}$



Propagating wave solutions of the Hodgkin-Huxley equations

## FitzHugh-Nagumo Equations

$$\begin{aligned} \frac{\partial V'}{\partial t'} &= -V'(V' - a)(V' - 1) - Y - \frac{\partial^2 V'}{\partial x'^2} \\ \frac{\partial Y'}{\partial t'} &= bV' \text{ all quantities dimensionless} \\ \frac{d}{d\zeta} \left[ \frac{d^2 V'}{d\zeta^2} - c \frac{dV'}{d\zeta} - V'(V' - a)(V' - 1) \right] - \frac{b}{c} V' &= 0, \text{ with } \zeta = x' + ct' \\ c(a, b = 0) &= \frac{1}{\sqrt{2}}(1 - 2a) - \text{Propagation of the Pulse Front} \\ c(a, b) < c(a, 0) &\text{ Front slows down to shape the back of the pulse} \end{aligned}$$

The good news: The pulse, stationary in the frame of  $\zeta$ , is now described by an ordinary third order differential equation. The bad news is that the result is a structurally unstable homoclinic orbit, which is why calculations take a bit of care. We have found an analytic expression for  $c(a, b)$ .

Connection to Hodgkin-Huxley:  $[h, n]$  are relatively slow variables, so keep them at resting values. This results in a contracted two-dimensional Hodgkin-Huxley  $[V, m]$  system which describes the pulse front at the full speed  $\theta$  to very close approximation. The two descriptions are now equivalent under the transformations

$$\begin{aligned} D &= \frac{r}{2R_2 C_m} \left[ 336 \frac{\text{cm}^2}{\text{sec}} \right] \quad \Gamma = \sqrt{\frac{G_{\text{Na}} h(0)}{C_m D}} \left[ 14.5876 \text{ cm}^{-1} \right] \\ V' &= \frac{V}{E_{\text{Na}}}, E_{\text{Na}} = 115 \text{ mv}, \quad \zeta = \Gamma z, \quad c = \frac{\theta}{\Gamma D} \left[ \Gamma D = 4903 \frac{\text{cm}}{\text{sec}} \right] \end{aligned}$$

Advantage: Provides a bridge between between detailed Hodgkin-Huxley based conductance models and formal spiking models which dispense with such details [a pulse is regarded as a delta-function-like spike].

Web-Page for further information:

<http://www.tbi.univie.ac.at/~pks>

