

# **Aligning Circularly Ordered Lists**

**Why would anyone want to do that?**

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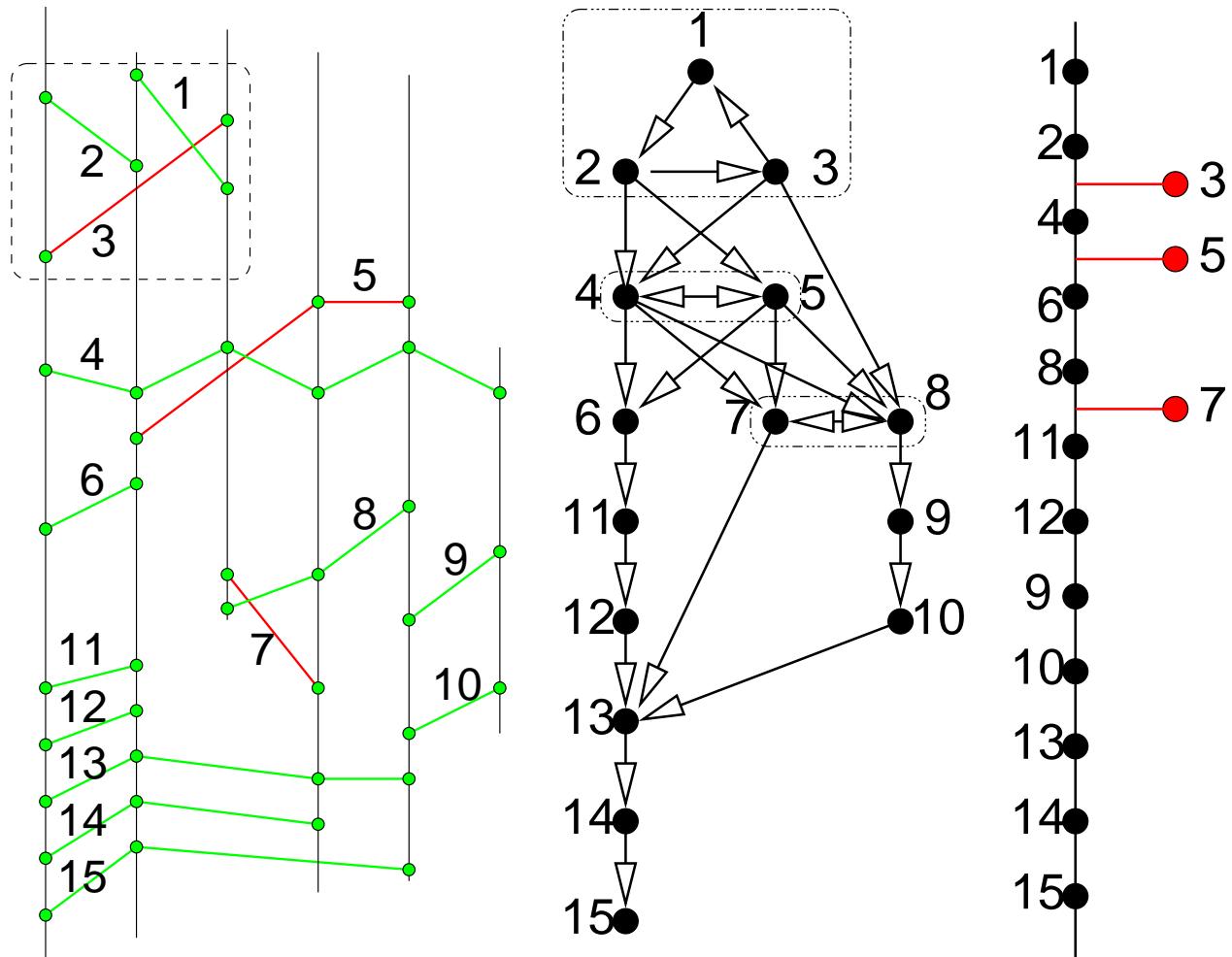
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## The Footprint Sorting Problem

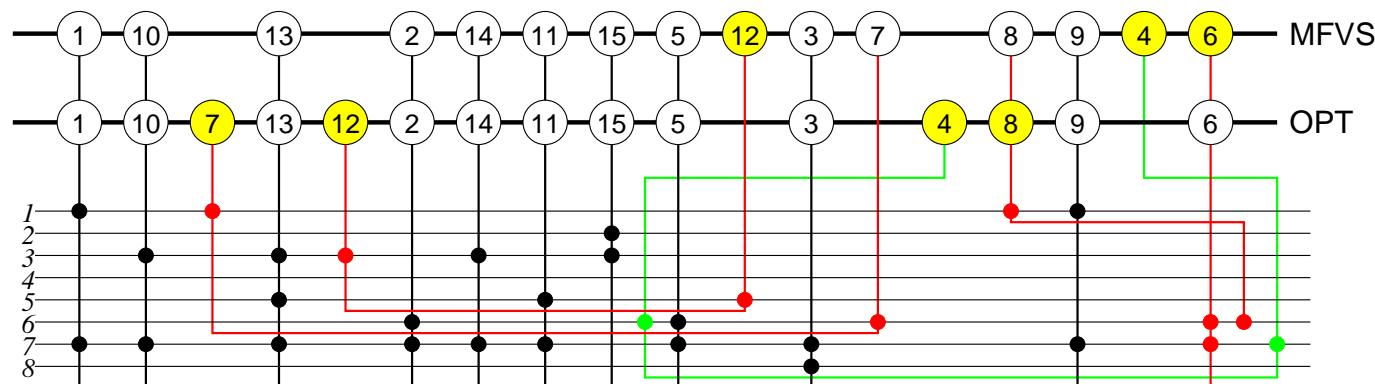


Linearly ordered list of objects (footprints in this case).

No unique solution to sorting problem  
 $\Rightarrow$  different heuristic approximations.

Comparison of linearly ordered lists  $\pi'$  and  $\pi''$ :  
 Insertion, deletion, exact match as only edit operations:  
 $\Rightarrow$  Simple Alignment Problem

$$D_{ij} = \min \begin{cases} D_{i,j-1} + 1 \\ D_{i-1,j} + 1 \\ D_{i-1,j-1} & \text{whenever } \pi'(i) = \pi''(j) \end{cases}$$



## The Circular Case: Mitochondrial Genomes

Arrangement of 13 protein coding genes, 2 rRNAs and 22tRNAs

```
> NC_000834.cgi Branchiostoma floridae
C01 -S2 D C02 K ATP8 ATP6 C03 ND3 R ND4L ND4 H S L1 ND5 G -ND6
-E CYTB T -P 12S F V 16S L2 ND1 I M -Q ND2 -N W -A -C -Y
```

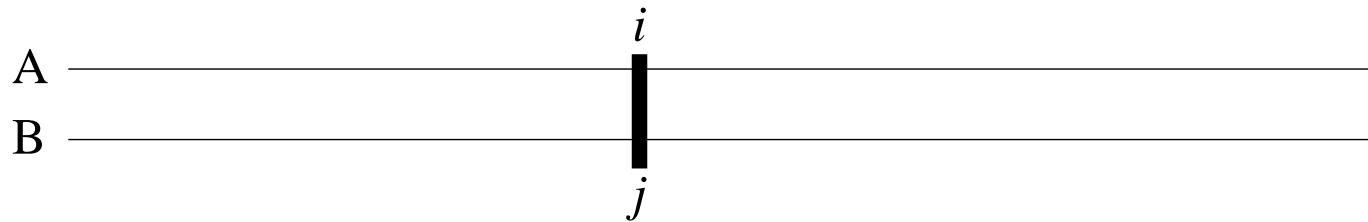
Traditional approaches:

- Break-point distances
- “Sorting by reversals”

$\xi = \pi' \circ \pi''^{-1}$  and compute a *length function* of  $\xi$ , i.e. number of reversals necessary to convert  $\xi$  to the identity permutation

## Circular Alignments

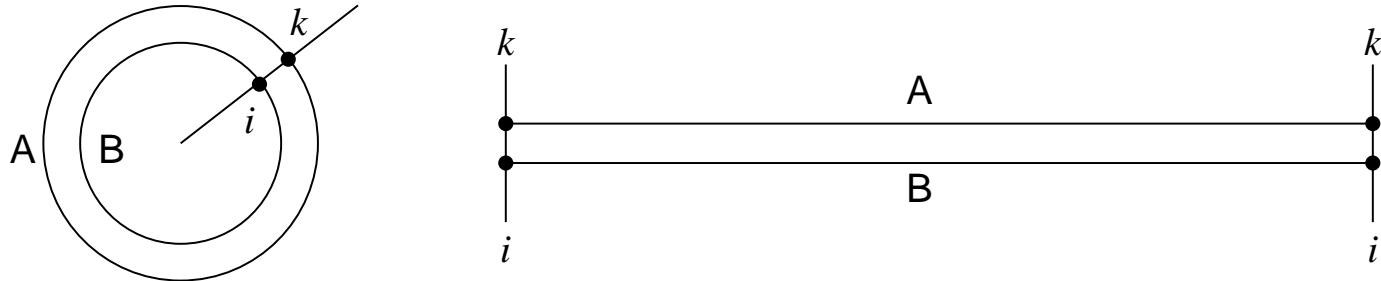
Basic Property of Alignments:



Match  $(i, j)$  separates alignment into two independent parts.

If  $(i, j)$  is part of the optimal alignment, then the two parts can be optimized independently.

## Computing Circular Alignments



Rotate sequence  $A$  such that it starts with  $i + 1$ :  $\rightarrow A'$

Rotate sequence  $B$  such that it starts with  $k + 1$ :  $\rightarrow B'$

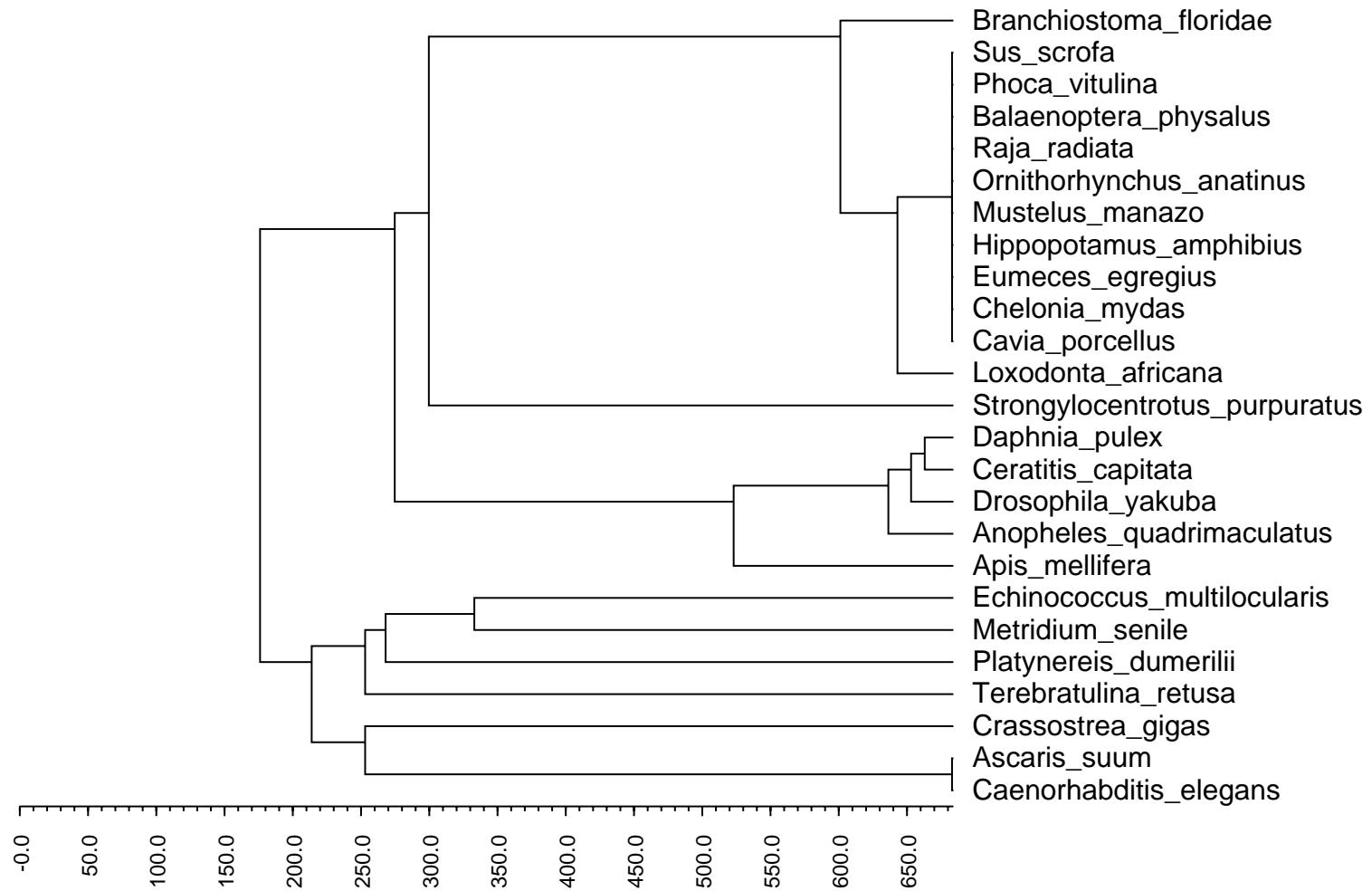
Compute Alignment of  $A'$  and  $B'$  with the restriction that (a) initial gaps have full costs and (b) the last entries (i.e.,  $i$  and  $k$ ) match.

Polynomial algorithm that runs in  $n^2 \times \text{Alignment}(A', B')$  time with  $\mathcal{O}(n^2)$  memory.

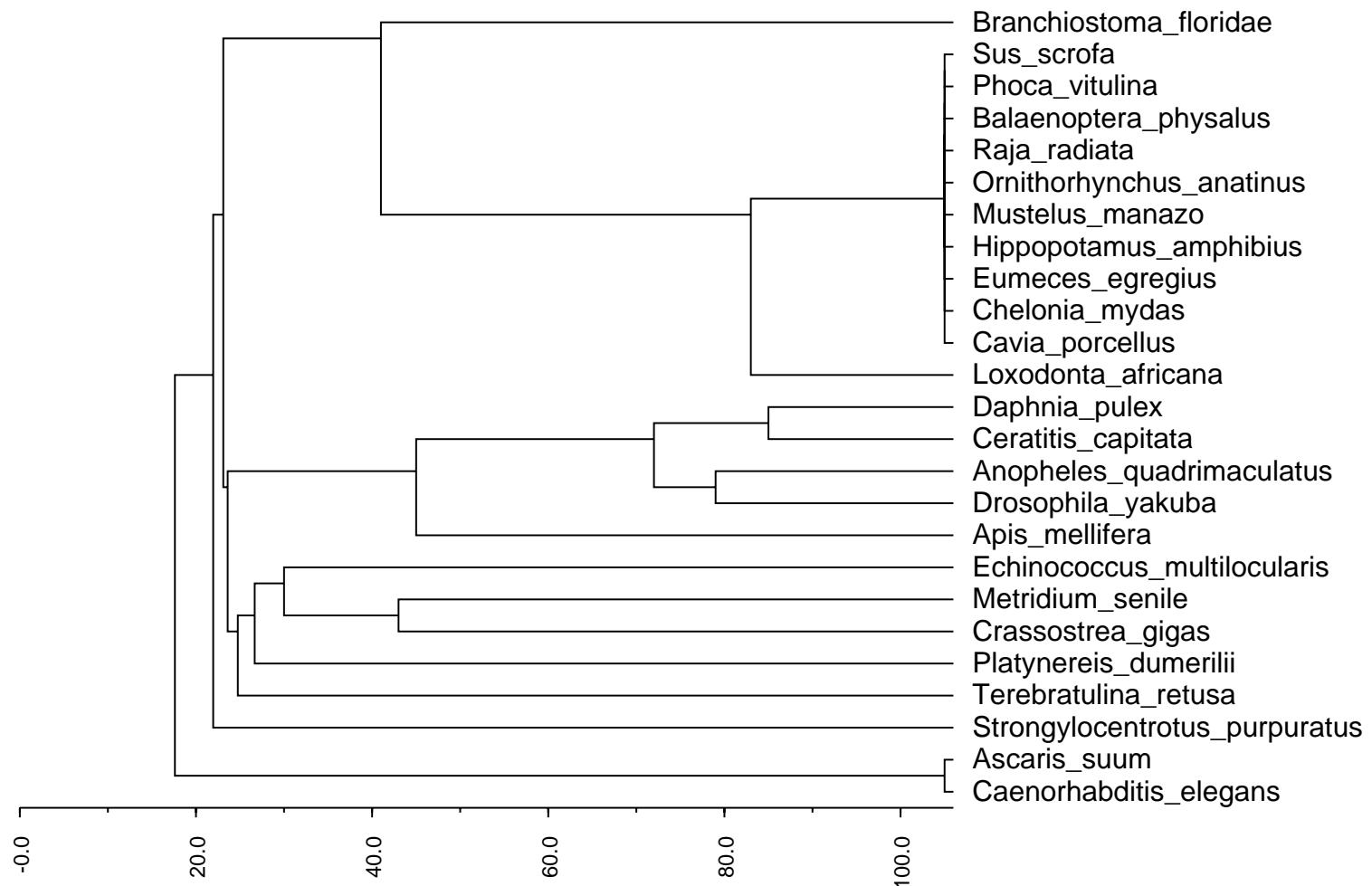
## Resulting Alignments:

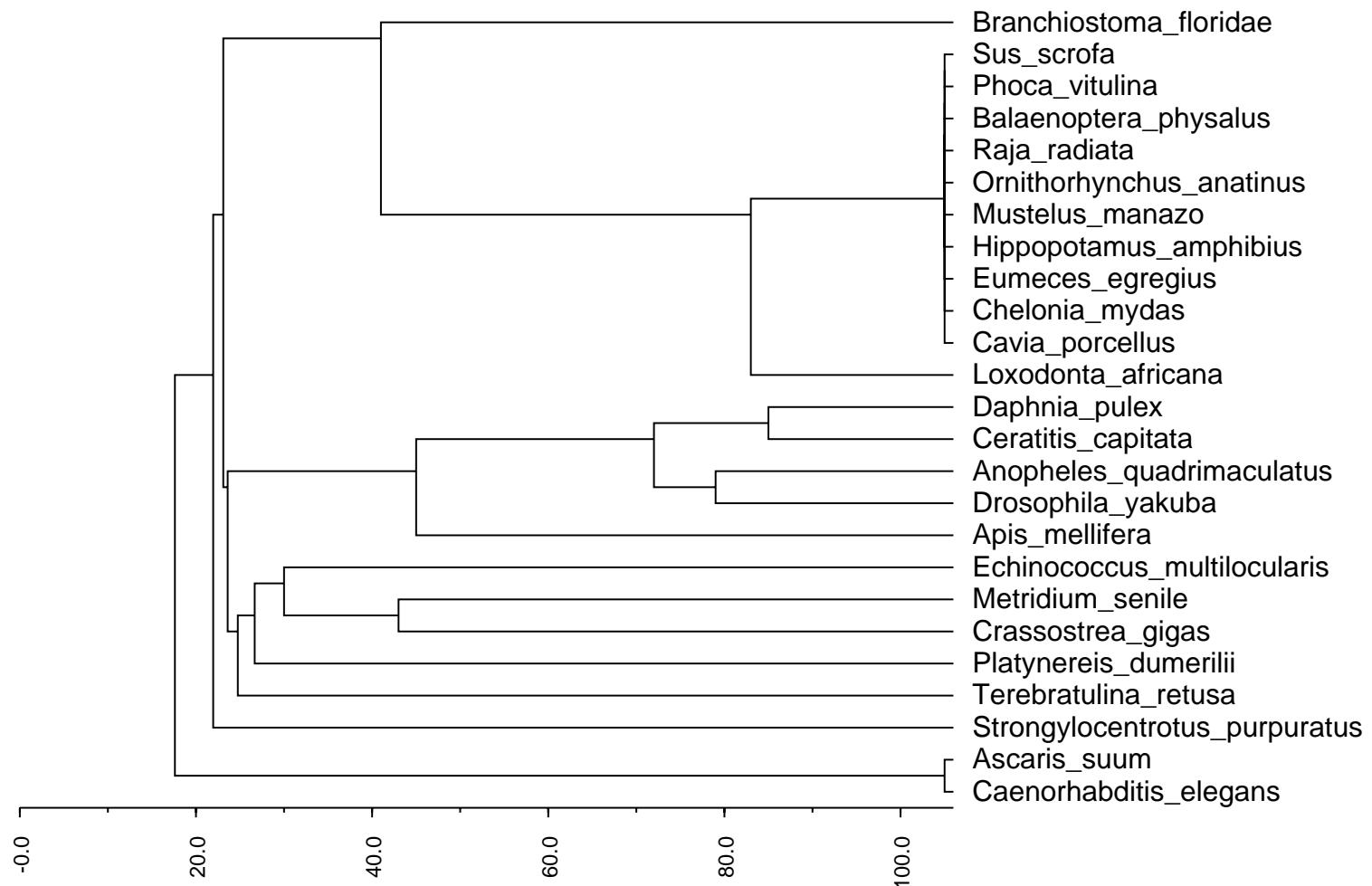
'NC_000834'	< <i>Branchiostoma_floridae</i>	>	111011011101011111111000000000011101111000000
'NC_000845'	< <i>Sus_scrofa</i>	>	11101101111011111011100000000011011111000000
'NC_000884'	< <i>Cavia_porcellus</i>	>	11101101111011111011100000000011011111000000
'NC_000886'	< <i>Chelonia_mydas</i>	>	11101101111011111011100000000011011111000000
'NC_000888'	< <i>Eumeces_egregius</i>	>	11101101111011111011100000000011011111000000
'NC_000889'	< <i>Hippopotamus_amphibius</i>	>	11101101111011111011100000000011011111000000
'NC_000890'	< <i>Mustelus_manazo</i>	>	11101101111011111011100000000011011111000000
'NC_000844'	< <i>Daphnia_pulex</i>	>	10011111111100000000000110111111000000011101
'NC_000857'	< <i>Ceratitis_capitata</i>	>	100111111111000000000001101111110000000110111
'NC_000875'	< <i>Anopheles_quadrimaculatus</i>	>	100111111110100000000001101111111000000011101

(For simplicity: representing presence/absence of genes only)



gap open = 10, gap extend = 10





gap open = 10, gap extend = 1

## Outlook

tRNAs and proteins “move” with different frequencies:  
⇒ more sophisticated scoring model: Gap costs depends on contents and length

## Acknowledgements

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Wim Hordijk (Canterbury, NZ)

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Guido Fritzsch, Martin Schlegel