

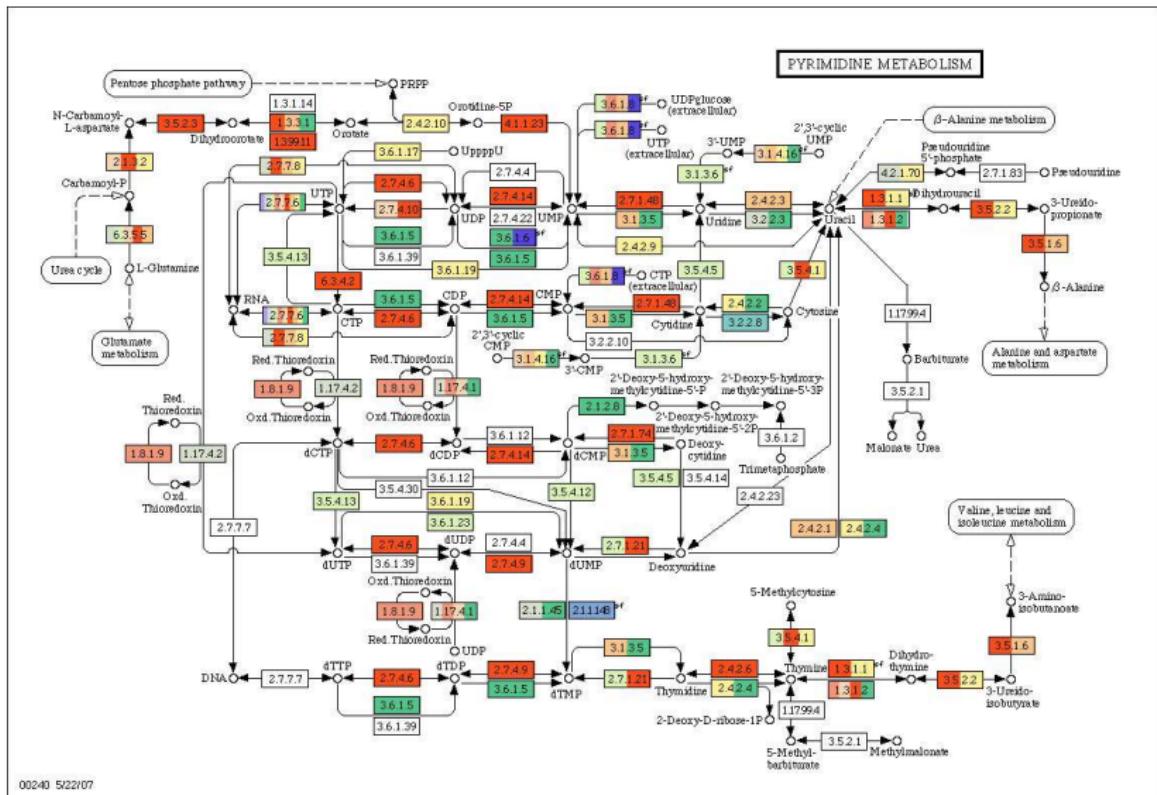
# Robustness and Modularity in Metabolic Networks

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Bioinformatics  
University of Leipzig

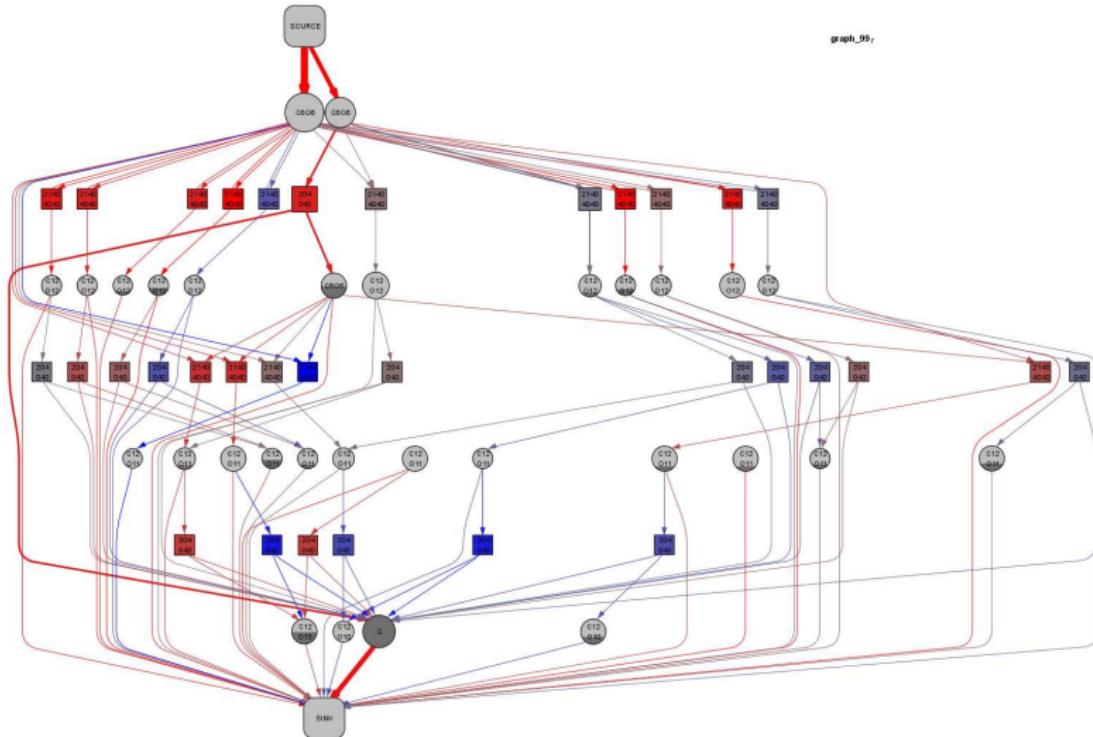
Bled, February 17

# Metabolic Networks



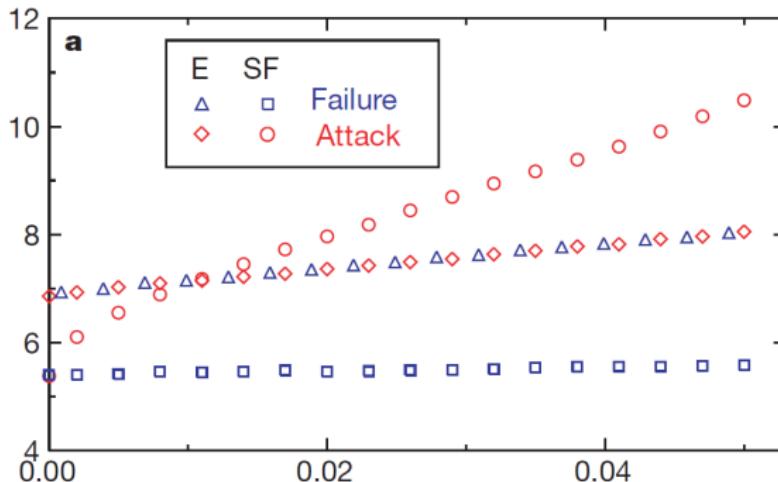
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# Metabolic Networks



# Robustness

- Ability to function despite changes
- Genetic Changes: Mutations,...
- Epigenetic Changes: Fluctuations in Molecule concentrations
- Complex Systems are highly robust
- Scale-free Networks are particularly robust



# Modularity

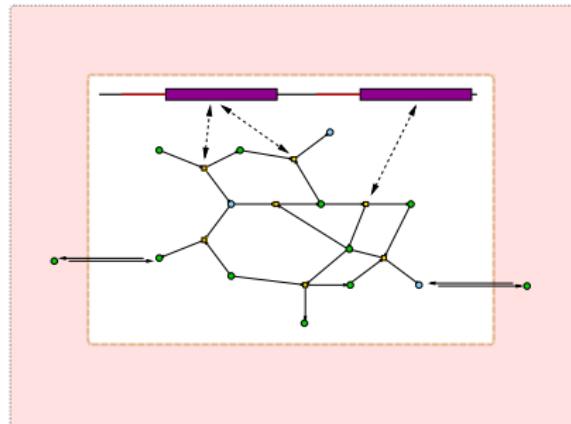
- Module: (structural) subsystem with distinct function
- Key organizing principle of biological networks
- High Clustering Coefficient suggests Modularity
- However, Origin and Preservation of Modularity not understood
- Changing Goals or Environments

# Motivation

- Biological systems develop desired properties
  - Robustness, Flexibility, Modularity, Evolvability, ...
  - Properties are connected
- Well studied, but their emergence is less well understood
  - Investigate the evolution of metabolic networks
  - Analyse network structure and metabolic functions
- Answers beyond analyzing real-world data

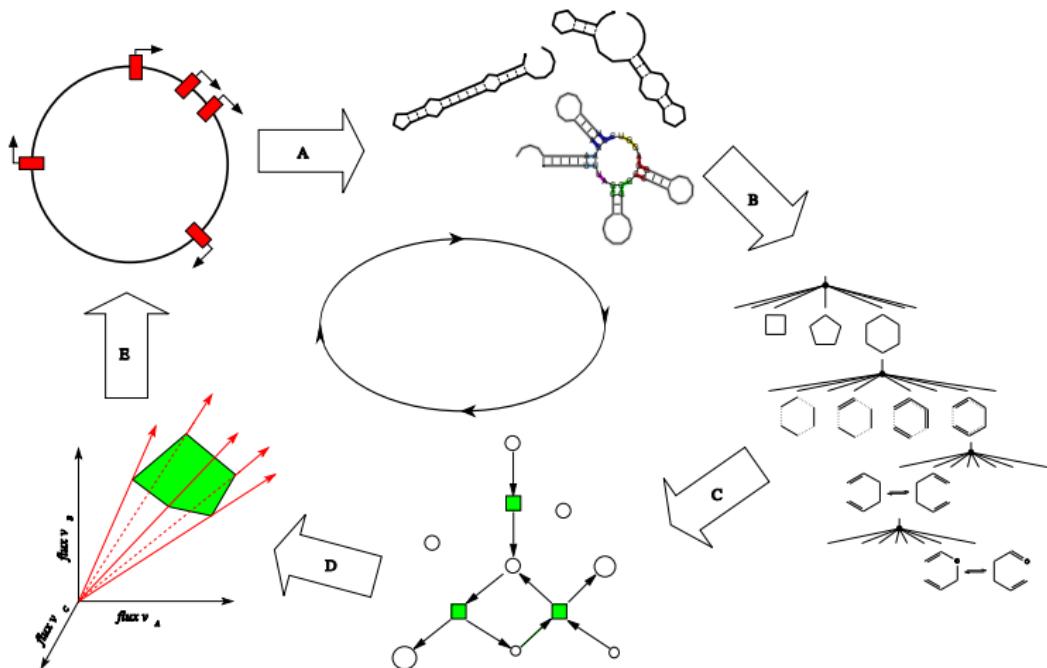
→ a multi-scale computational model of early metabolism  
→ appropriate measures for network properties

# Simulation

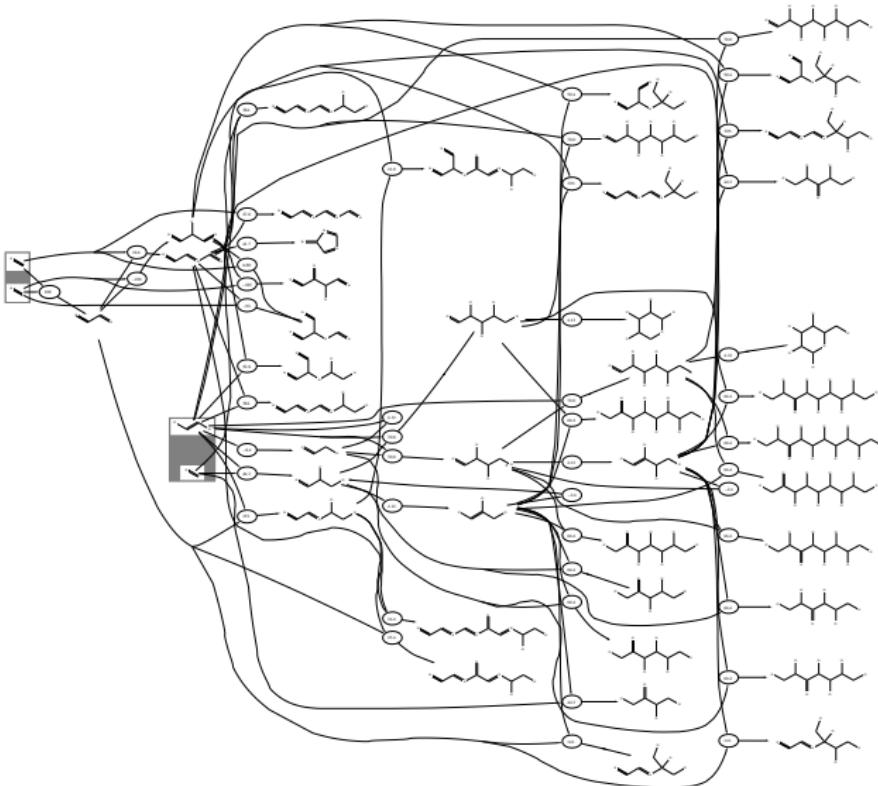


- Protocellular entity
- Bag of ribozymes
- Algebraic chemistry model
- Exchange of molecules with the environment

# Simulation - Overview

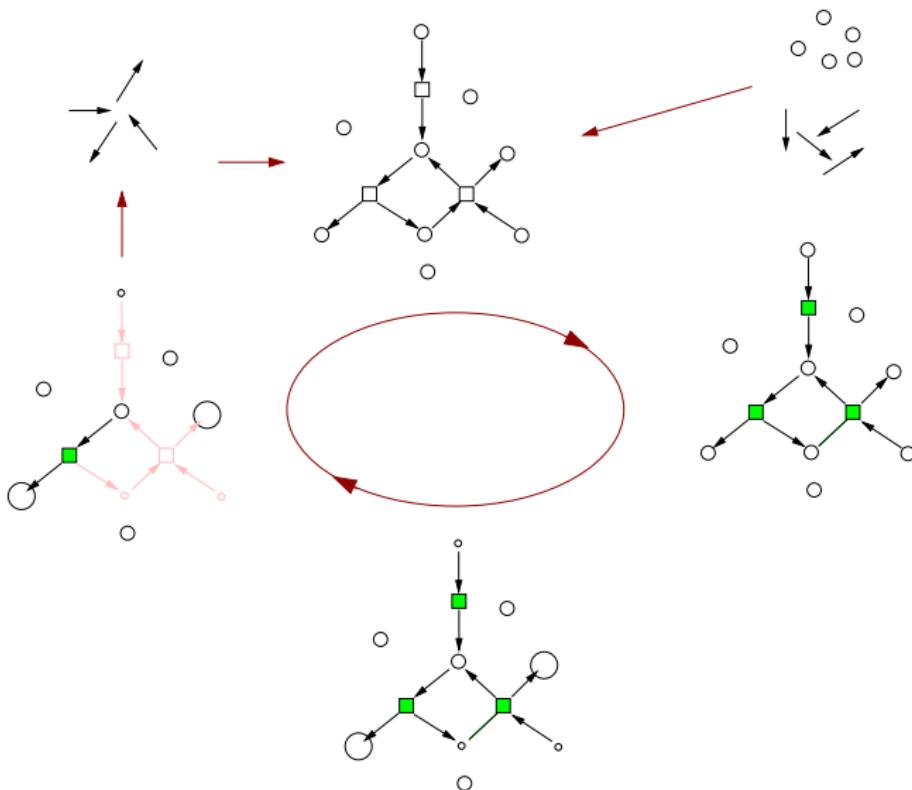


# Simulation - Growth



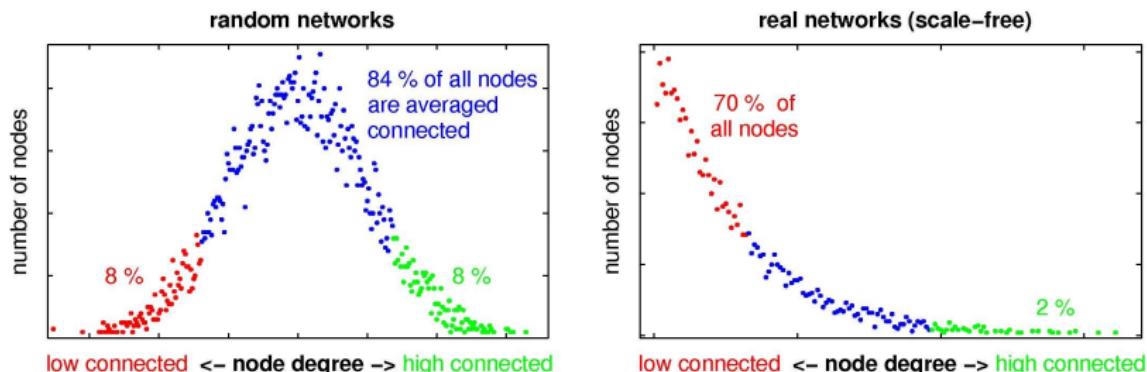
cyanide, formaldehyde glycol; aldolcondensation, tautomerization

# Simulation - Stochastic Network Generator

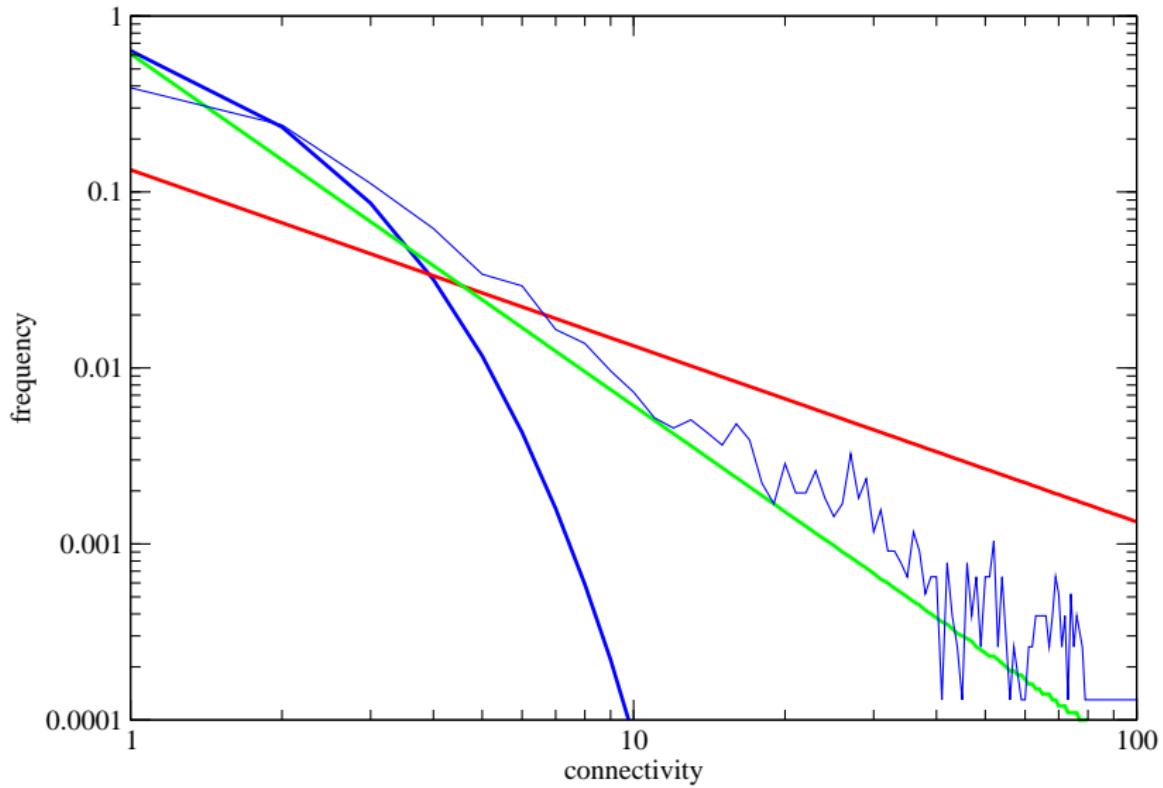


# General network analysis

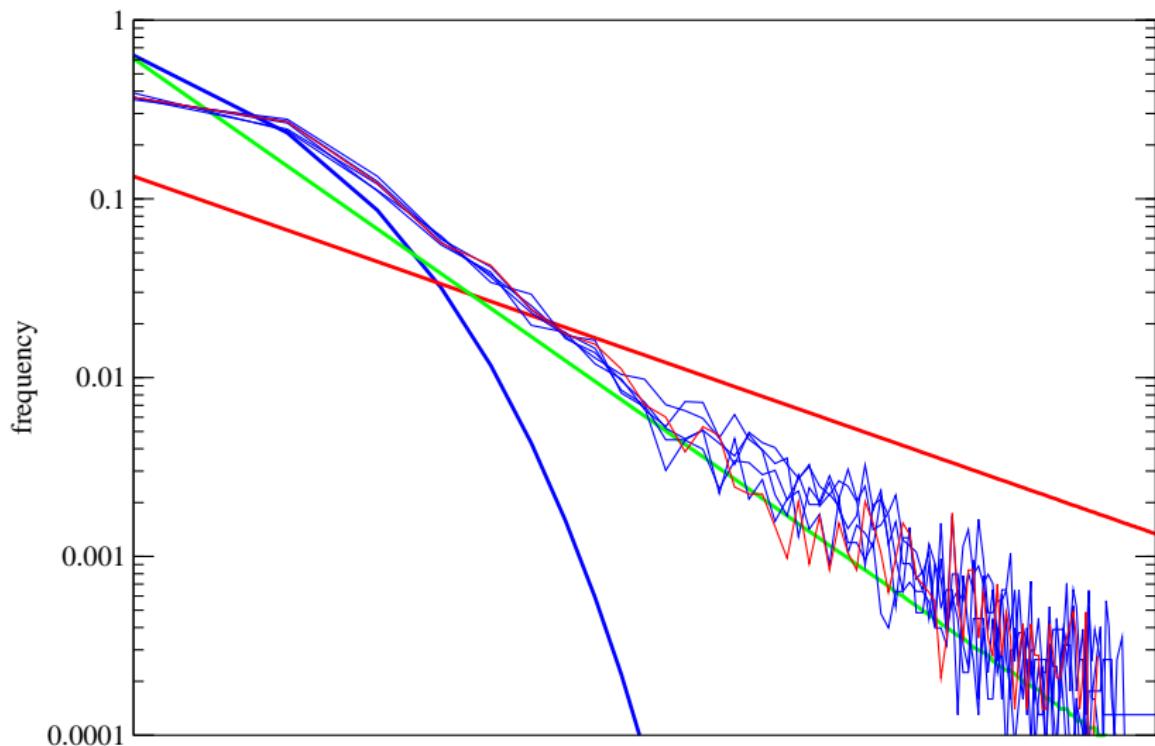
- Connectivity Distribution
  - small vs big
  - early vs evolved
- Clustering Coefficient, Centrality, Entropy, ...
  - simulated vs real world



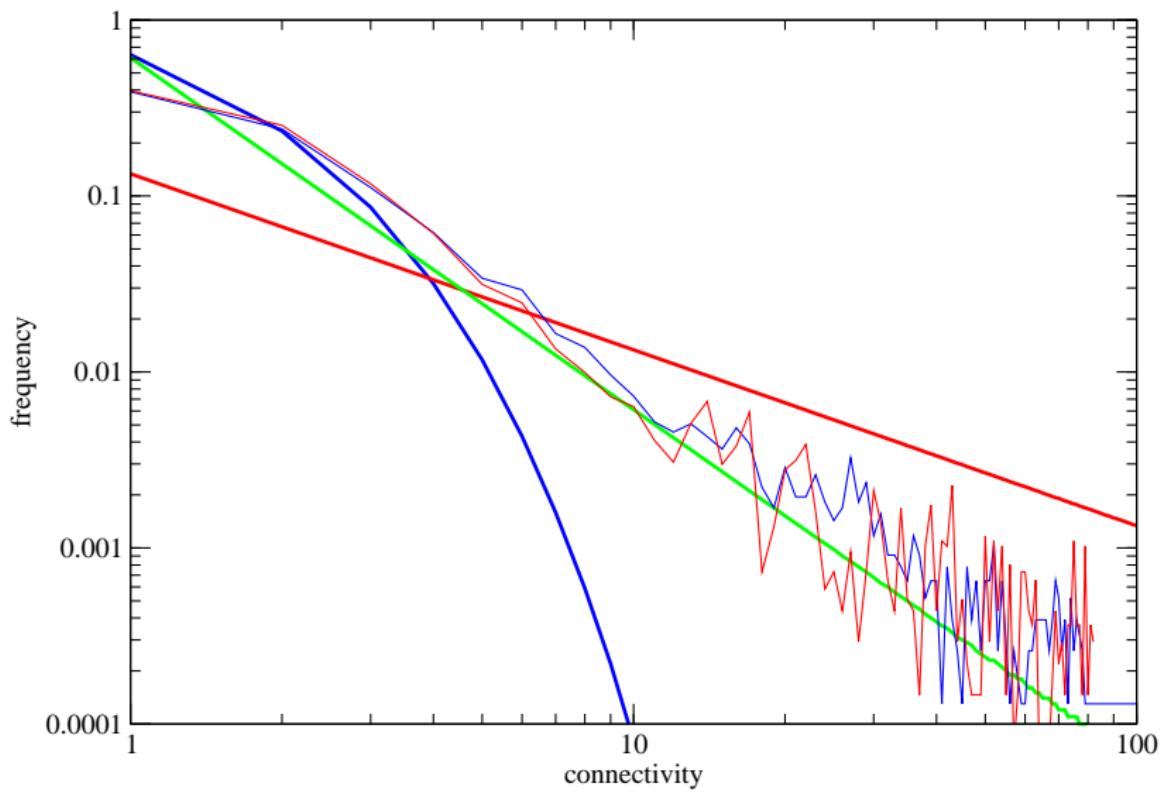
## after 10 Generations



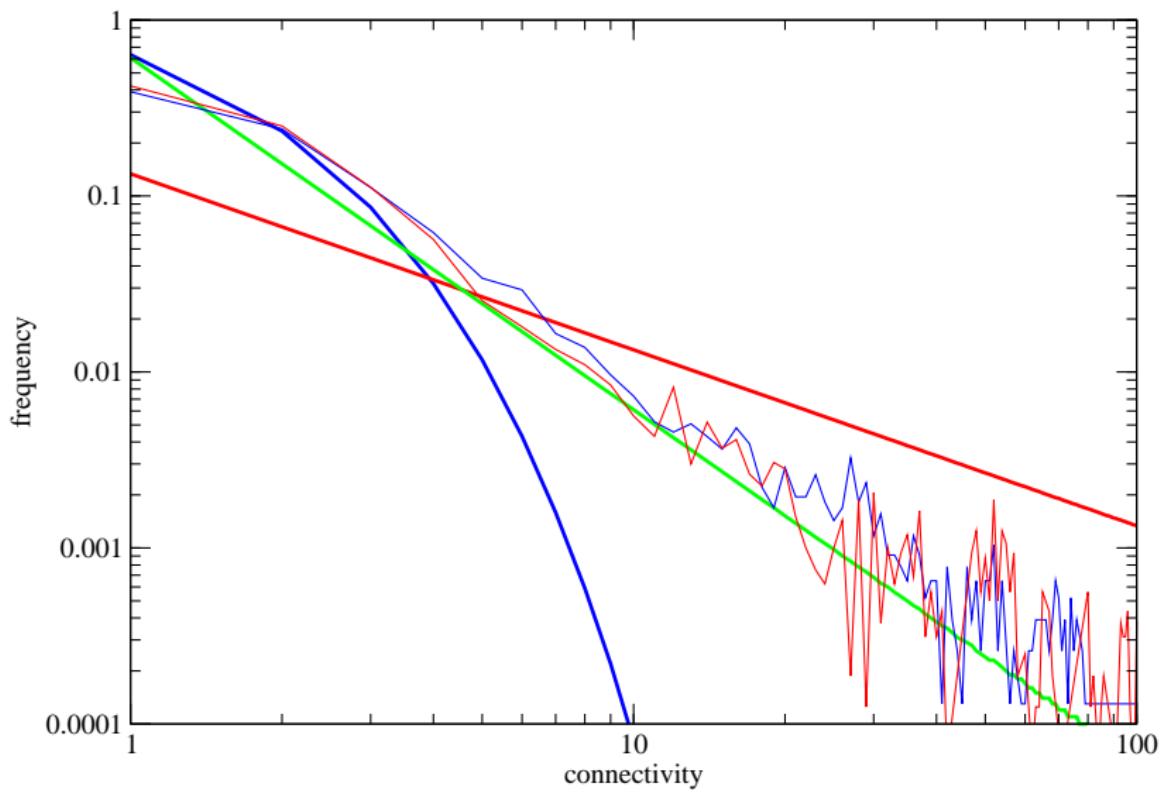
## after 50 Generations



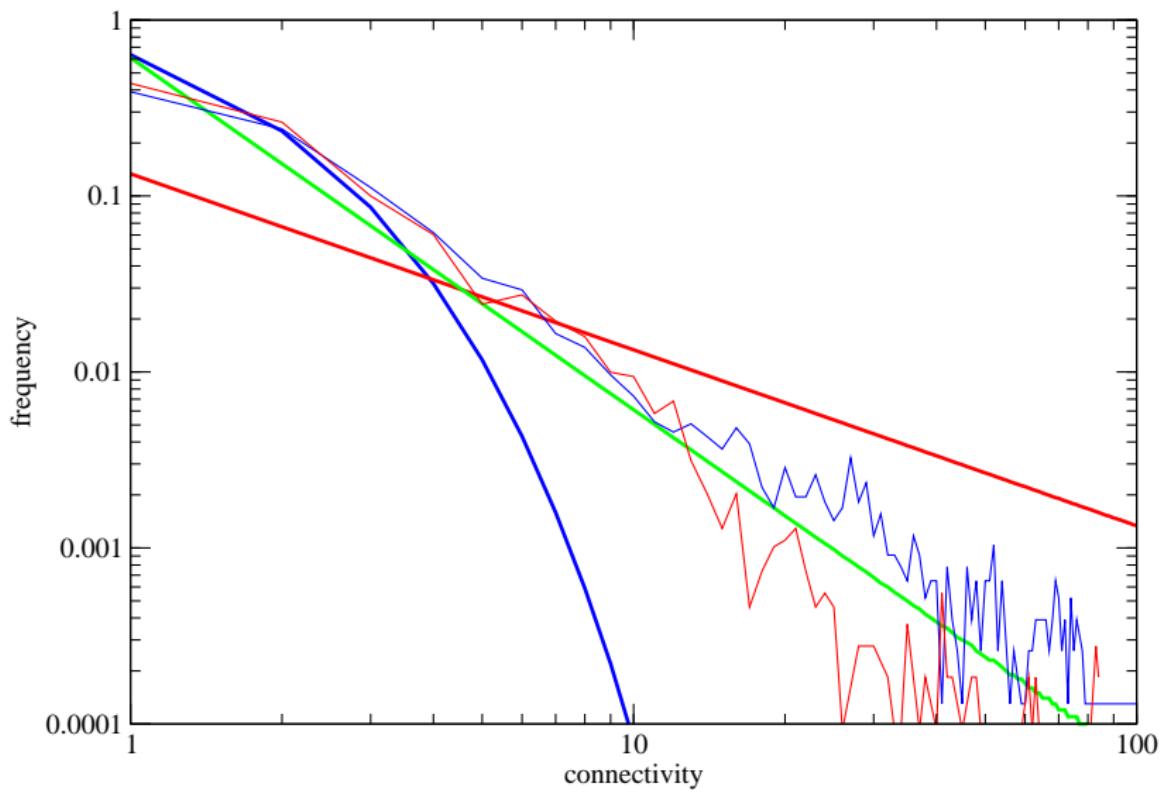
after 100 generations



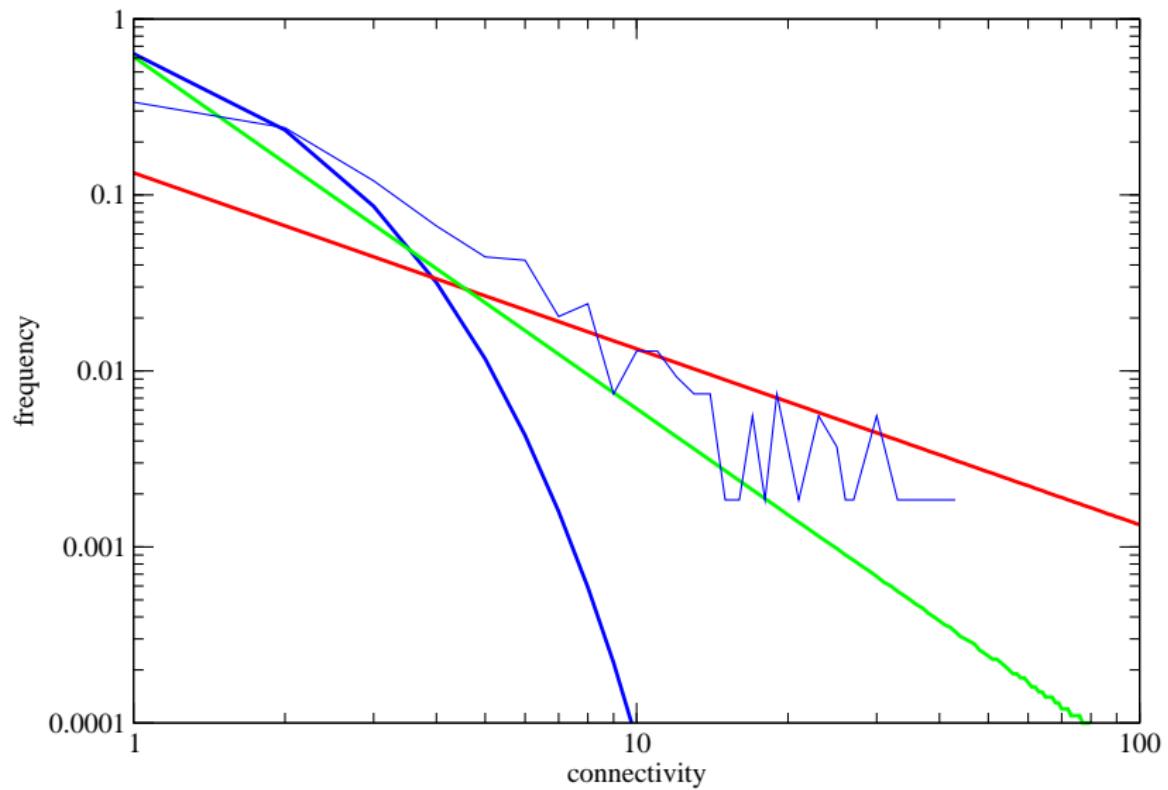
after 250 generations



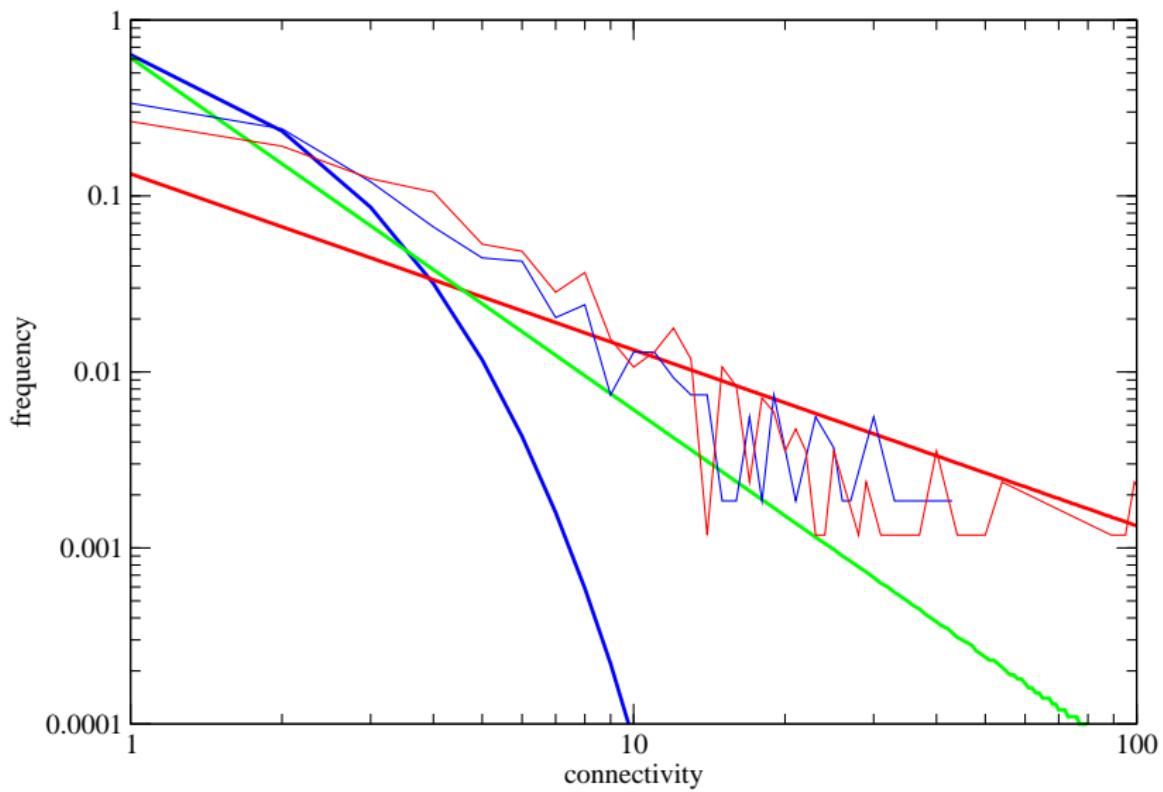
after 500 generations



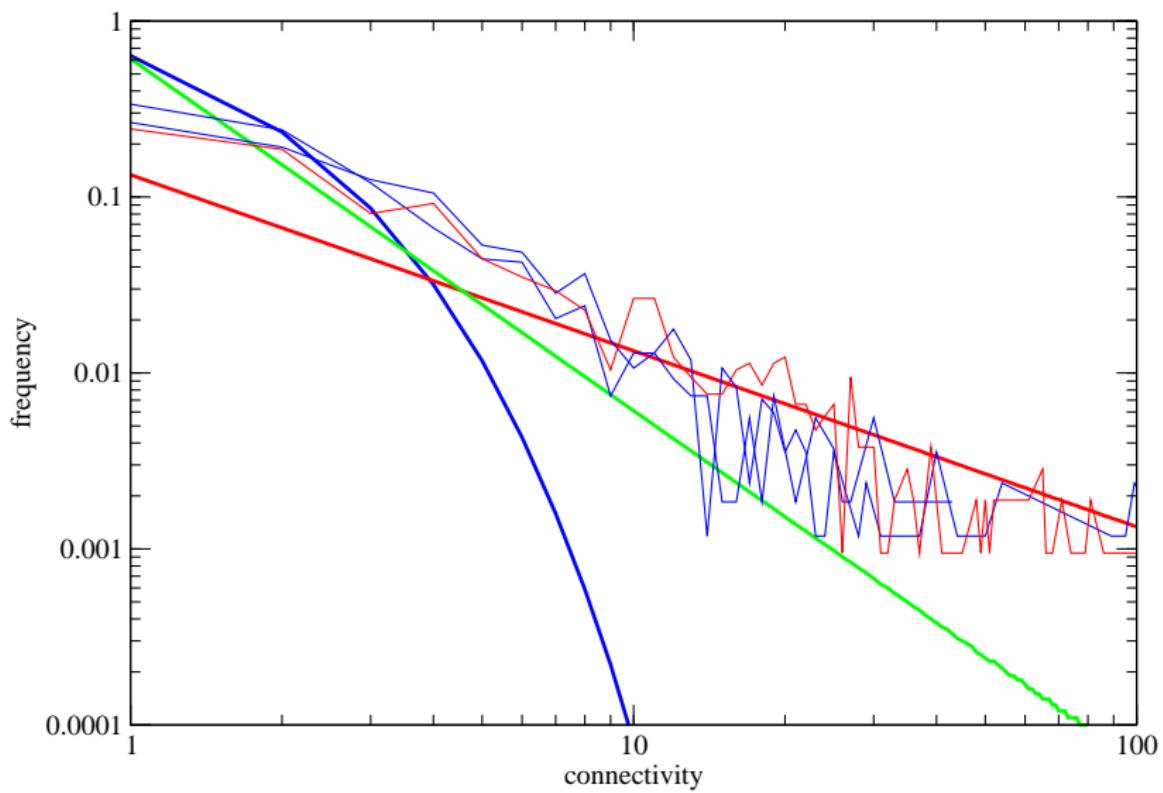
## Changing Environment - after 100 generations



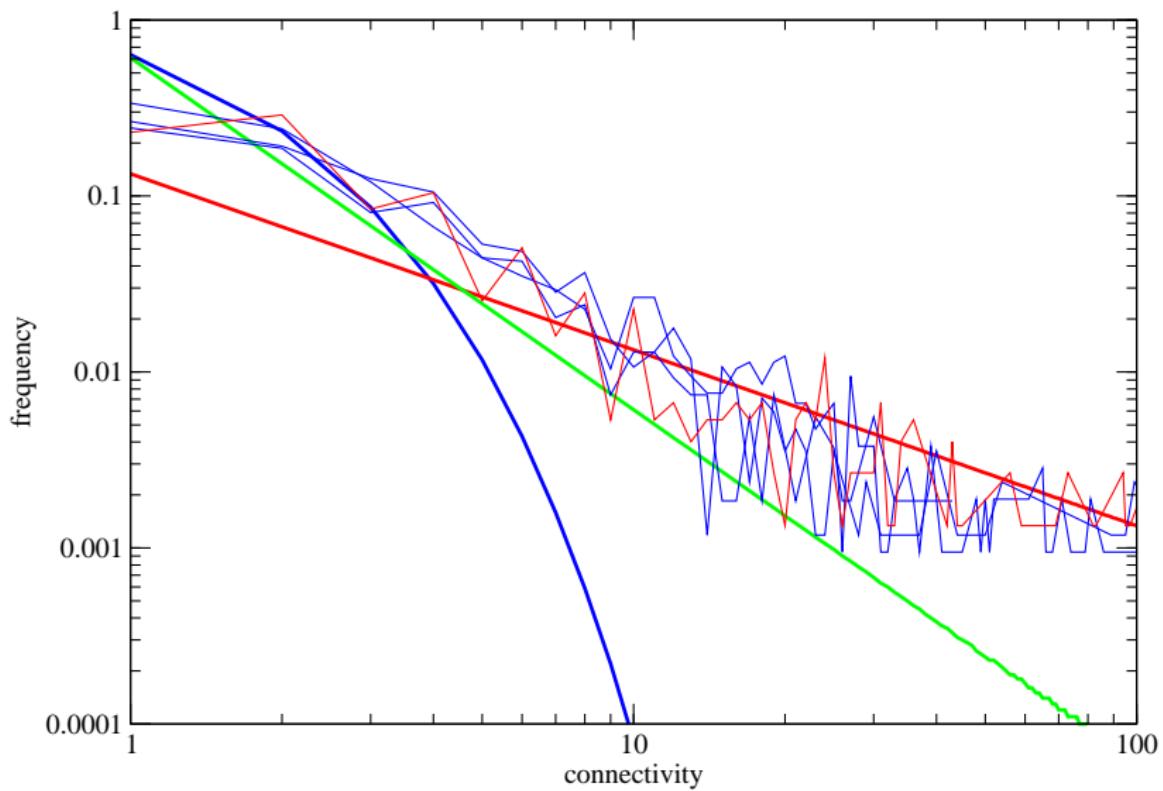
after 250 generations



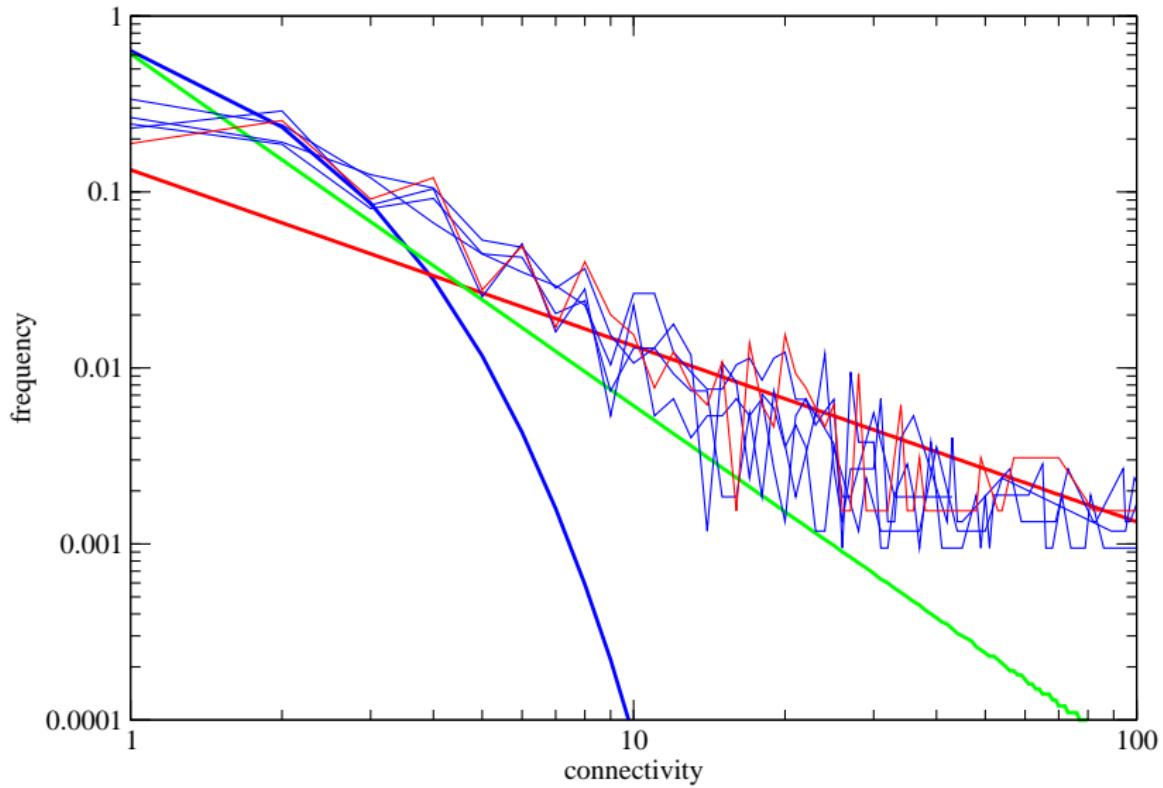
## after 500 generations



after 750 generations



## after 1000 generations



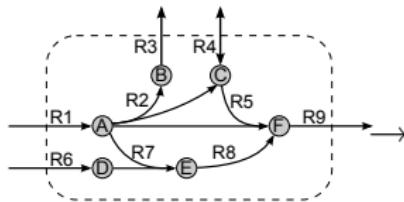
# Metabolic network analysis

We have sets of edges forming meaningful complex entities



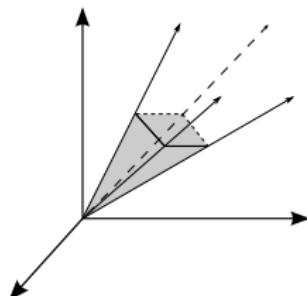
- number of pathways → flexibility
- change in case of single/multiple knockouts → robustness
- number of acceptable knockouts → robustness

# Metabolic Pathway Analysis



$$\begin{array}{l}
 \begin{matrix} & R1 & R2 & R3 & R4 & R5 & R6 & R7 & R8 & R9 \end{matrix} \\
 \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} \left( \begin{array}{ccccccccc}
 1 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\
 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1
 \end{array} \right)
 \end{array}$$

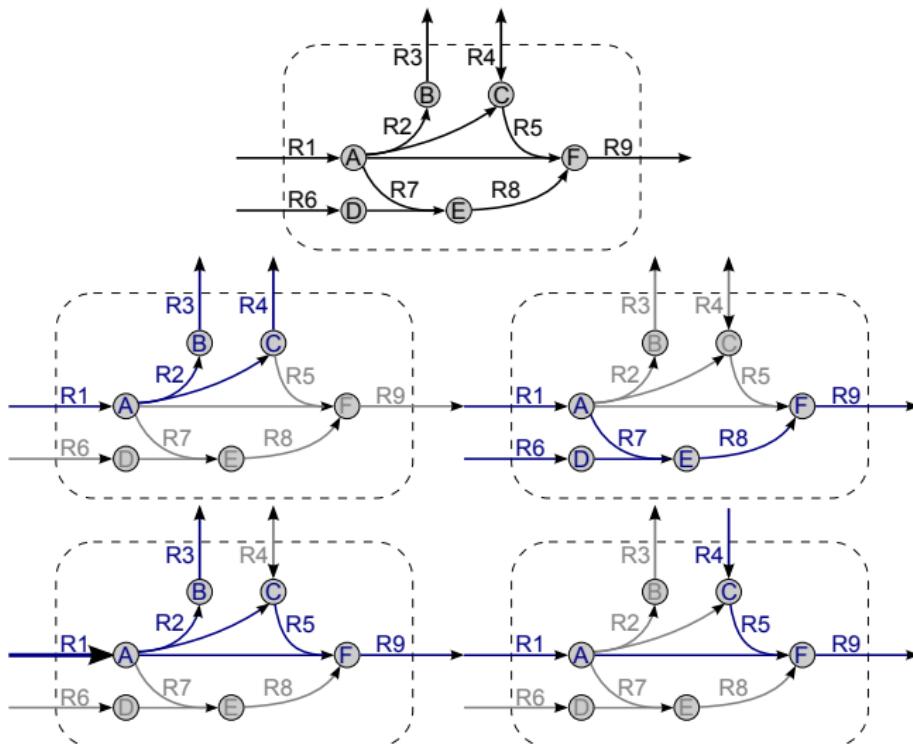
↓



←

$$\begin{array}{l}
 S * v_i = 0 \\
 \begin{array}{lllll}
 v_1 & v_2 & \dots & v_{n-1} & v_n
 \end{array} \\
 \begin{array}{l}
 R1 \\
 R2 \\
 R3 \\
 R4 \\
 R5 \\
 R6 \\
 R7 \\
 R8 \\
 R9
 \end{array} \left( \begin{array}{lllll}
 2 & -1 & \dots & 0 & 0 \\
 1 & -1 & \dots & -1 & -1 \\
 1 & -1 & \dots & -1 & -1 \\
 0 & 1 & \dots & 1 & 0 \\
 1 & 0 & \dots & 0 & -1 \\
 0 & 0 & \dots & 1 & 2 \\
 0 & 0 & \dots & 1 & 2 \\
 0 & 0 & \dots & 1 & 2 \\
 1 & 0 & \dots & 1 & 1
 \end{array} \right)
 \end{array}$$

# Metabolic Pathway Analysis

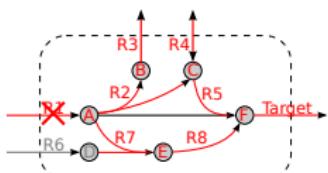
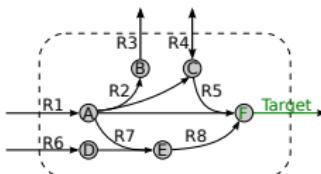


# Knockout effects

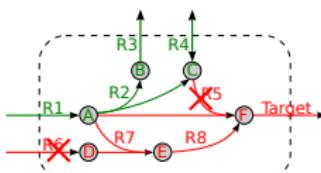
single	$R_1 = \frac{\sum_{i=1}^r z^i}{r * z}$	depletion	$R_2 = \frac{\sum_{i=1}^n R_1^i}{n}$
multiple	$R_3(k) = \frac{\sum_{i=1}^{s(k)} z^i}{s(k) * z}$	overall	$R_3(\leq K) = \sum_{k=1}^K R_3(k)p_k$

Example system	Number of reactions	Number of elementary modes	$R_1(1)$	$R_1(2)$	$R_1(3)$	$R_1(\leq 3)$	
1		4	2	$1/2 = 0.5$	$1/6 \approx 0.167$	0	0.414
2		4	2	$1/2 = 0.5$	$1/4 = 0.25$	$1/8 = 0.125$	0.436
3		4	2	$3/8 = 0.375$	$1/12 \approx 0.083$	0	0.302
4		4	2	$1/4 = 0.25$	0	0	0.189
5		8	2	$7/16 \approx 0.438$	$3/8 = 0.375$	$5/16 \approx 0.313$	0.418
6		8	2	$1/2 = 0.5$	$3/14 \approx 0.214$	$1/14 \approx 0.071$	0.416
7		5	4	$13/20 = 0.65$	$3/8 = 0.375$	$7/40 = 0.175$	0.573
8		5	3	$2/3 \approx 0.667$	$2/5 = 0.4$	$1/5 = 0.2$	0.592

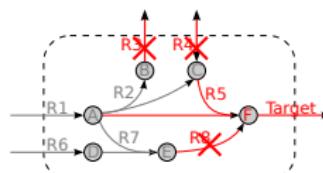
# Minimal Knockout sets



$\{R1\}$

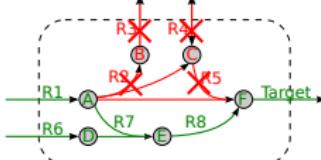


$\{R5, R6\}$

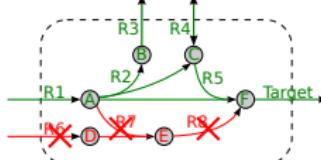


$\{R3, R4, R8\}$

## NO knockout sets



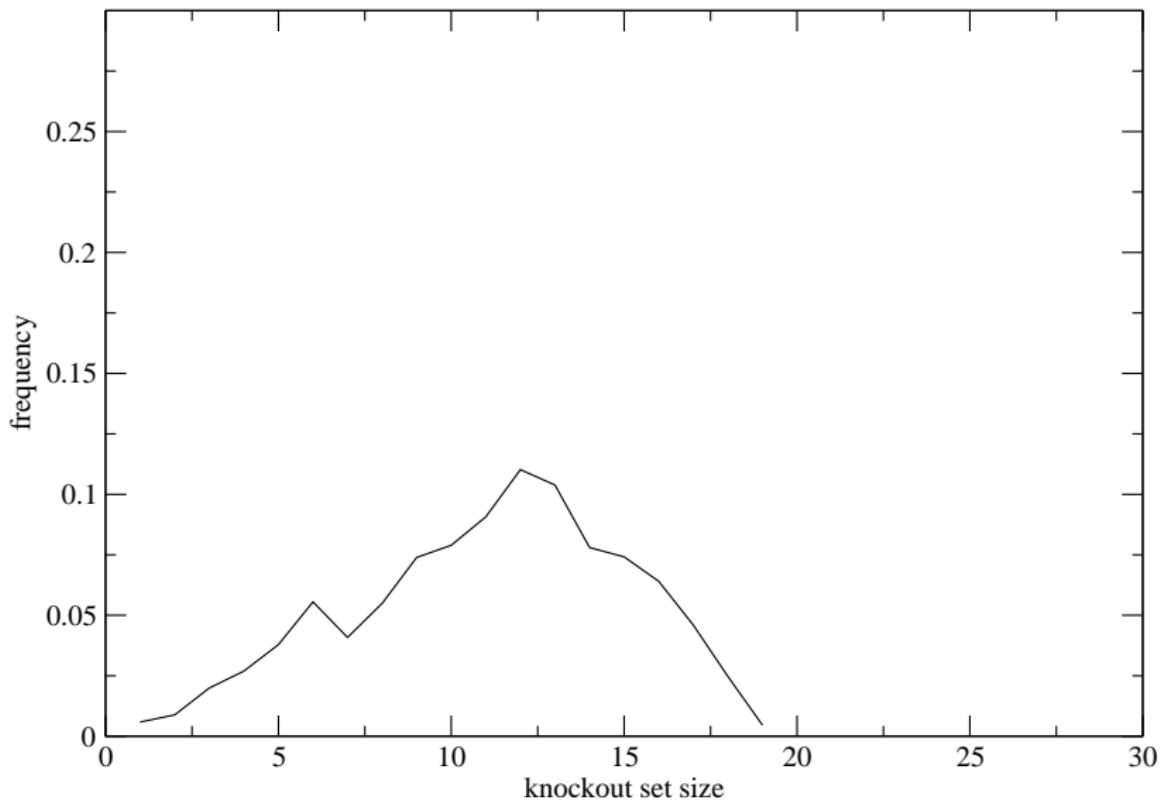
$\{R2, R3, R4, R5\}$



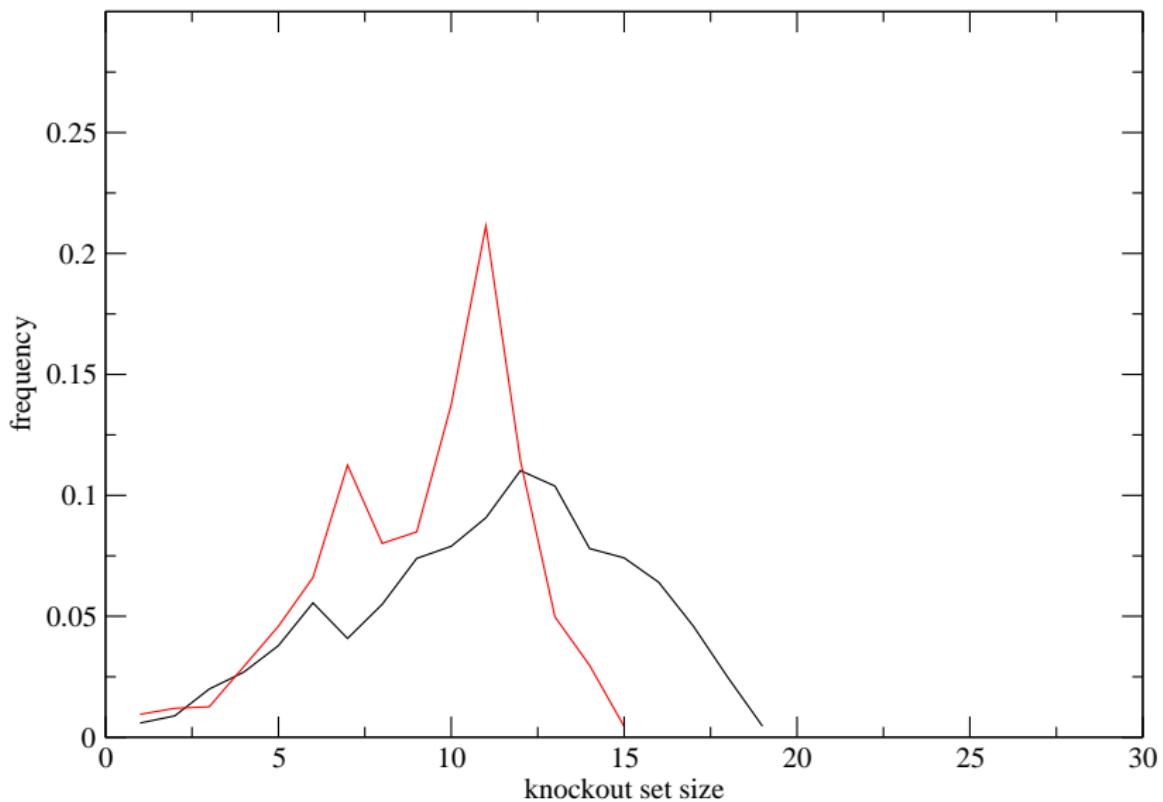
$\{R6, R7, R8\}$

Knockout set size distribution → Robustness (bigger is better)

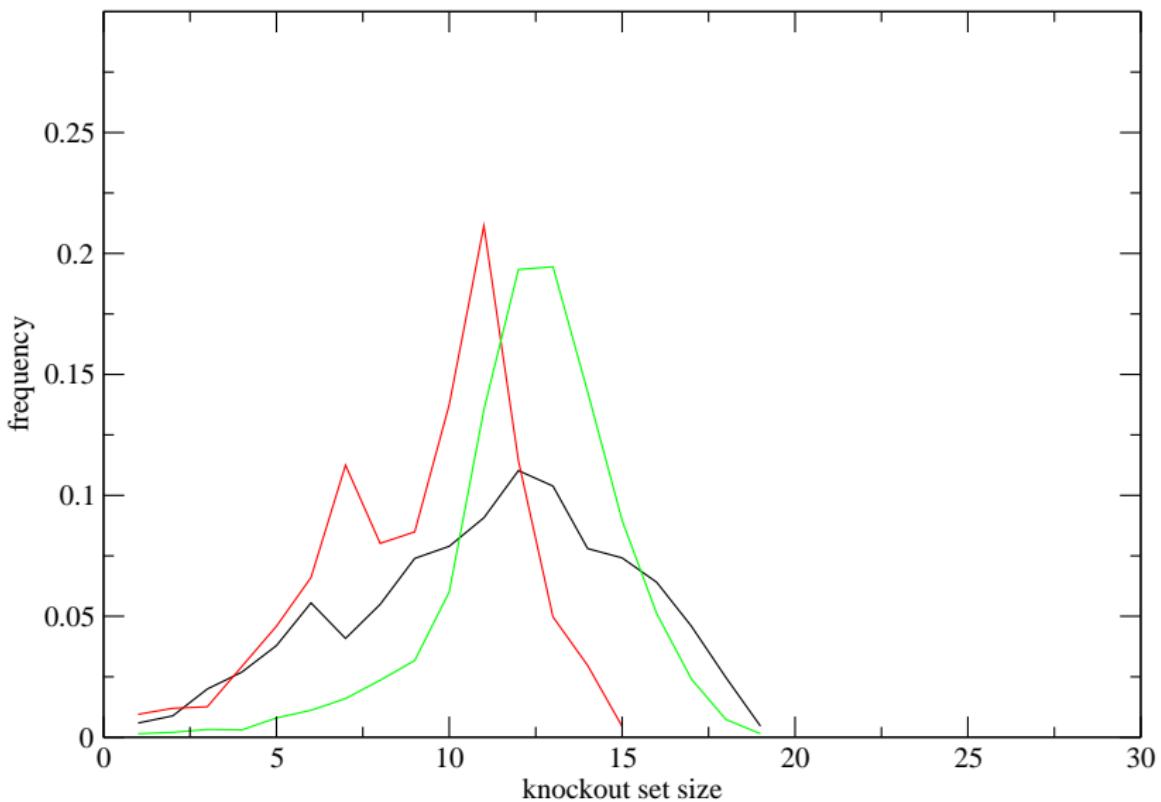
after 10 generations



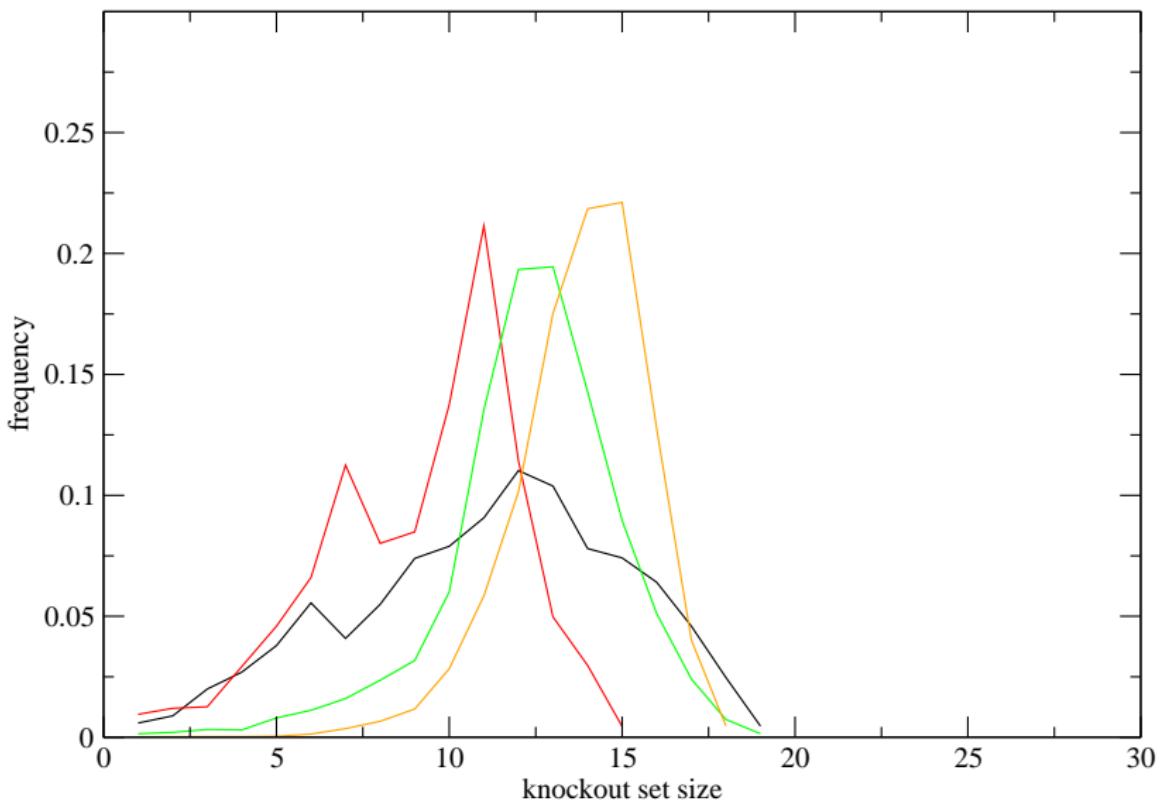
after 20 generations



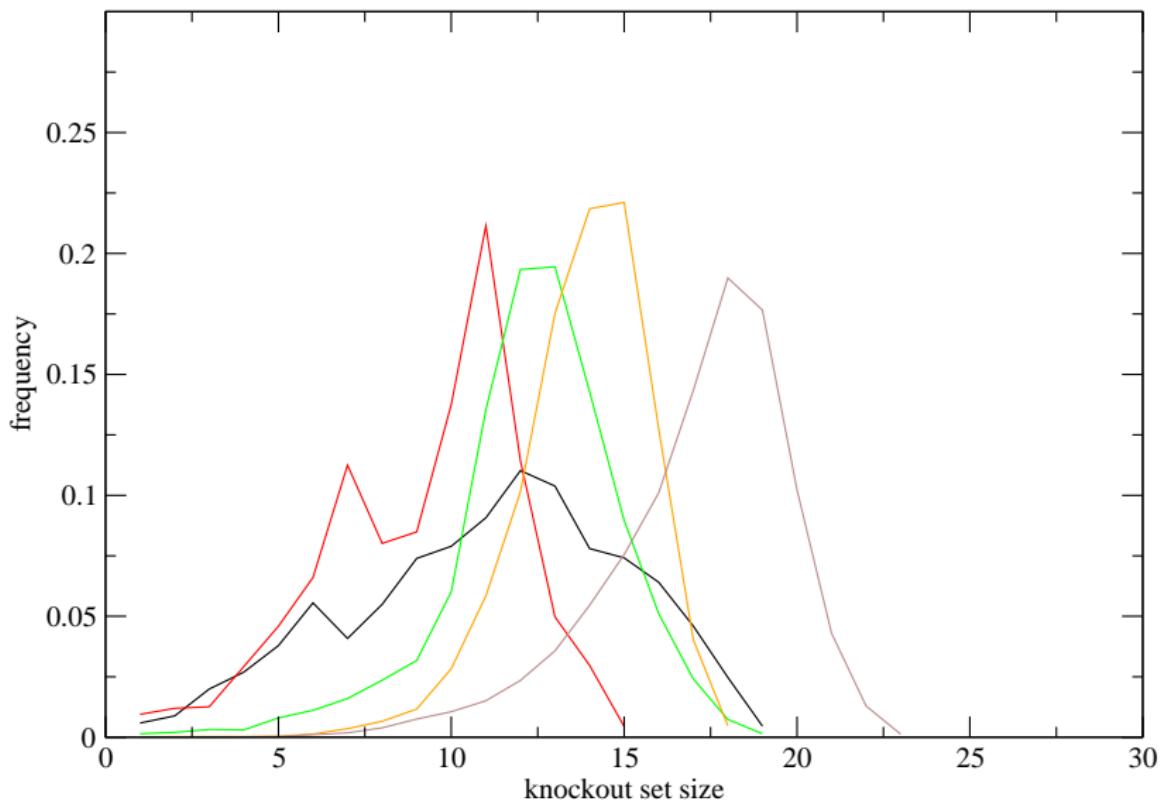
after 50 generations



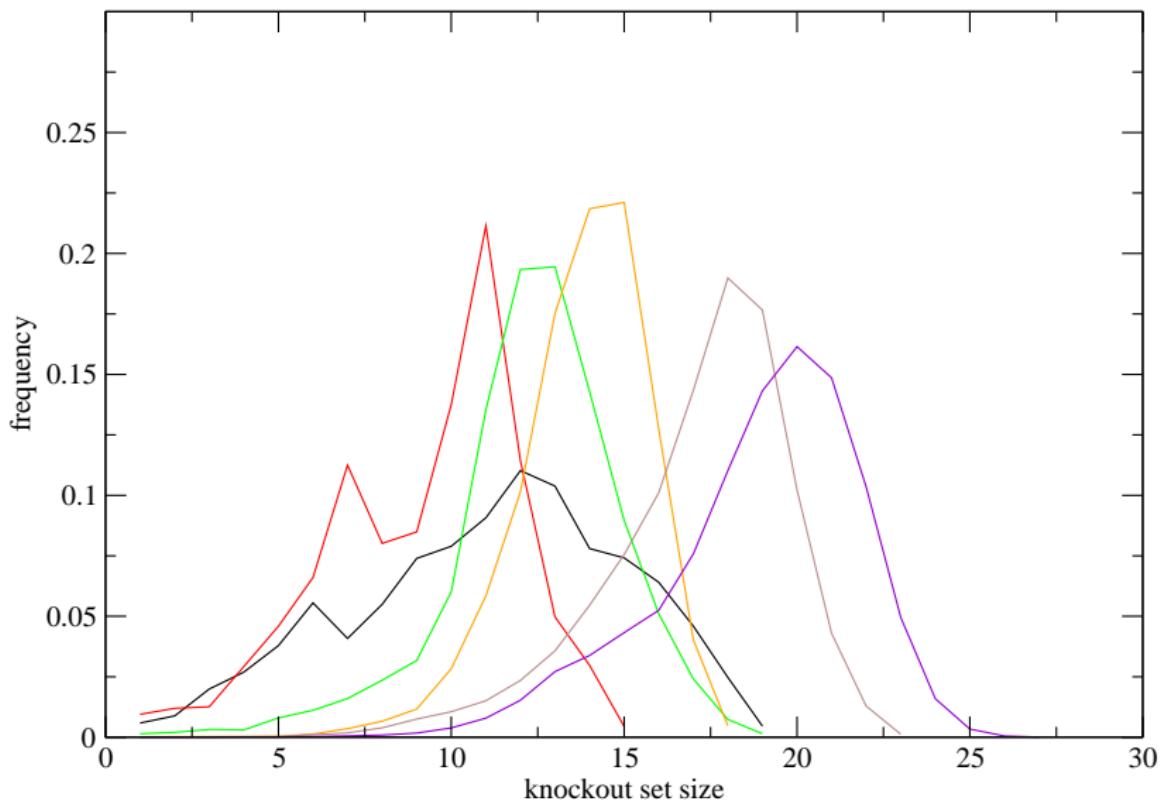
after 100 generations



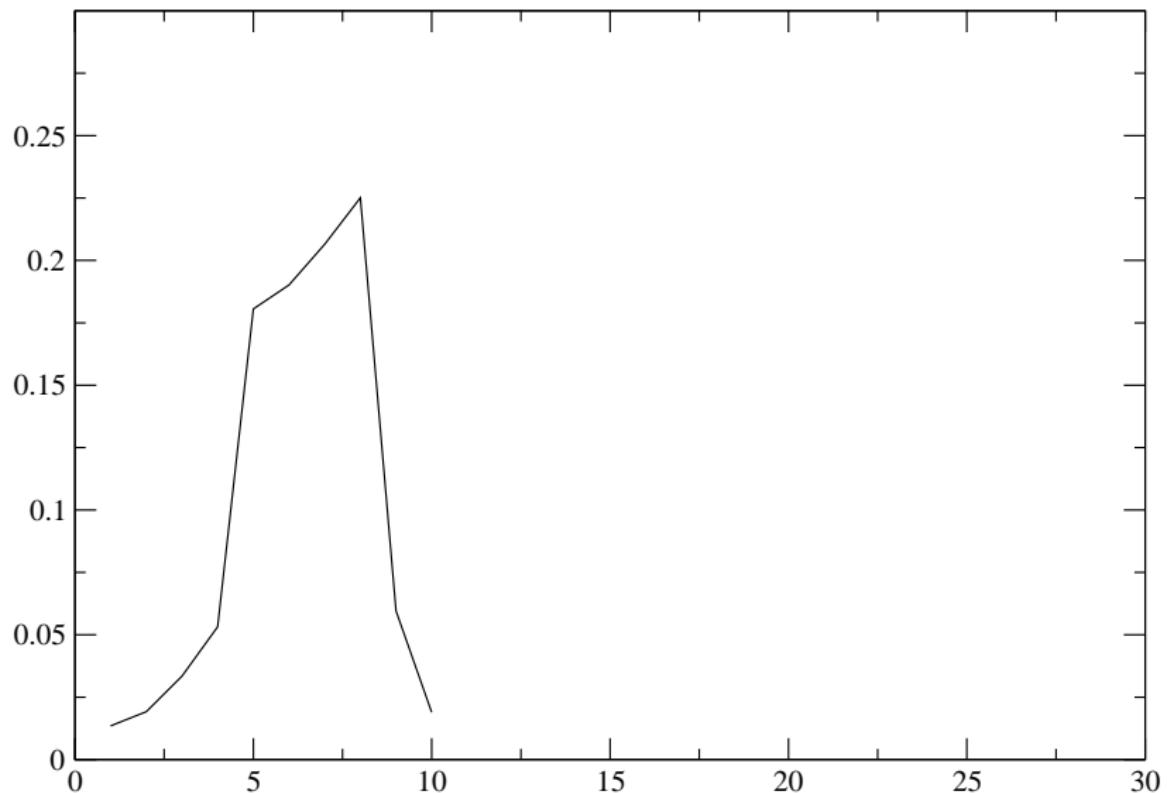
after 250 generations



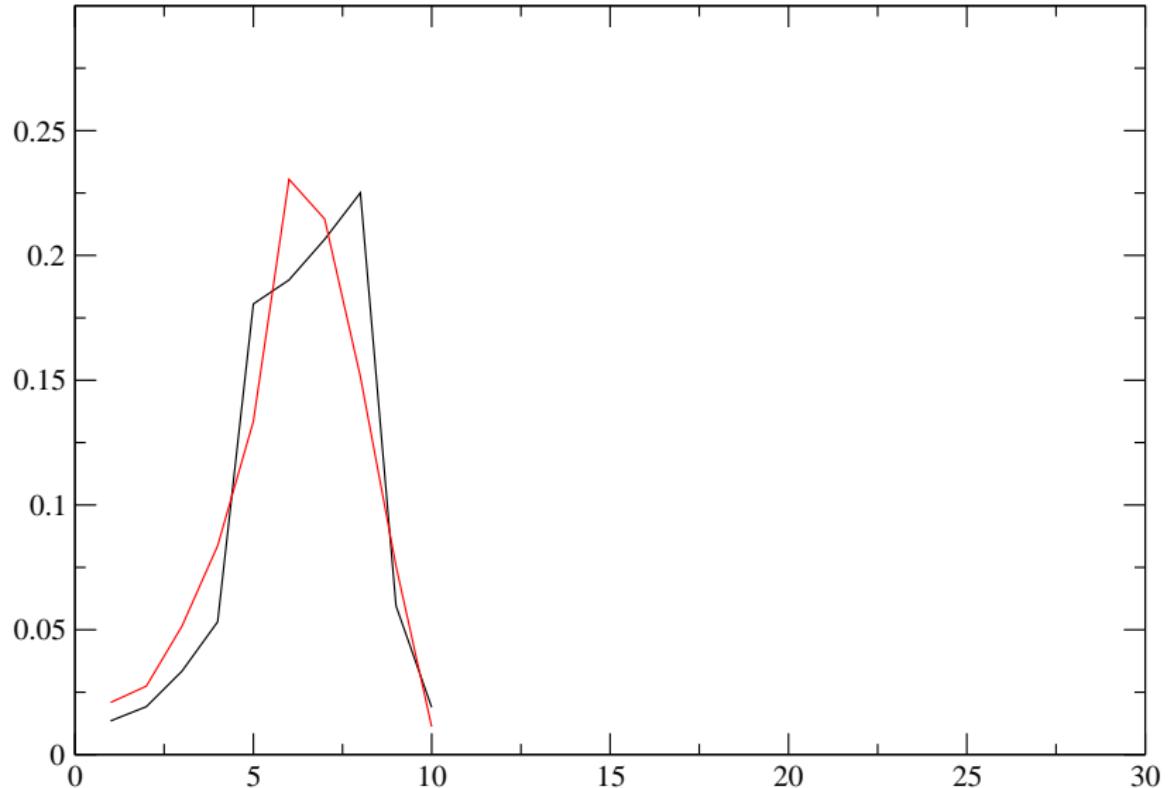
after 500 generations



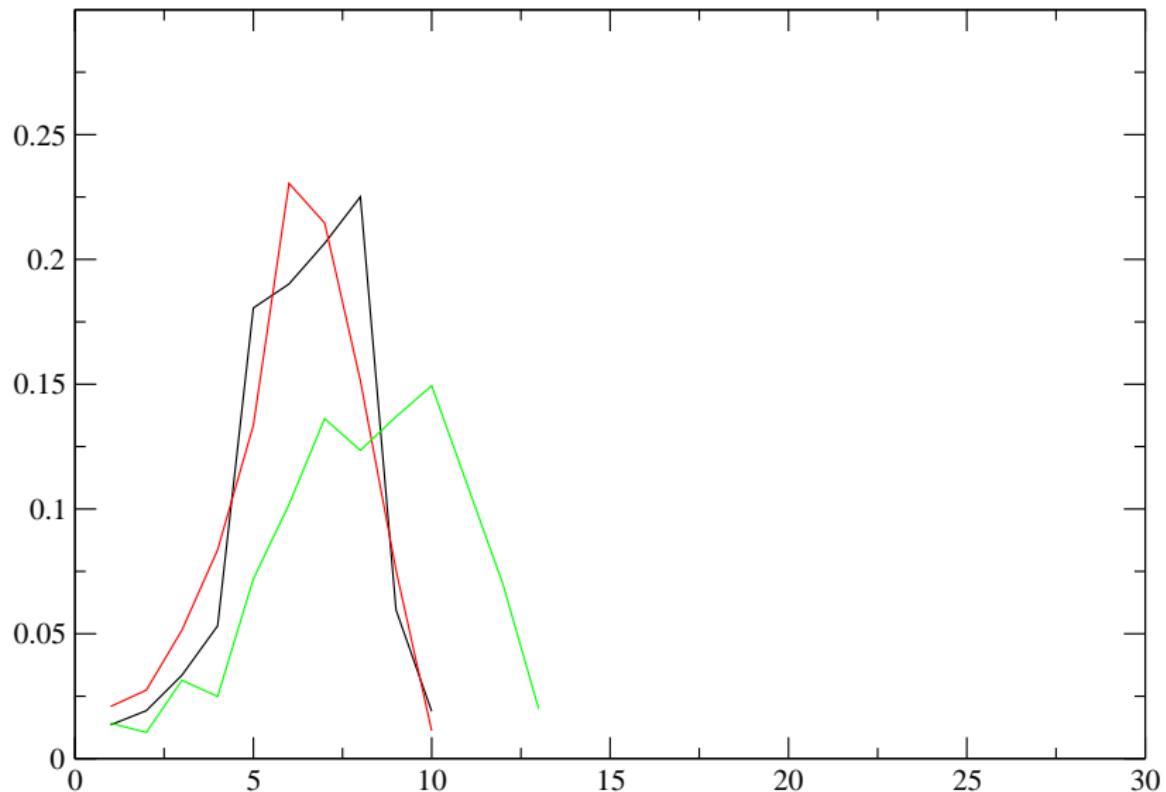
## Changing Environment - after 10 generations



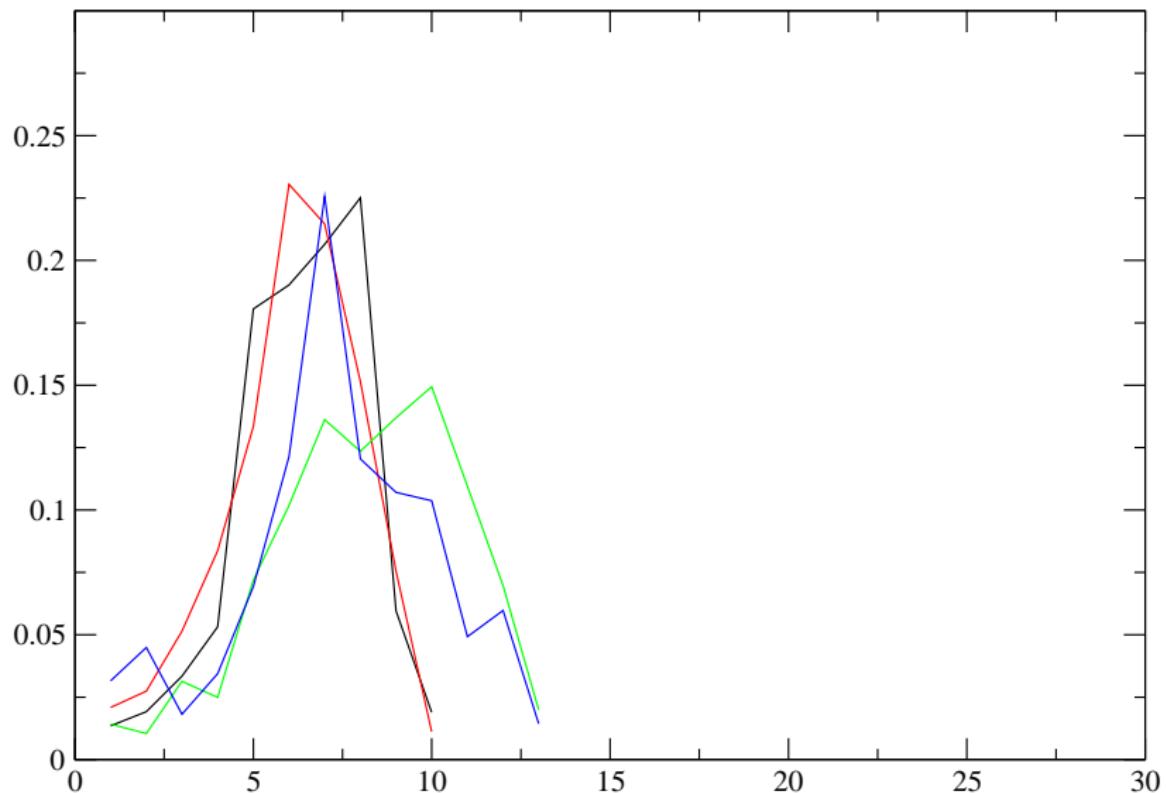
after 50 generations



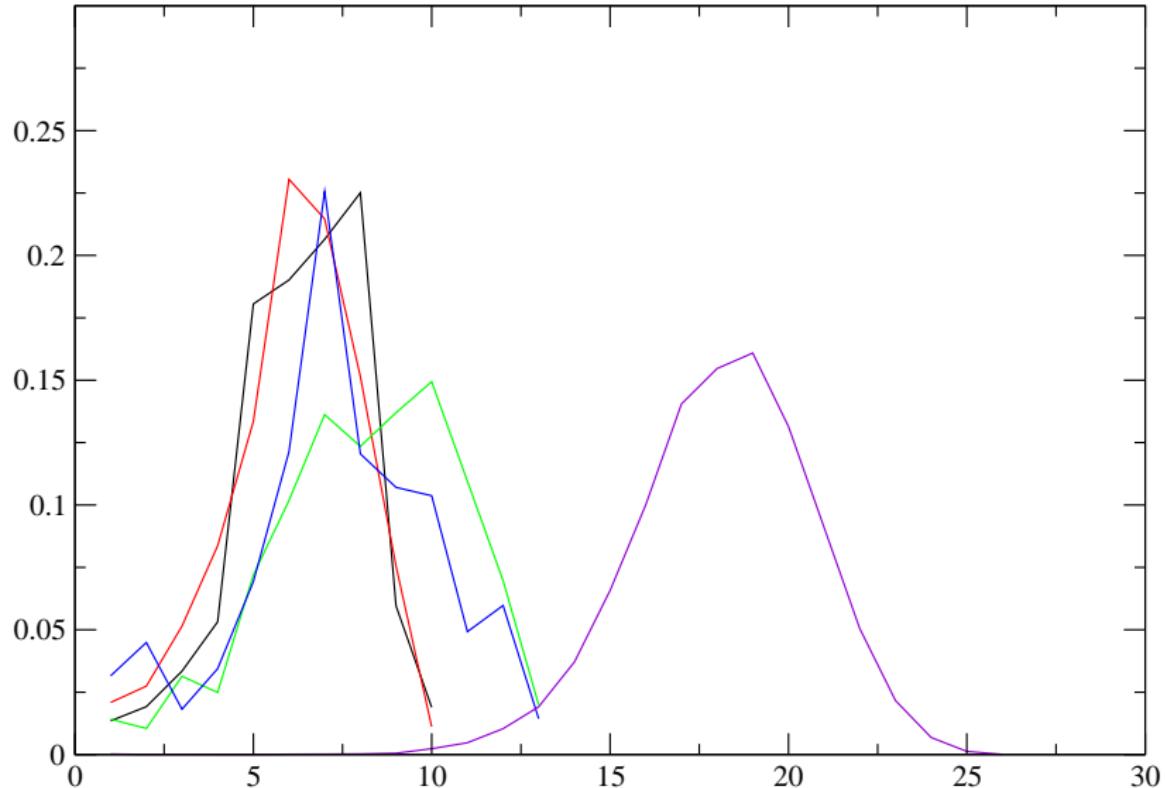
after 250 generations



after 500 generations



after 1000 generations



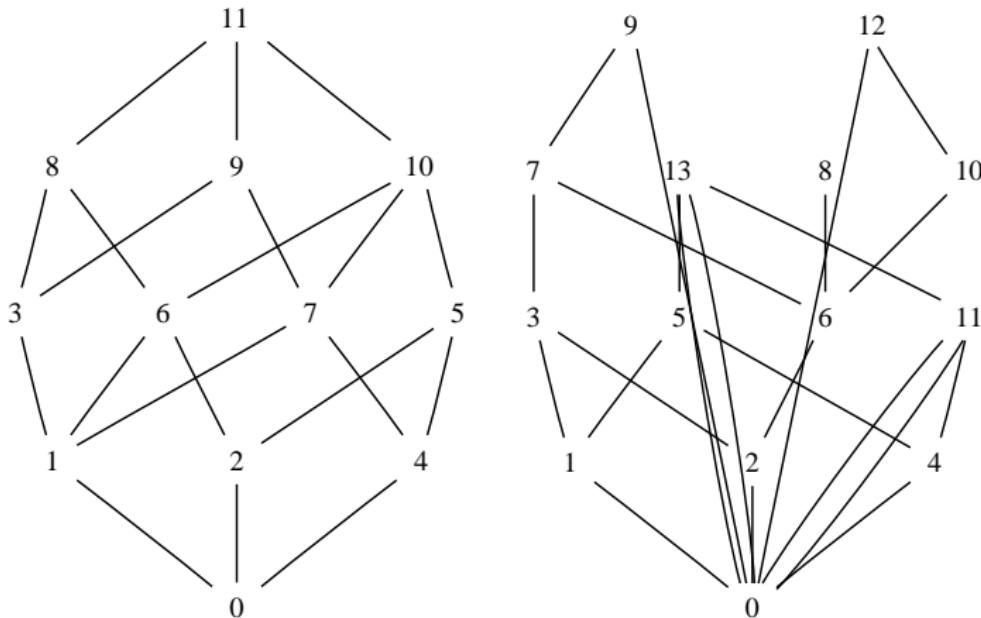
# Chemical Organizations

Self-maintaining and closed sets of molecules and reactions

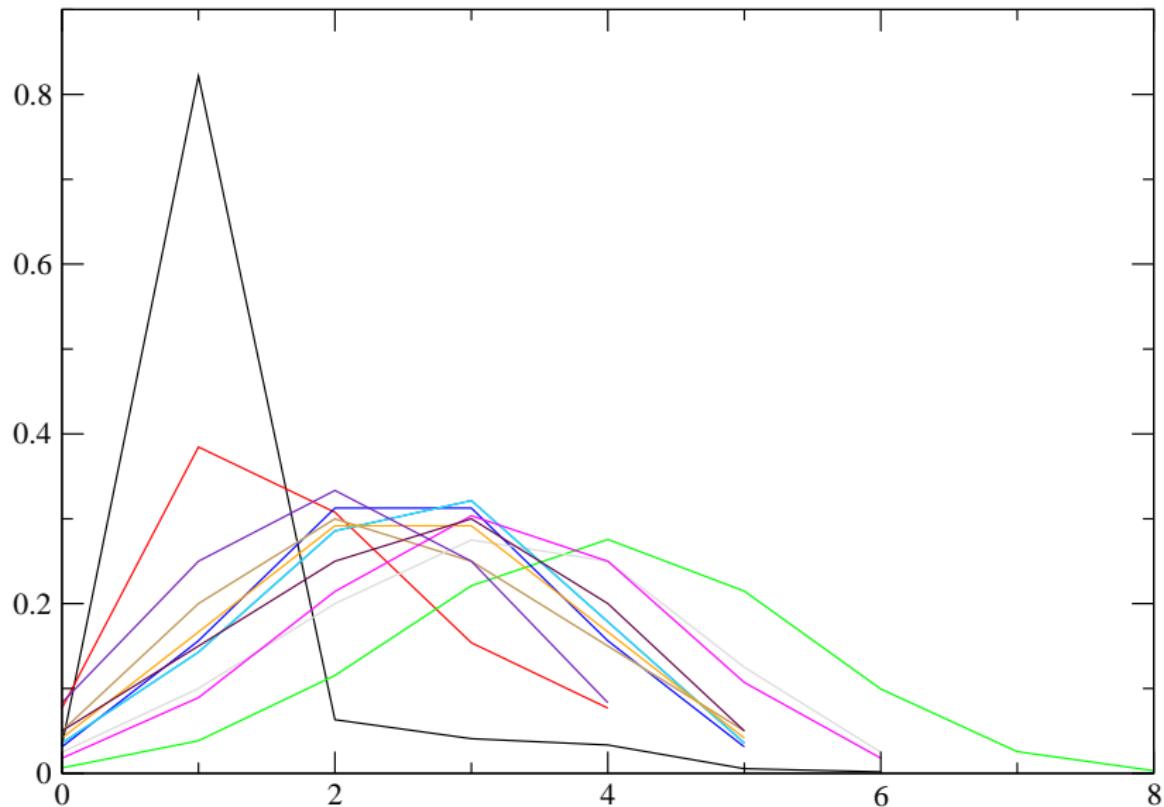
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chemical organizations

- Hierarchies of organizations
- Shape of Hierarchies → robustness
- Size distribution of organizations → robustness, modularity

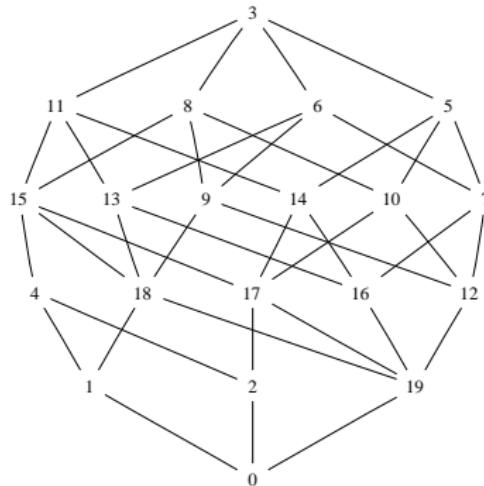
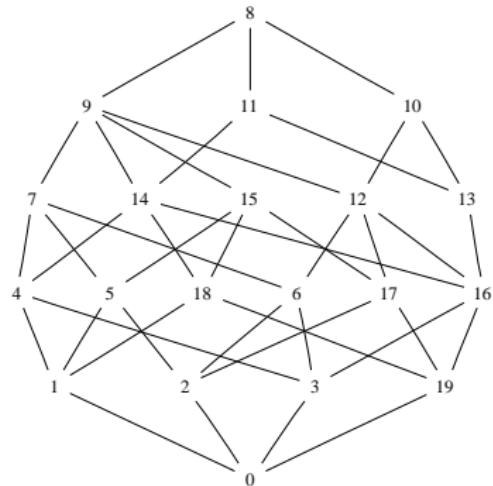
# Chemical Organizations



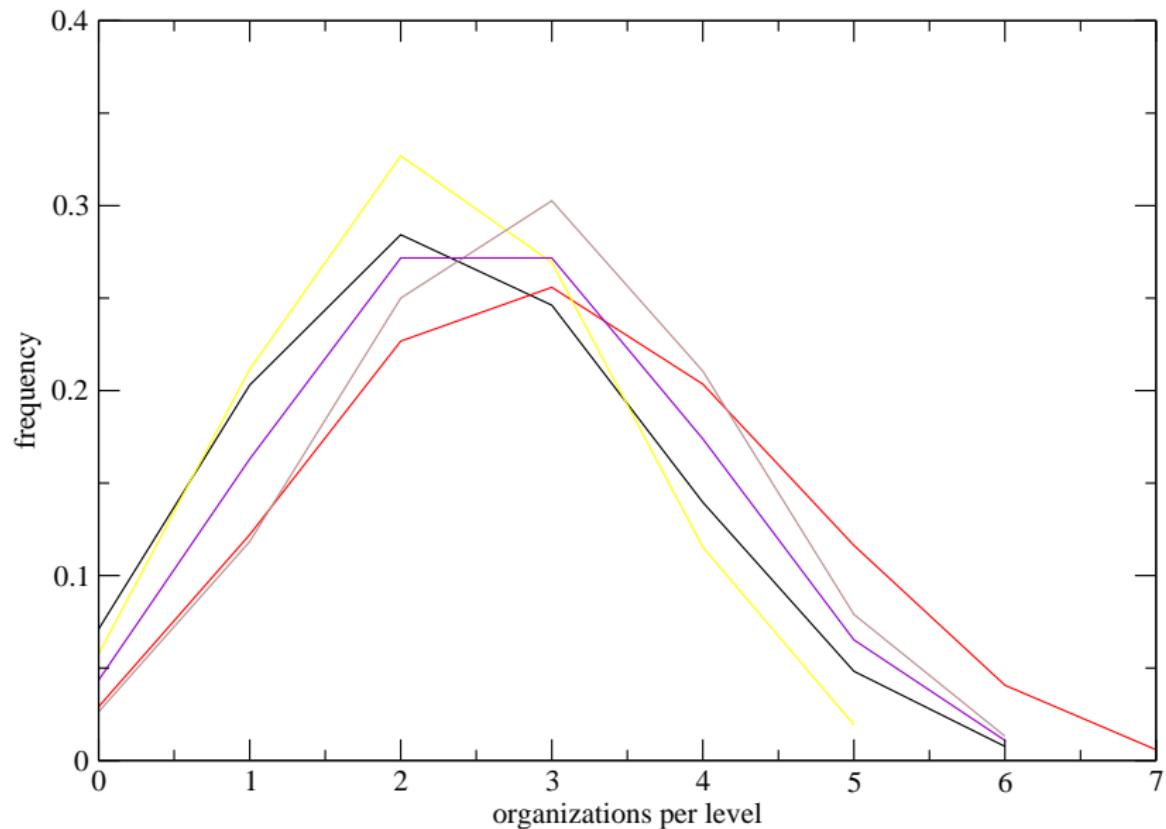
## Level Size Distribution - after 500 generations



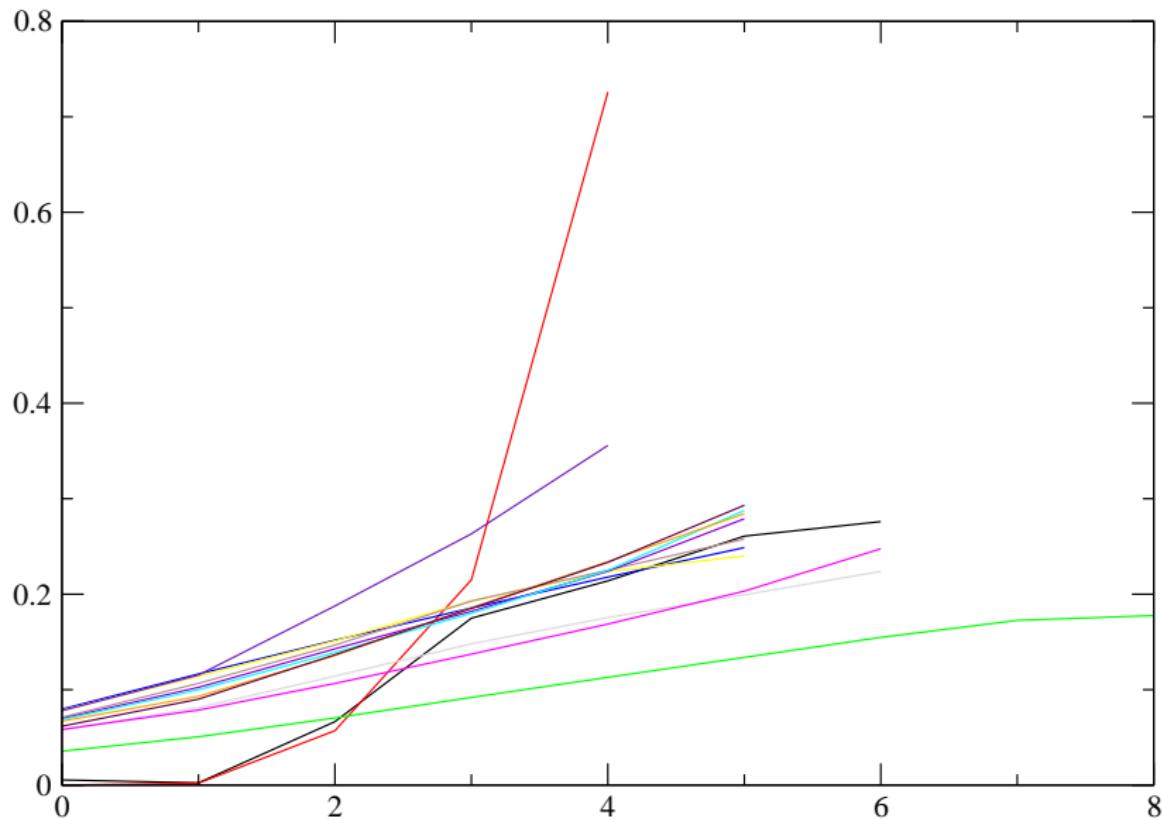
# Chemical Organizations



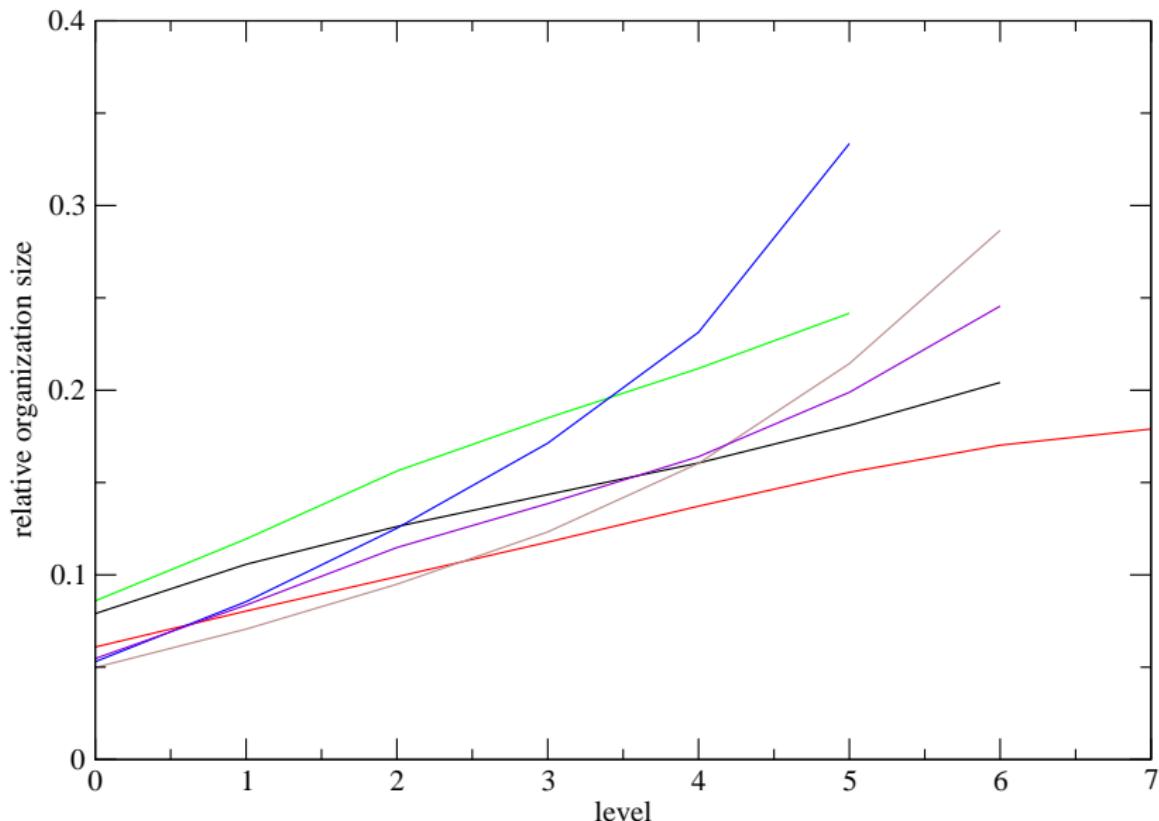
## Changing Environment - after 500 generations



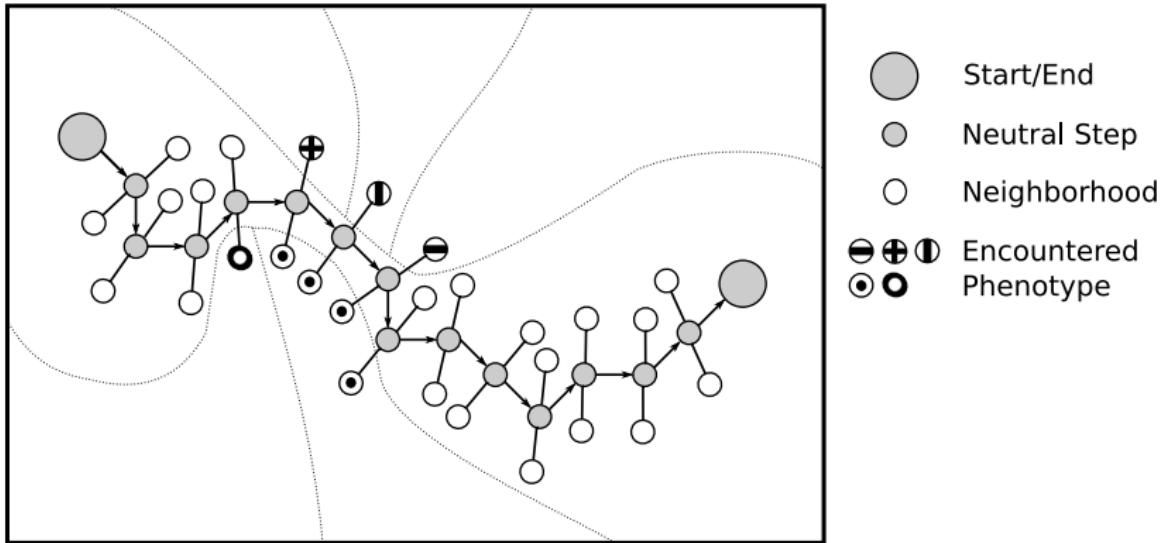
## Organization Size Distribution - after 500 generations



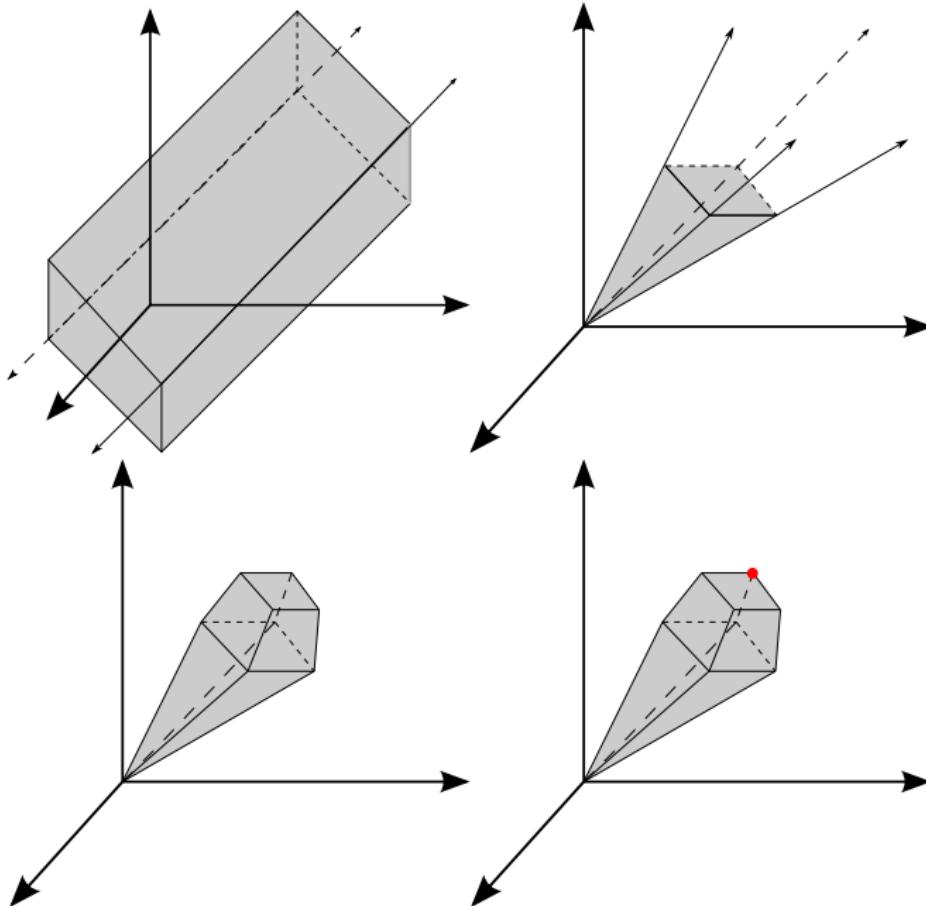
## Changing Environment - after 1000 generations



# Work in Progress

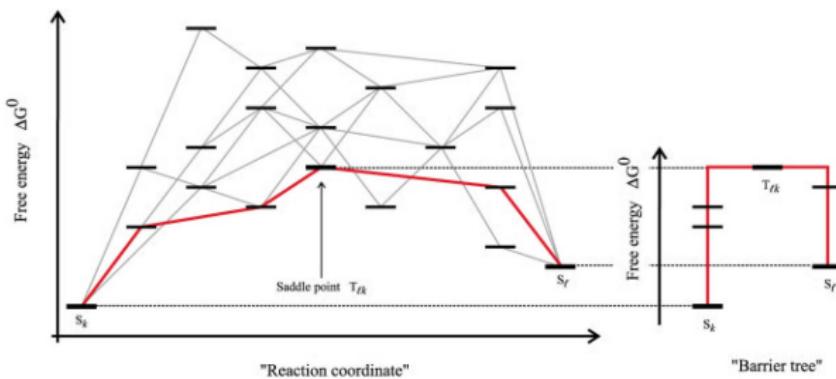


FBA



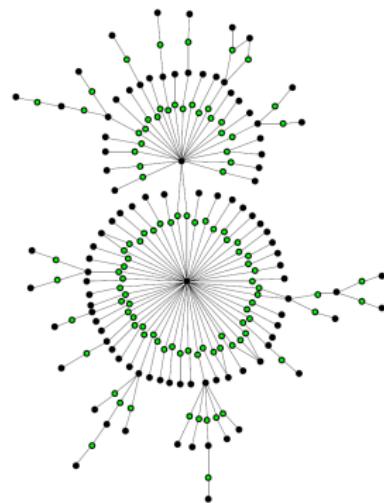
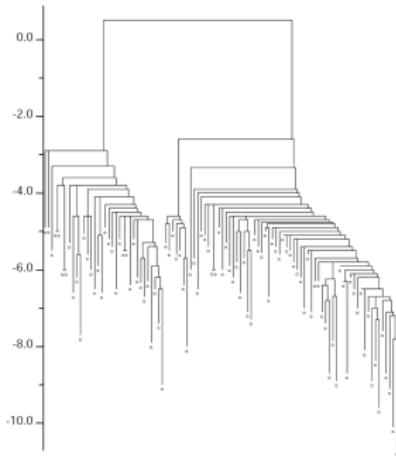
# Flux barrier analysis

- linear optimization: EMs modeled as system of linear equations
- constraints: limits on reactions, exclusion of combinations of EMs
- barrier tree



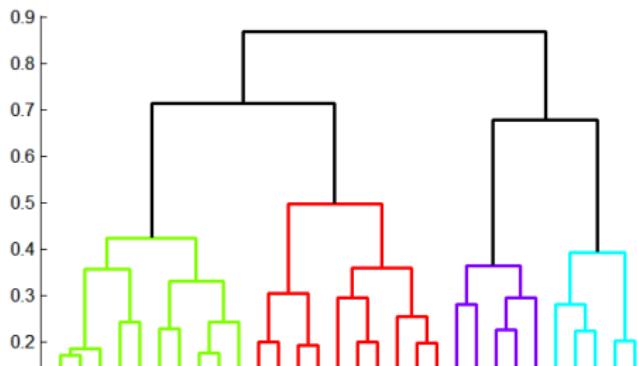
# Reaction barrier analysis

- linear optimization: stoichiometry matrix
- constraints: limits on reactions, exclusion of combinations of reactions
- barrier tree



## Flux similarity

- Compute pairwise similarity of elementary modes
- similarity between metabolites (in+out / all) through topological indices
- similarity between enzymes/reactions by comparing transition state structure



# Conclusion

- Summary
  - Structural and Functional measures for Robustness and Modularity
  - Follow the Law (Connectivity Distribution)
  - Size Matters (Knockout set Size)
  - Shape too (Chemical organization Hierarchy)
- Outlook
  - Investigate single networks (flux barriers, flux similarity)
  - Different scenarios (Horizontal Gene Transfer)
  - Structural modularity (Clustering Coefficient)

## Acknowledgements

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Dennis Goerlich