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# Axelrod's discrete model for the dissemination of Cultures: from dynamics on a network to its landscape

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# Axelrod's Model

## Motivation and Model's Description

- Culture was defined by Axelrod, in a simple fashion, as the set of people's characteristics subject to other individual's influence.
- The cultural state of an individual is therefore a multidimensional and discrete vector.
- The cultural vetor consists of $F$ cultural features, each of which can express one of $q$ possible traits:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\sigma_i^1$ | $\sigma_i^2$ | $\sigma_i^3$ | $\ldots$ | $\sigma_i^f$ | $\ldots$ | $\sigma_i^{F-1}$ | $\sigma_i^F$ |

- $\sigma_i^f \in \{0, 1, 2, \ldots q-1\}$
- These two parametres, $F$ and $q$, are related to cultural diversity, and the dynamics depend on their choice.

# Axelrod's Model

## Comparing Vectors

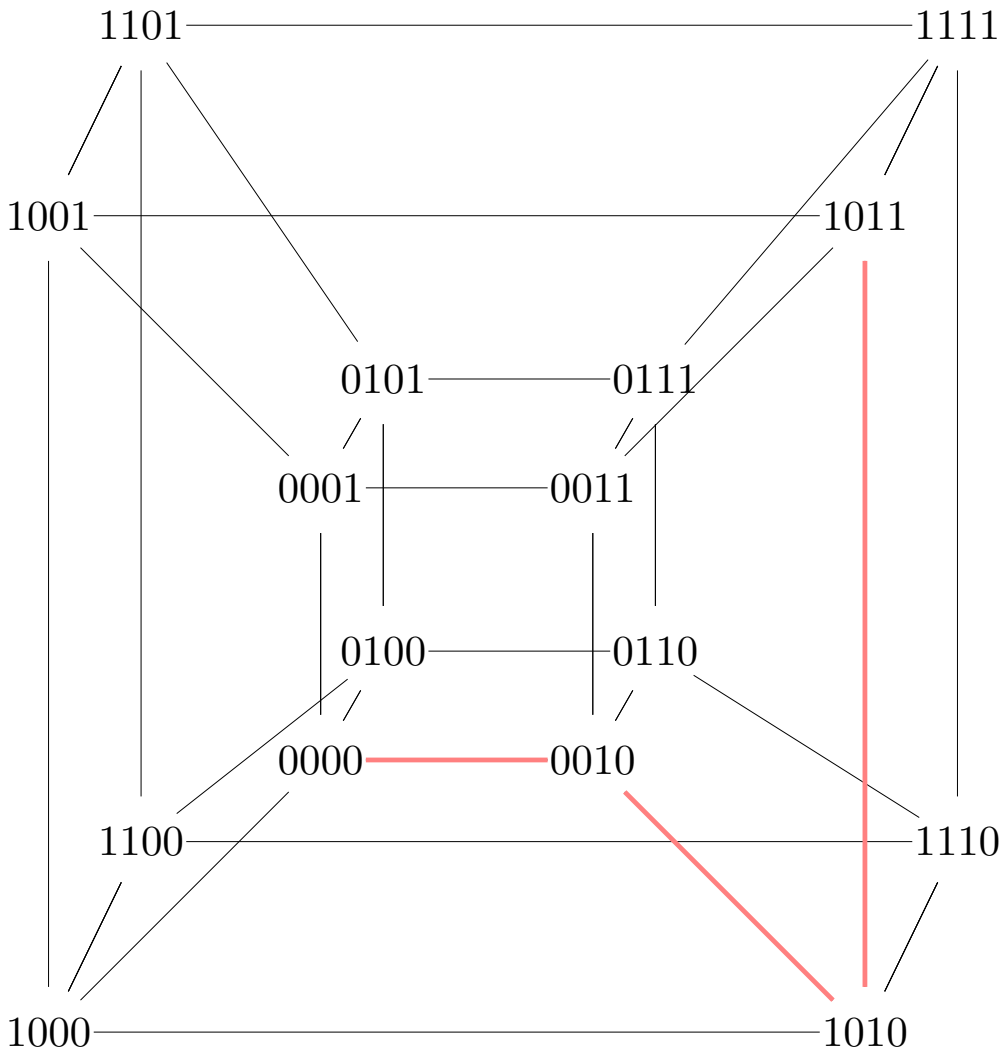
- "The transfer of ideas occurs most frequently between individuals [...] who are similar in certain attributes such as beliefs, education, social status, and the like"
- Hamming distance: $d_{\vec{\sigma_i},\vec{\sigma_j}} = F - \sum_{f=1}^{F} \delta_{\sigma_i^f,\sigma_j^f}$
- Similarity: $\omega_{ij} = \frac{1}{F} \sum_{f=1}^{F} \delta_{\sigma_i^f,\sigma_j^f}$
- Example: $F = 4$, $q = 2$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |

| | | | |
|---|---|---|---|
| 1 | 0 | 1 | 1 |

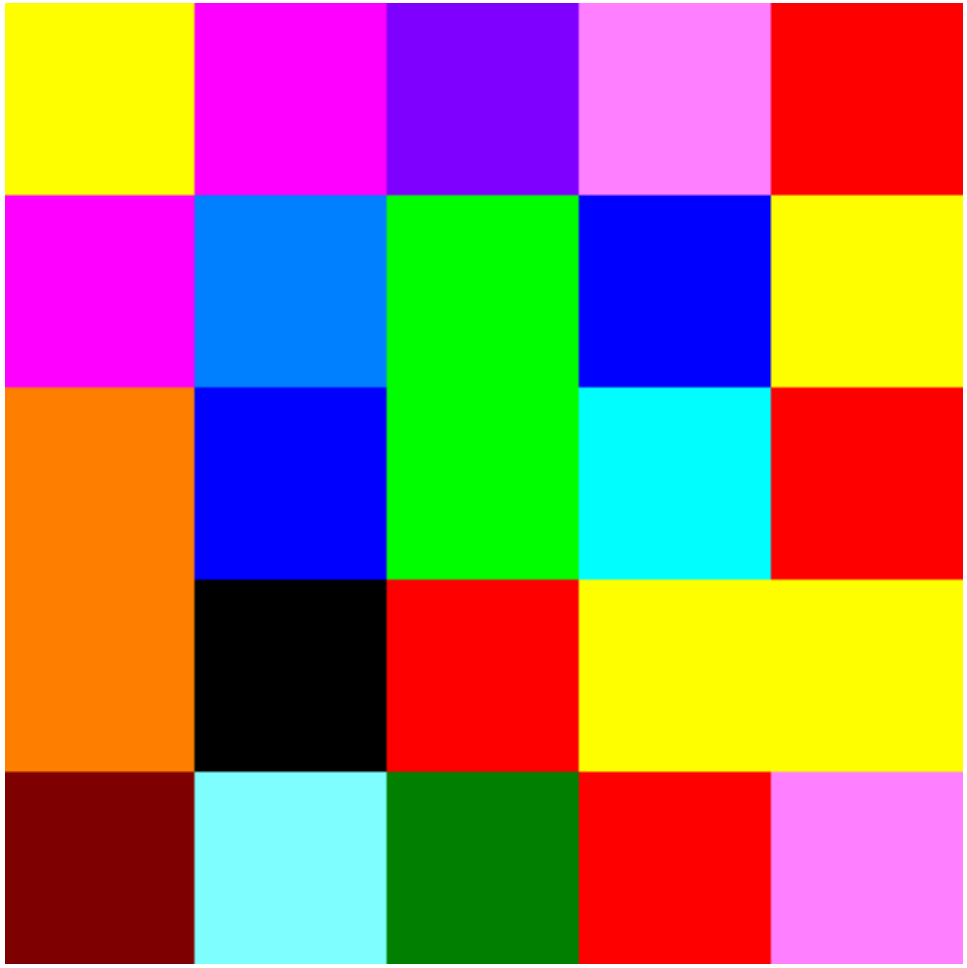
# Axelrod's Model

- Example of the cultural space, with $F = 4$ and $q = 2$:



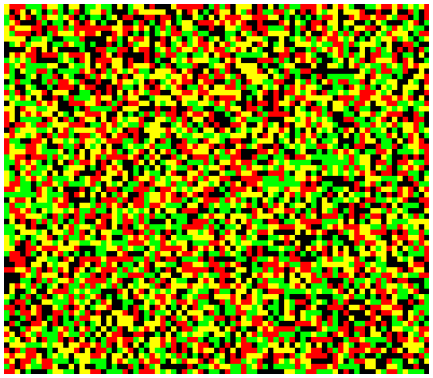
# Axelrod's Model

The node representation: Connection topologies: regular square lattice (periodic boundary conditions)

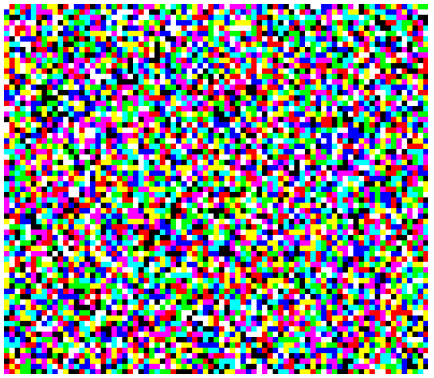


# Axelrod's Model

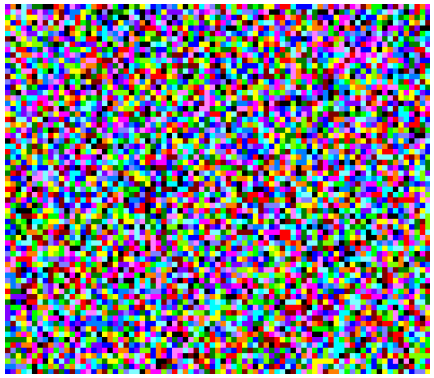
Initial conditions: Uniform distribution $L = 80$



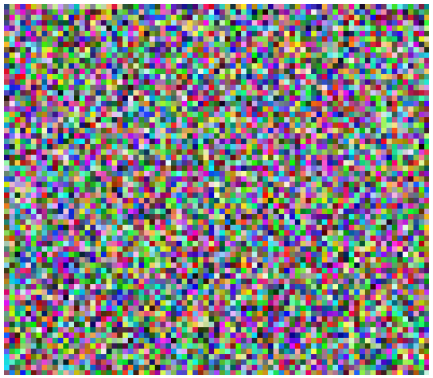
F=2 q=2



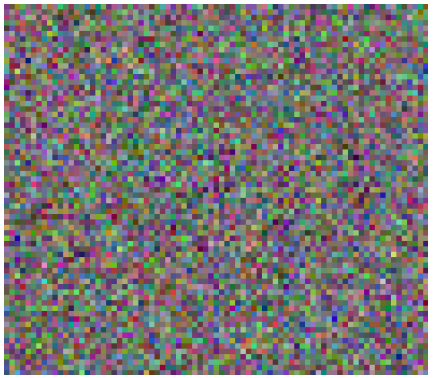
F=3 q=2



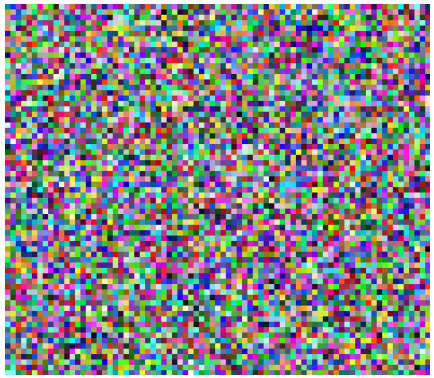
F=4 q=2



F=17 q=150



F=25 q=2



F=25 q=25

# Axelrod's Model

## Dynamics

- Asynchronous and stochastic model
- Iteration of the following steps:

  **Step 1:** Randomly choose one active site $i$ and one of its neighbours $j \in \nu_i$, where $\nu_i$ is the neighbourhood of site $i$.

  **Step 2:** With a probability equal to their similarity, there is an interaction. The interaction consists of randomly selecting an $f$ such that $\sigma_i^f \neq \sigma_j^f$ (if there is), and making site $j$ adopt $\sigma_i^f$.

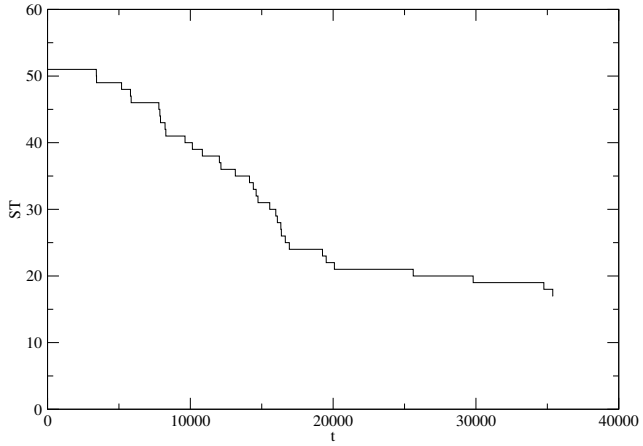
- A site $i$ is active if at least one of its neighbours $j$ is in a state such that $0 < \omega_{ij} < 1$
- This is not an 'energy-driven' process.
- Markov chain: The configuration space can be represented by a directed graph, where nodes are configurations and the directed edges are possible transitions associated with their probabilities.
- Time evolution is a random walk on this graph.

# Axelrod's Model

## Transient behaviour

- In every interaction one cultural trait is discarded, eventually becoming extinct
- One possible Lyapunov function is given by the counting of Surviving Traits in every time step

$$ST = qF - \sum_{f=1}^{F} \sum_{t=0}^{q-1} [\delta_{0,\sum_{i=1}^{N} \delta_{t,\sigma_i^f}}]$$




Time series of the Lyapunov Function ST $L = 60$, $F = 17$ and $q = 3$.

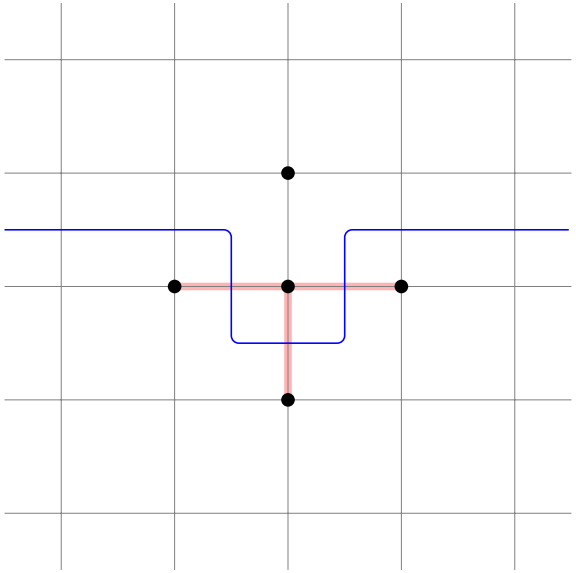
# The Model with Surface Tension

## Model's description and motivation:

- A subtle difference is introduced:

**Step 1:** Randomly choose one active site $i$ and one of its neighbours $j$ such that $0 < \omega_{ij} < 1$.

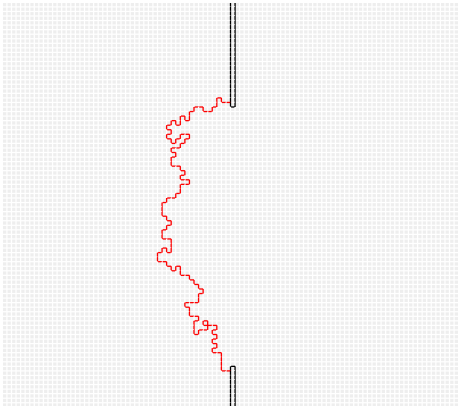
**Step 2:** With a probability equal to their similarity, there is an interaction. The interaction consists of randomly selecting an $f$ such that $\sigma_i^f \neq \sigma_j^f$ and making site $j$ adopt $\sigma_i^f$.



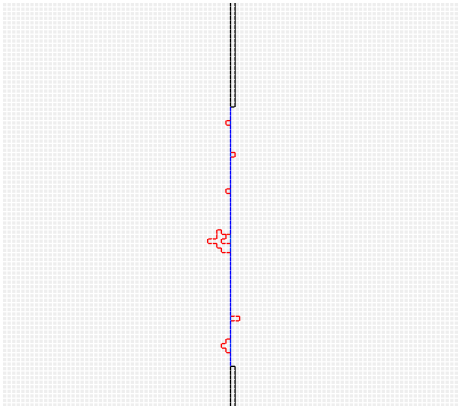
# The Model with Surface Tension

## Consequences:

- Cultural cohesion and spatial localization
- Cultural borders and membranes - cultural exchange and multicultural regions
- Simple or composed membranes - adhesion tendency
- Metastability
- This is achieved without changing topologically the configuration space.



$\omega_{ij} = \frac{29}{30}$

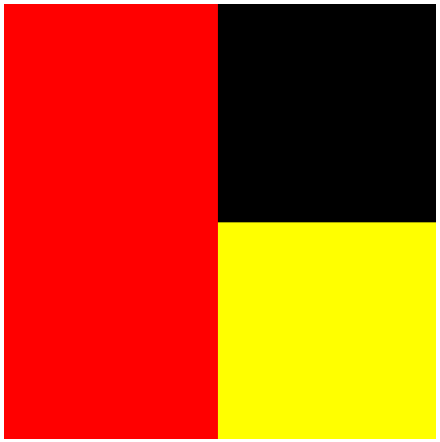


$\omega_{ij} = \frac{1}{30}$

# The Configuration Space

## Simplest case: $L = 2$, $F = 2$

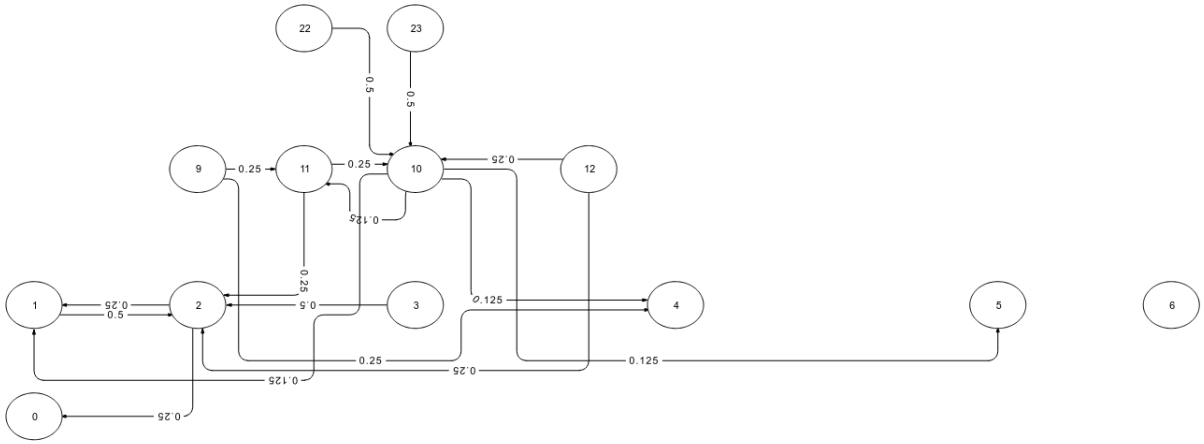
- Number of nodes in the configuration space: $q^8$
- Lyapunov function associated: landscape
- Collapsing nodes by symmetry - drastic reduction of landscape size.



One possible configuration for $q = 2$.

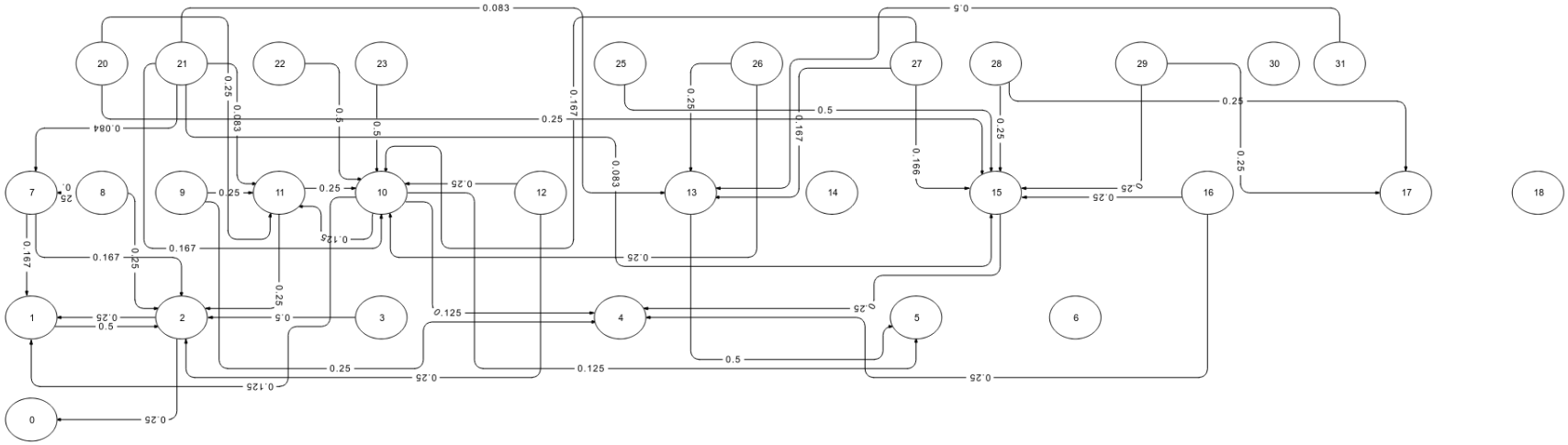
# The Configuration Space

Simplest case: $L = 2$, $F = 2$, $q = 2$ (13 nodes)



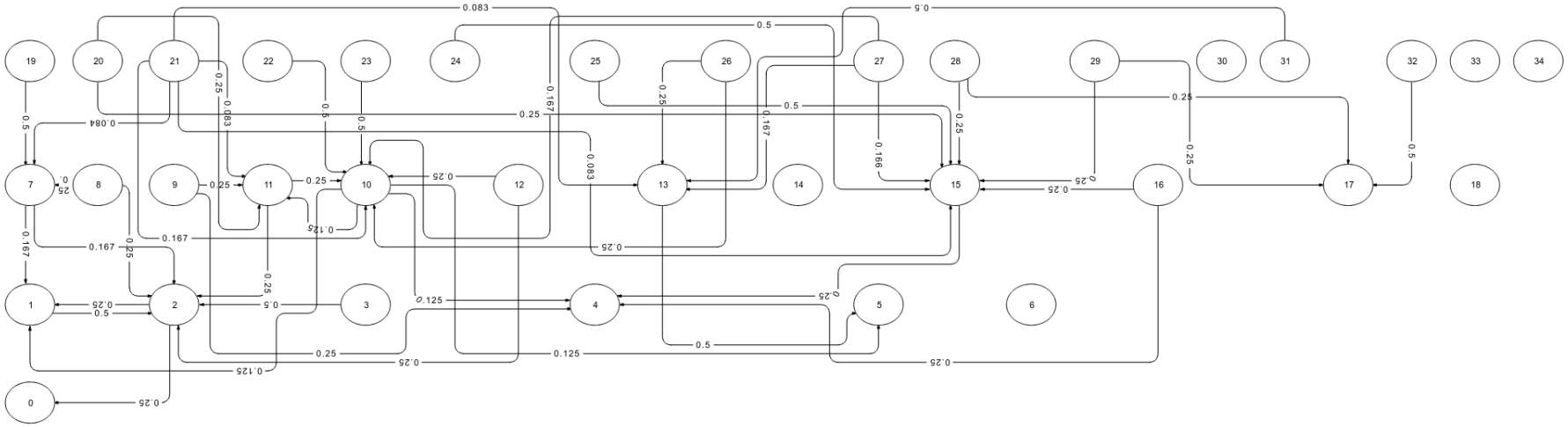
# The Configuration Space

Simplest case: $L = 2$, $F = 2$, $q = 3$ (30 nodes)



# The Configuration Space

Simplest case: $L = 2$, $F = 2$, $q \geq 4$ (35 nodes)



- Dynamics equivalent to random walk based on the weights.
- Distributing probabilities, time evolution and stationary distribution are obtained.
- The surface tension was introduced without changing the topology, only the weights.

# Conclusion

- Discrete dynamic model on a network, discrete configuration space.
- Subtle modification of Axelrod's Model - the topology of the configuration space is not altered, but the dynamics are.
- Landscape topology and transition probabilities drive dynamics.
- Distribution of weights create different attractor structures throughout landscape (metastability).

# Acknowledgement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