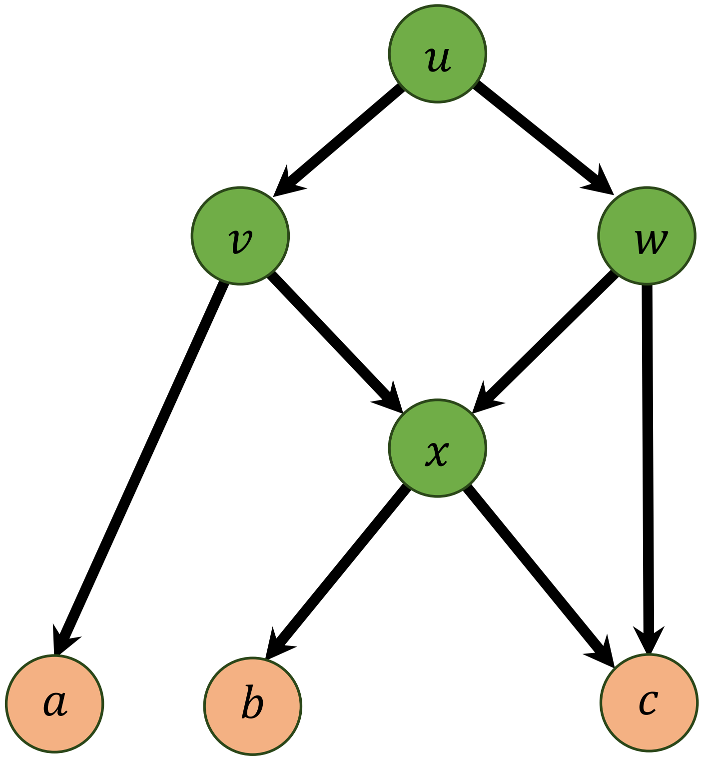
Descendant Clusters

Bruno J. Schmidt, Marc Hellmuth, Peter F. Stadler

Leaf & Descendant Clusters



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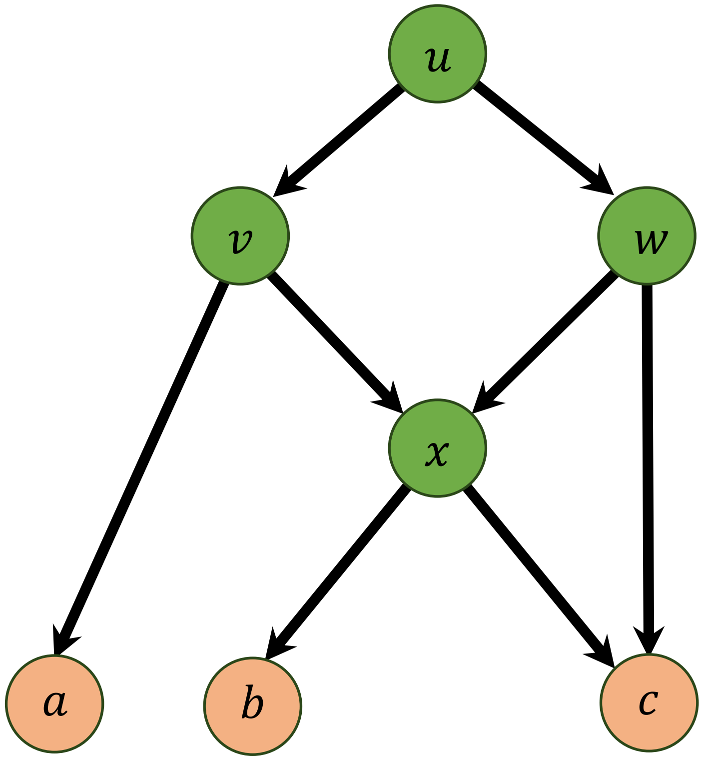

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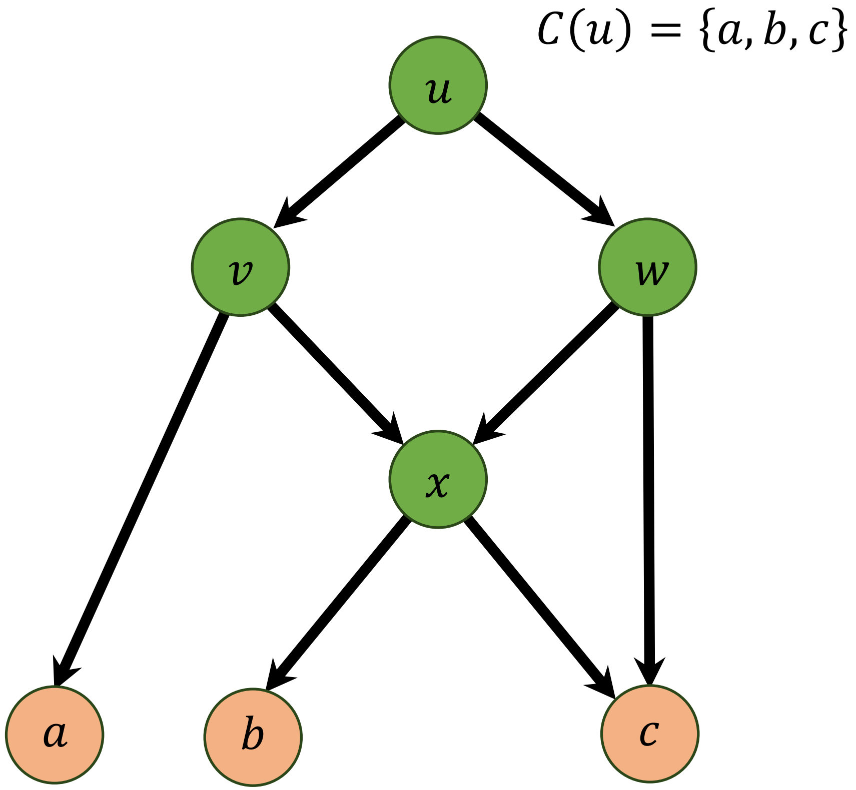

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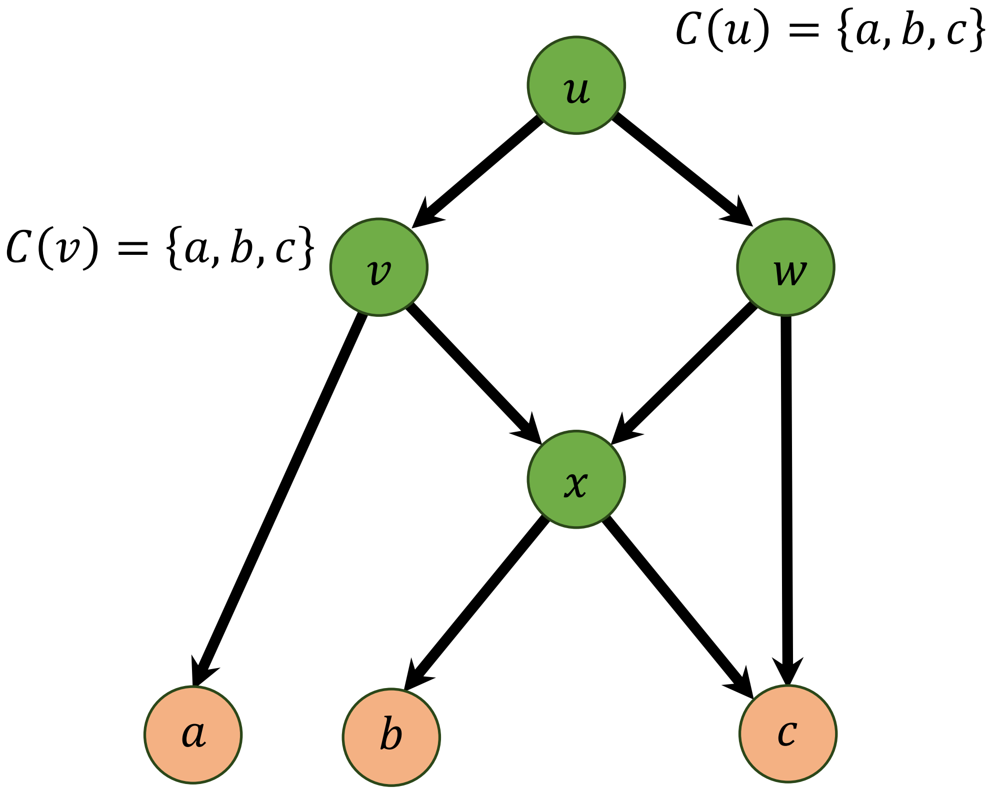

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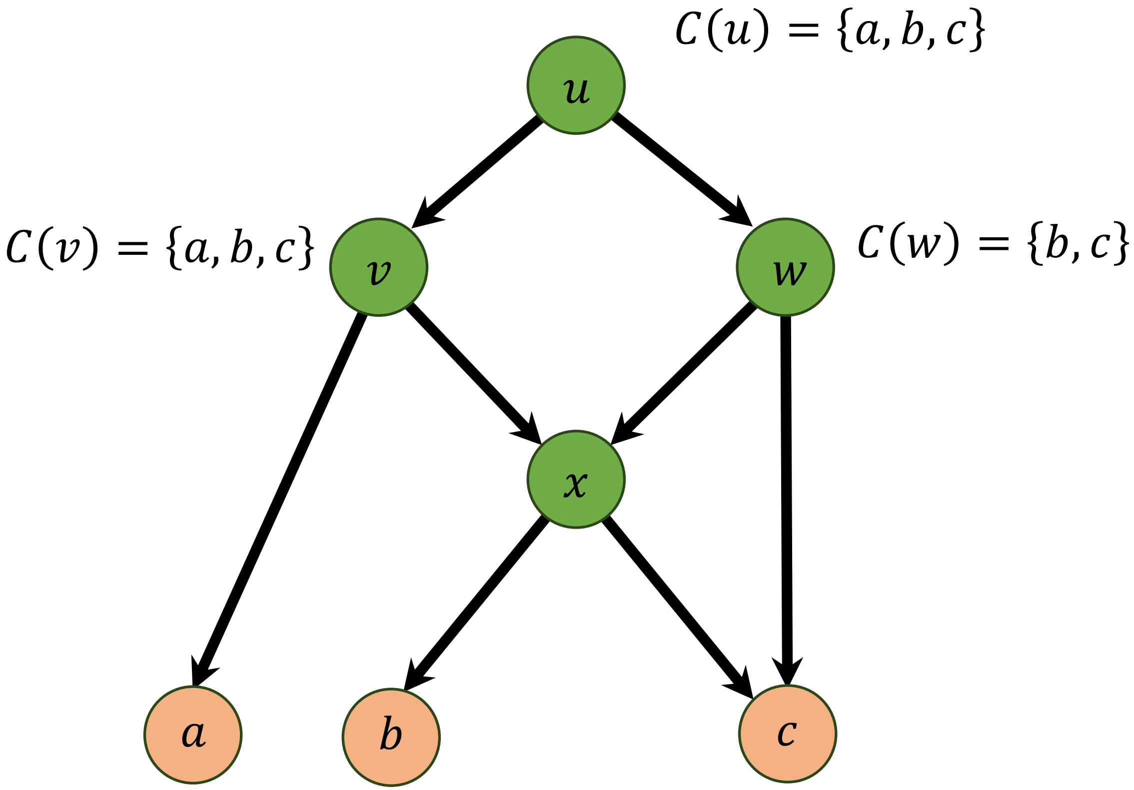

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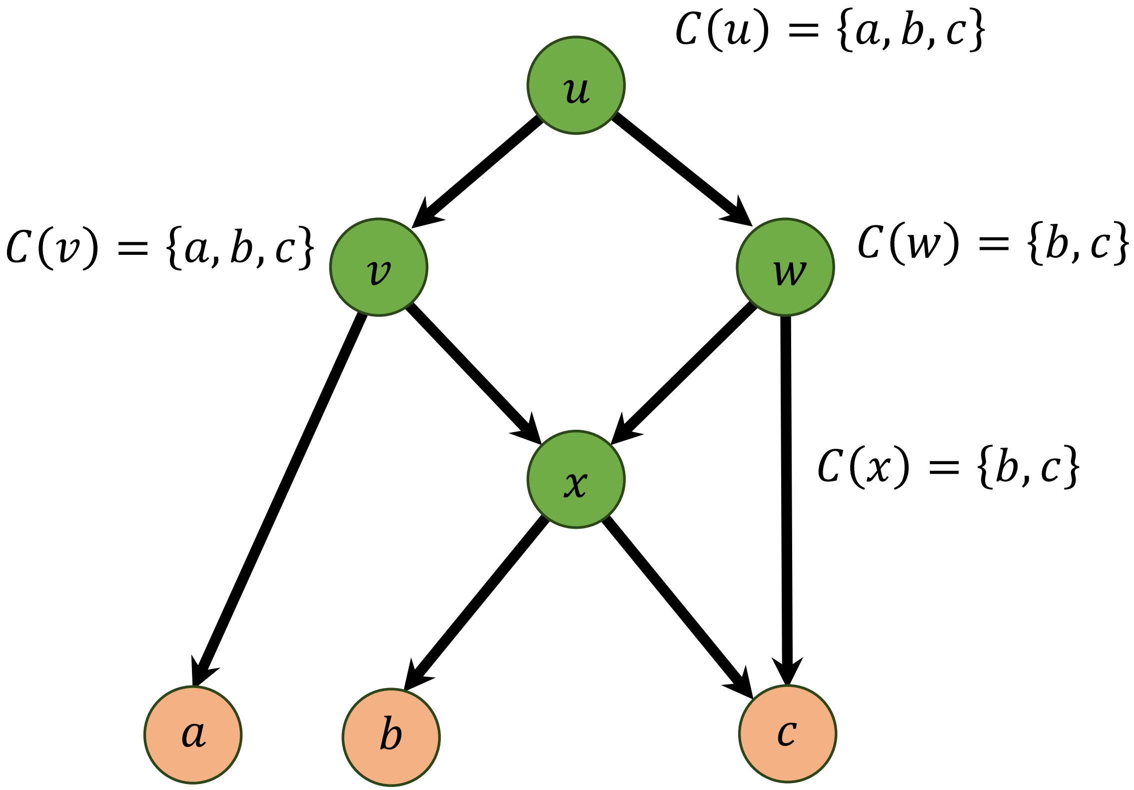

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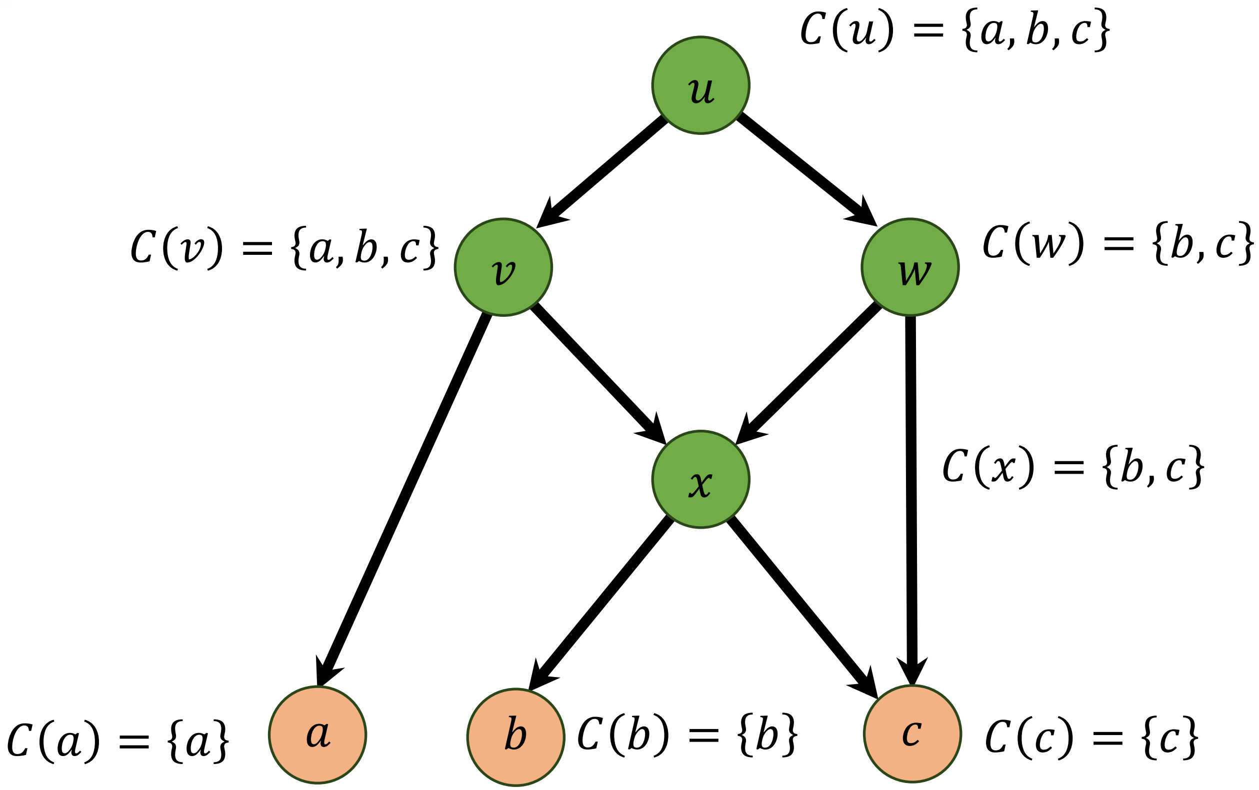

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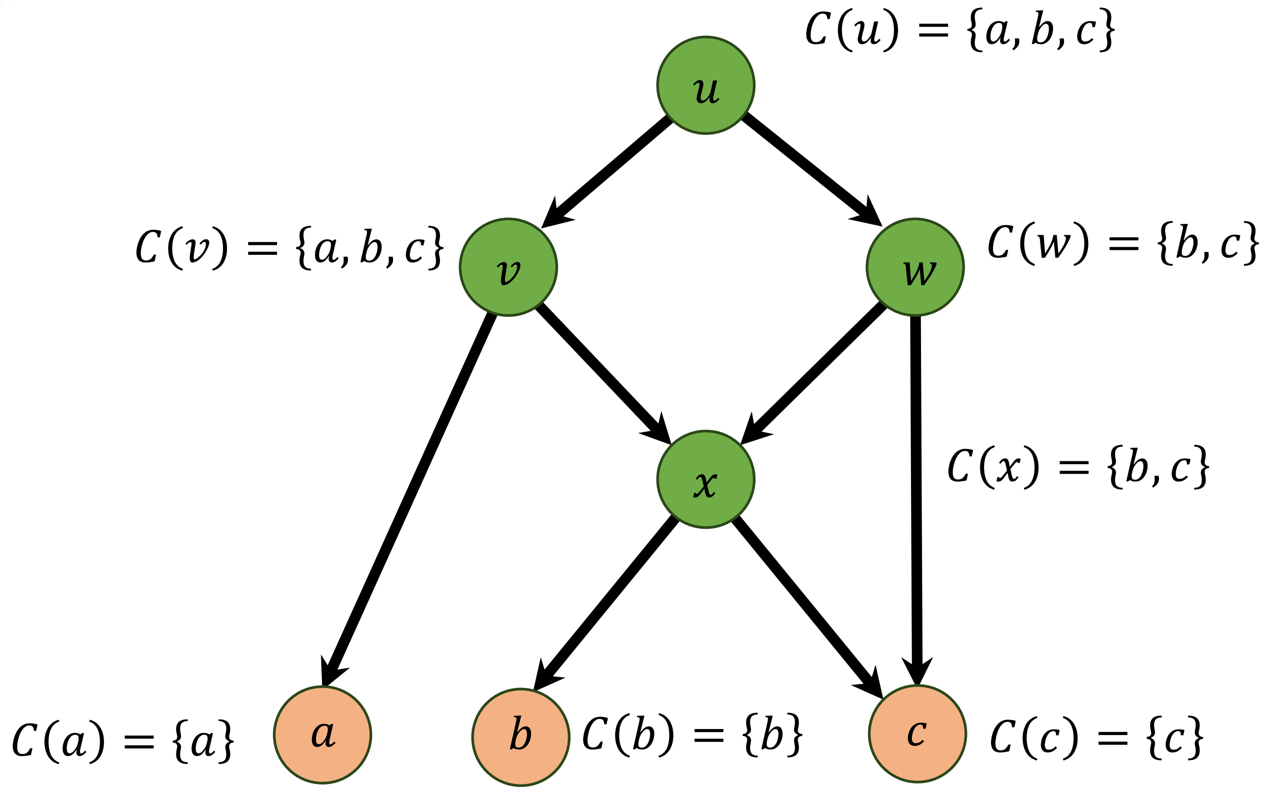

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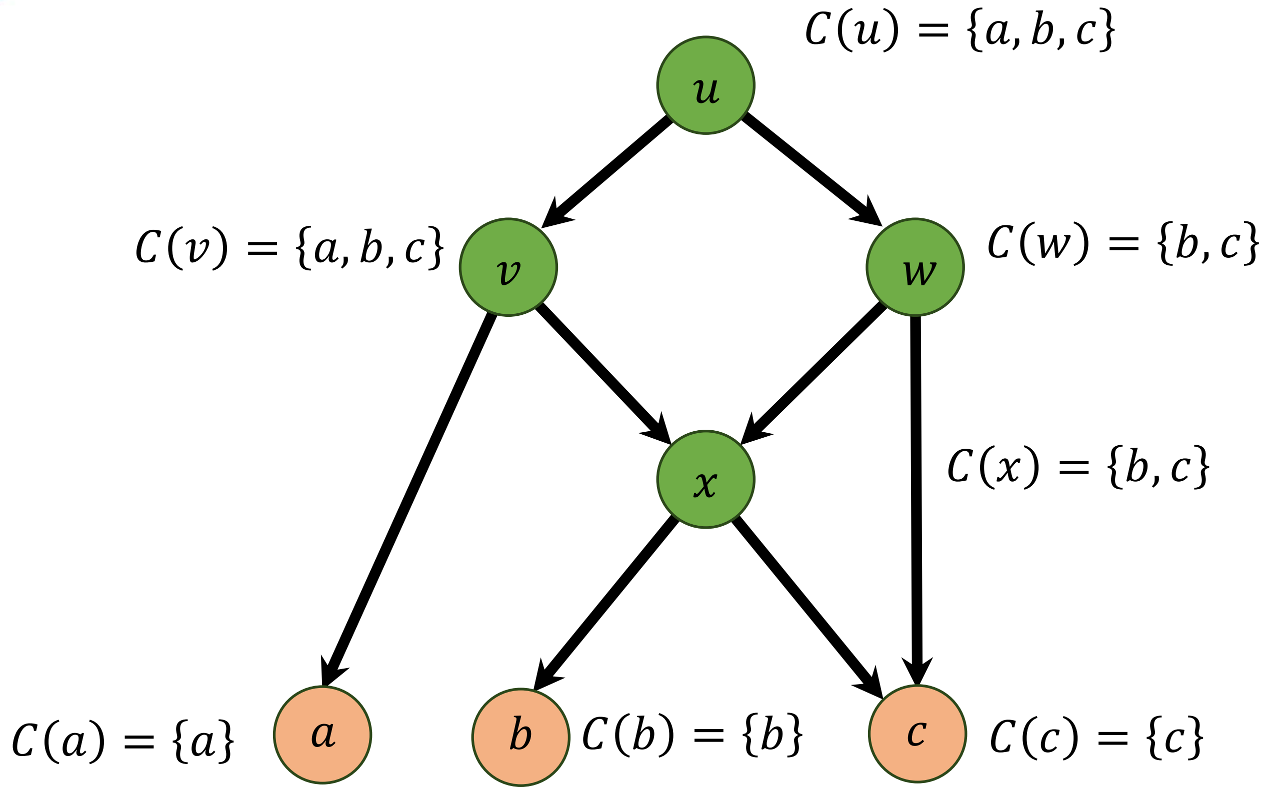

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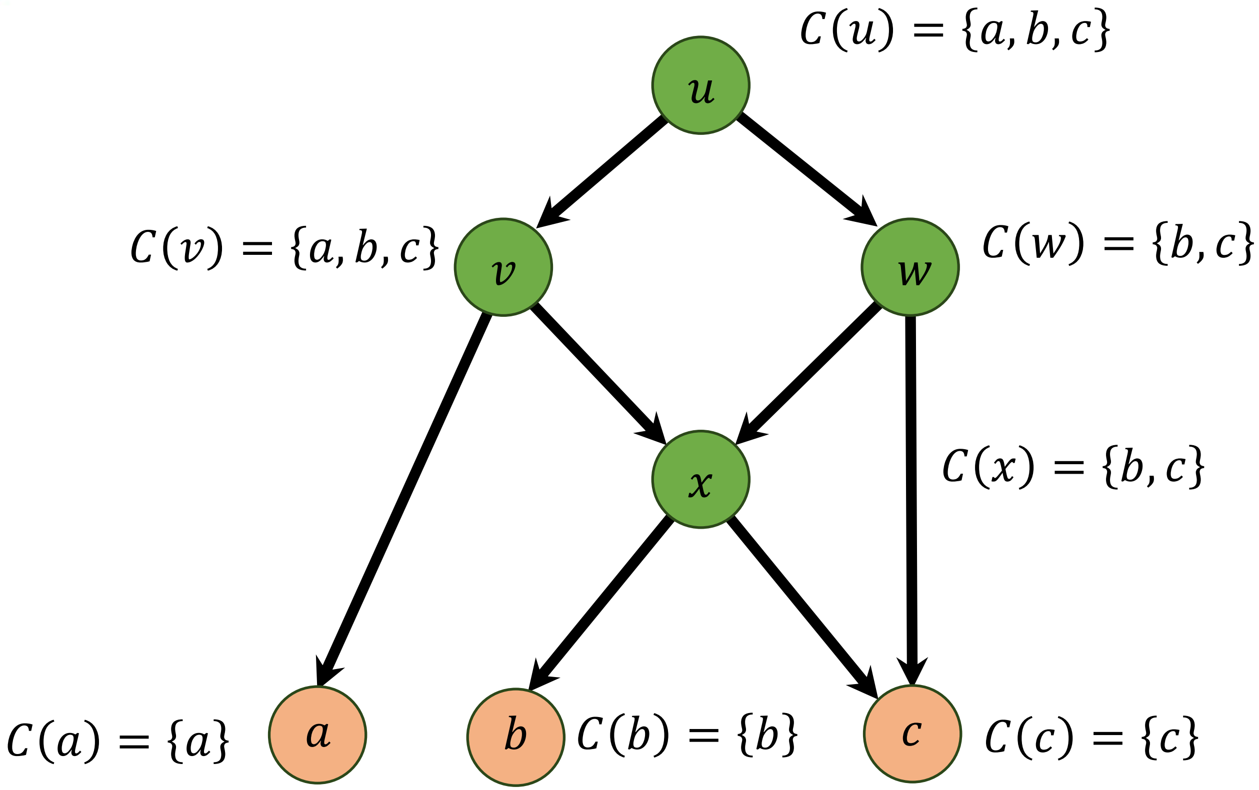

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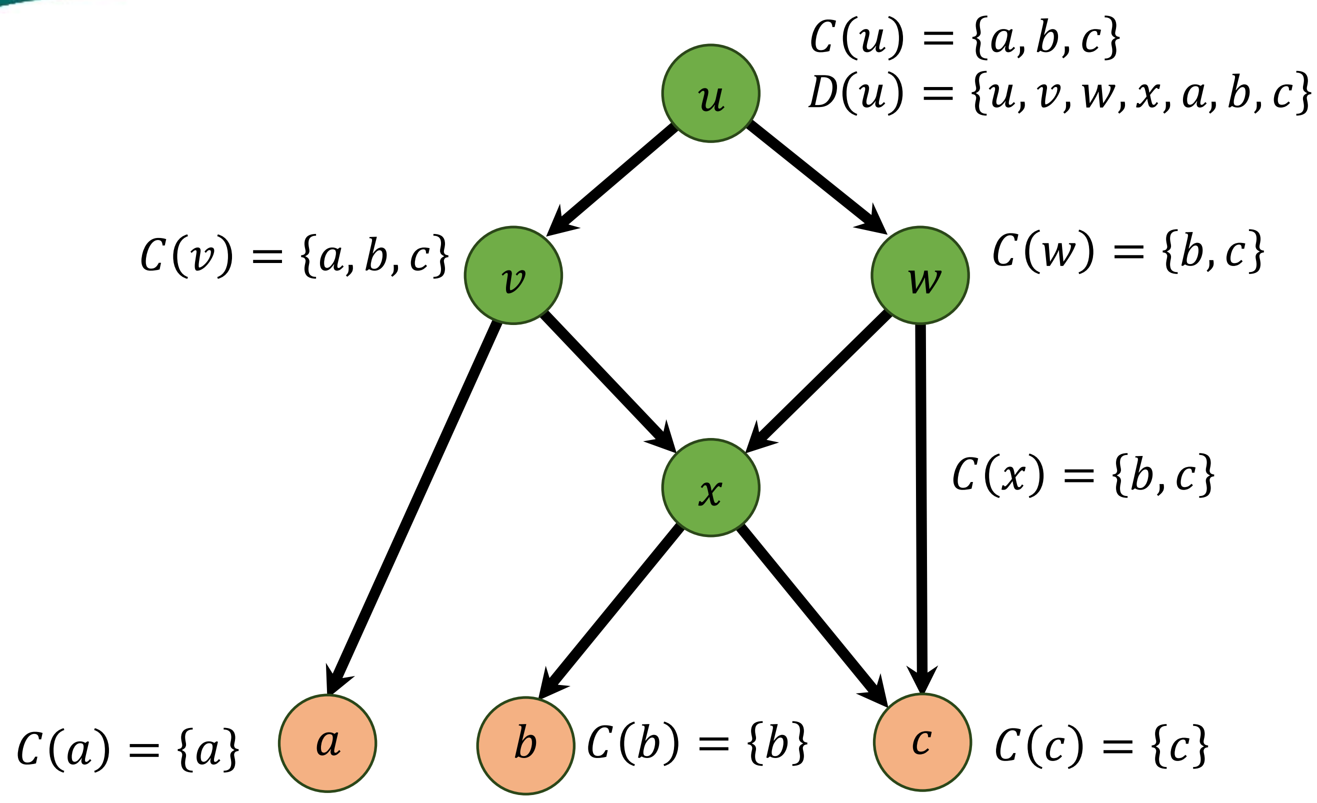

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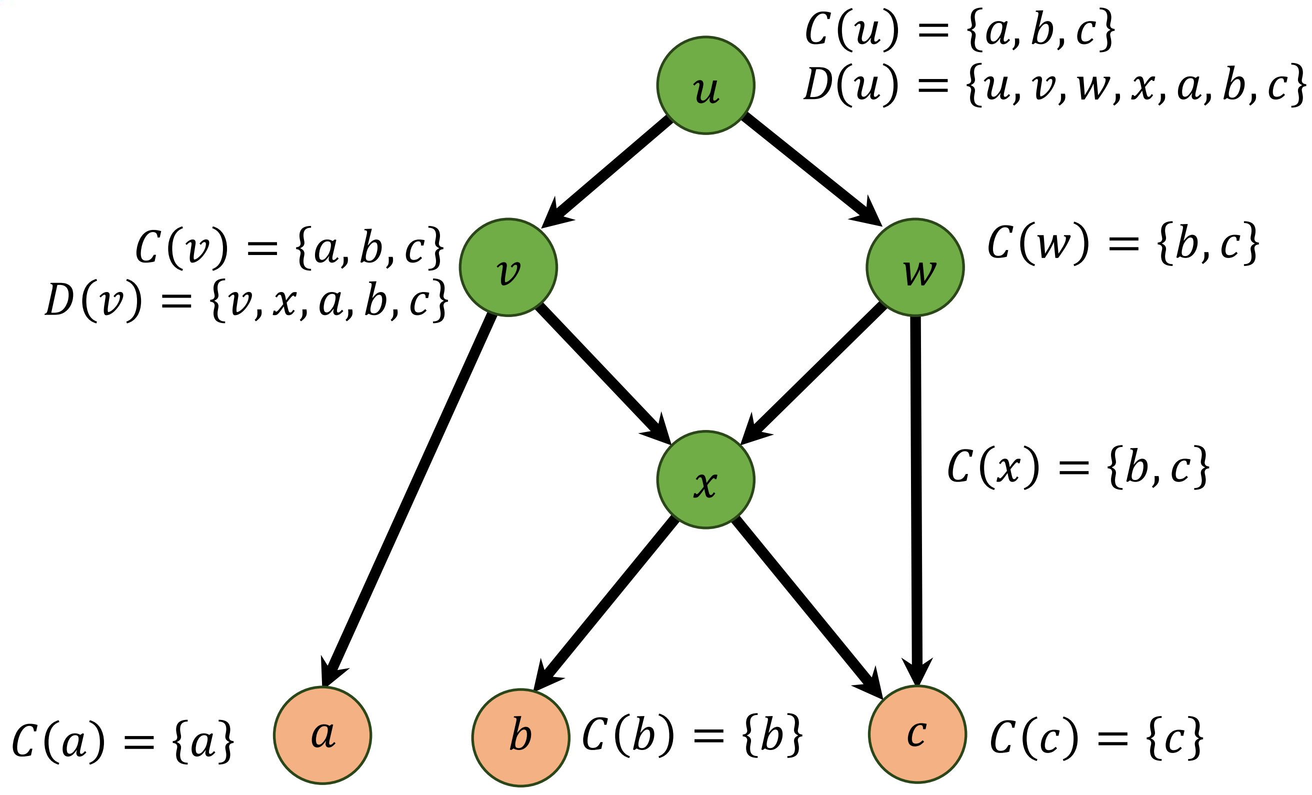

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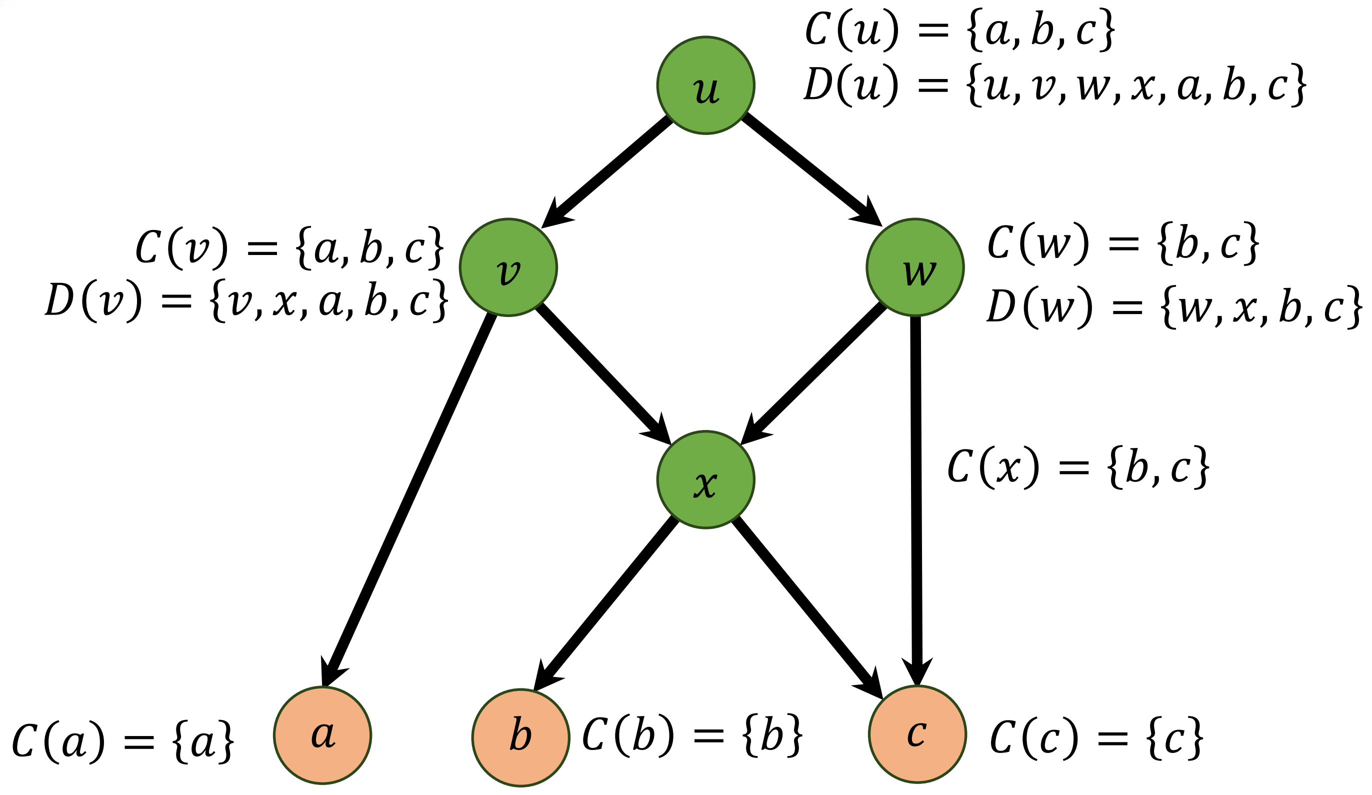

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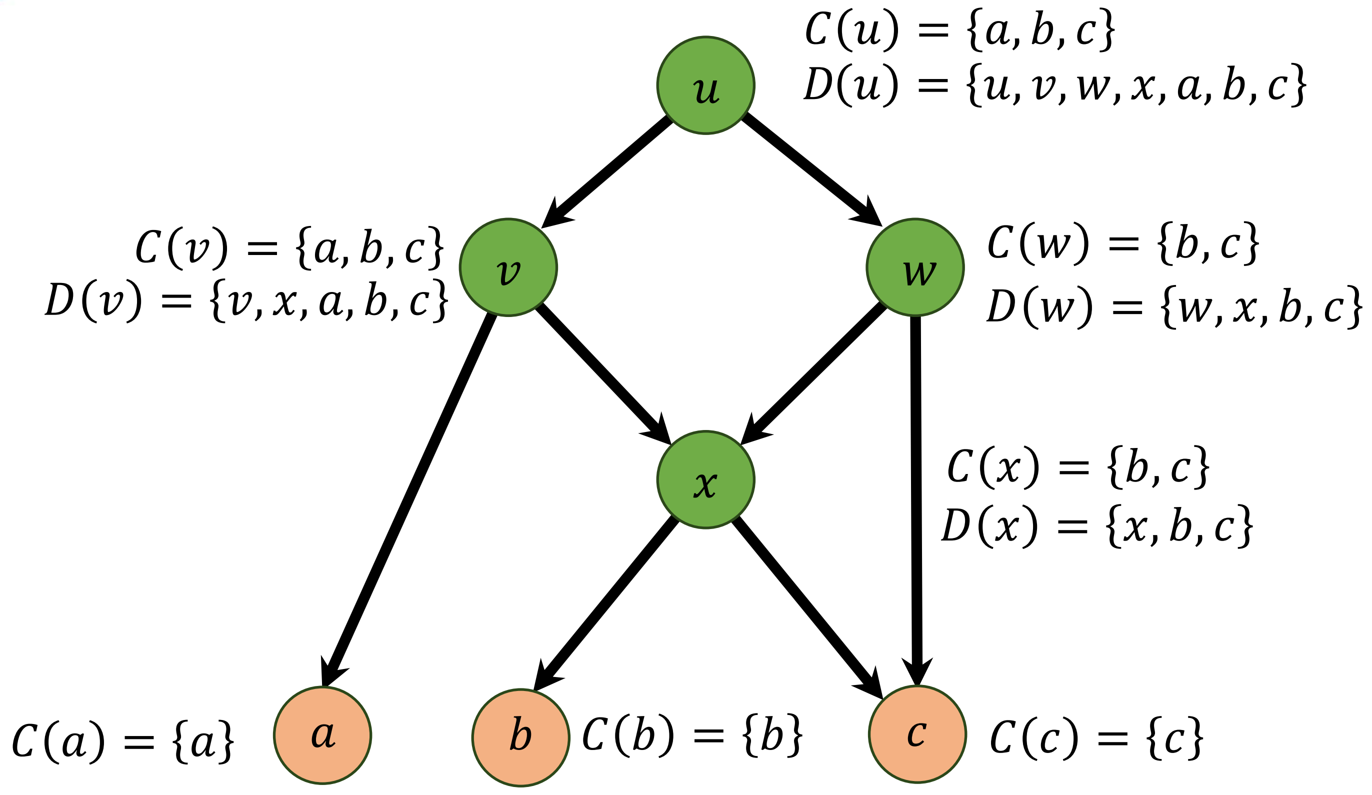

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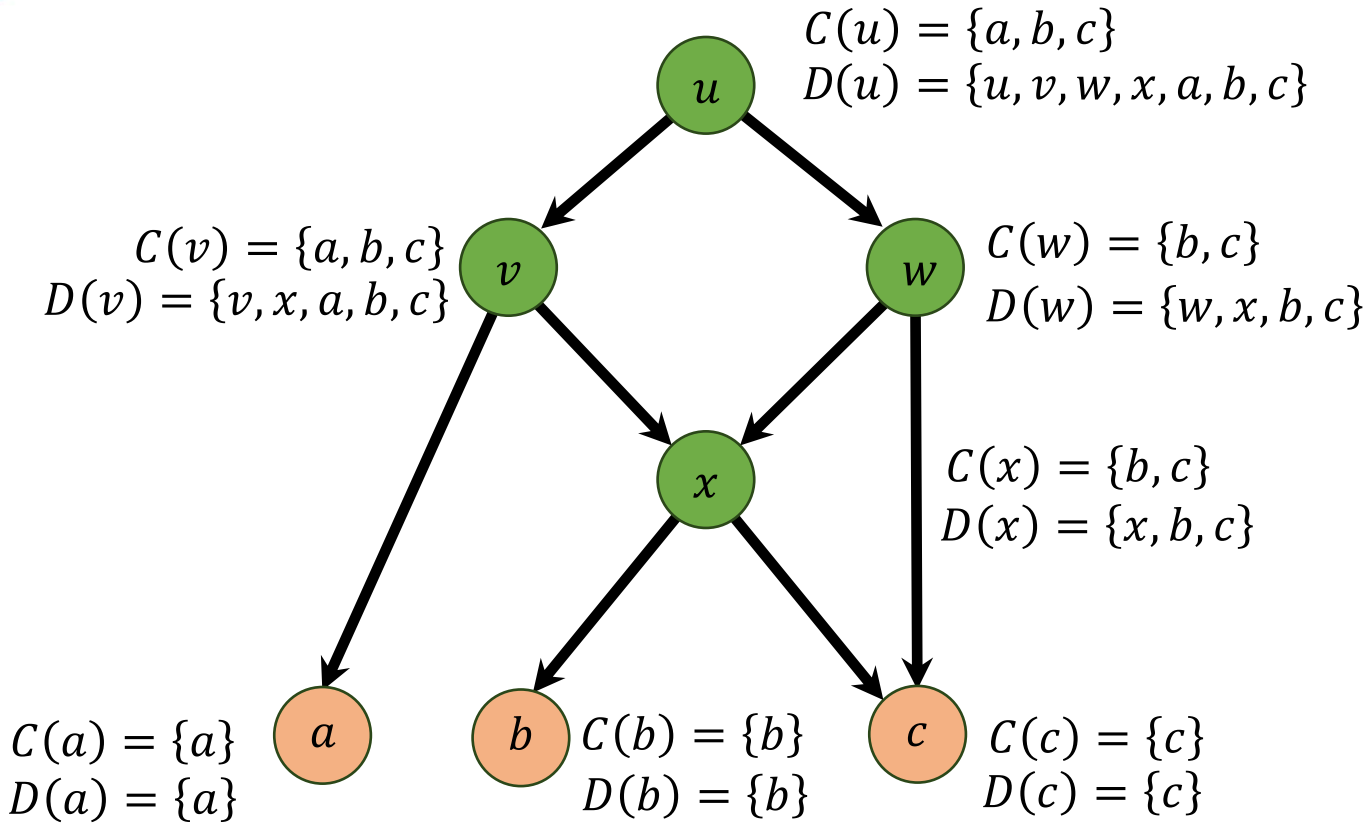

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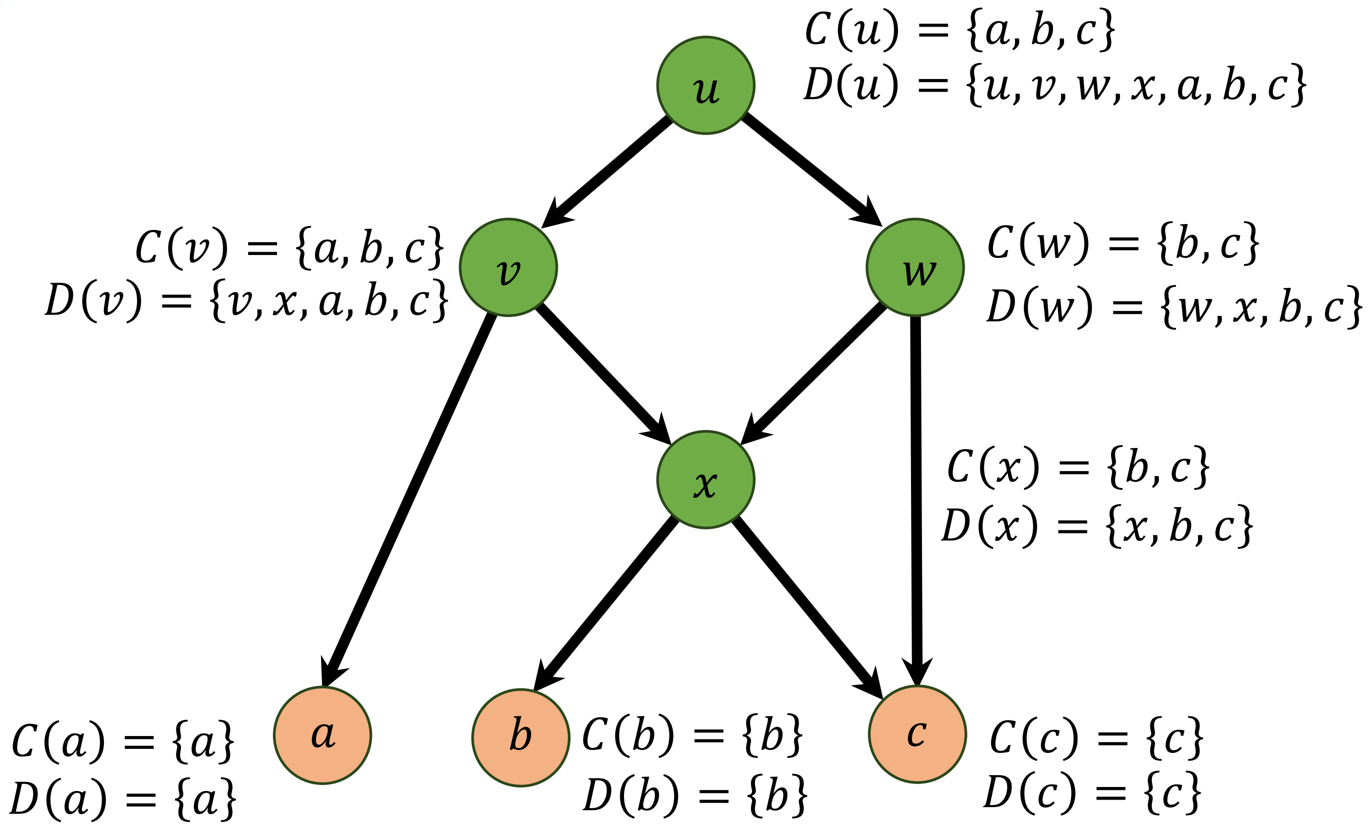

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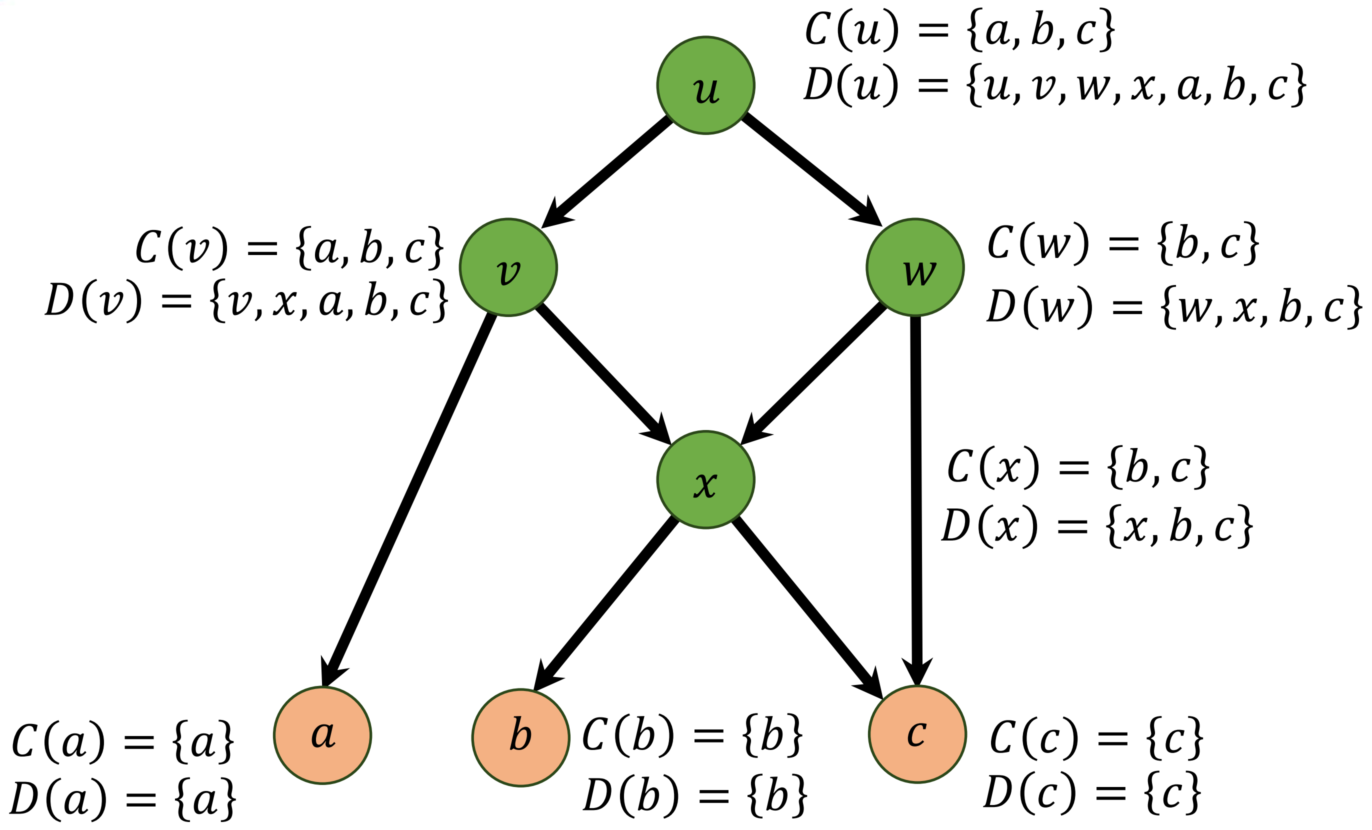

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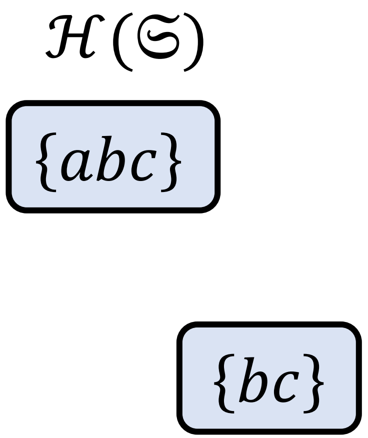

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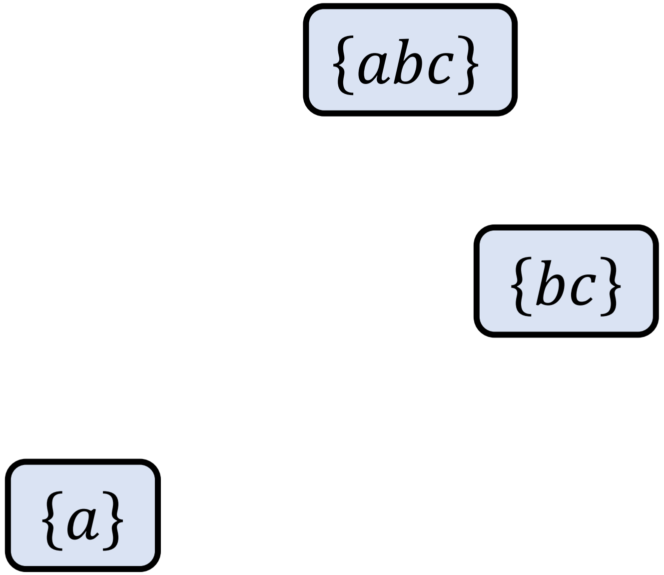

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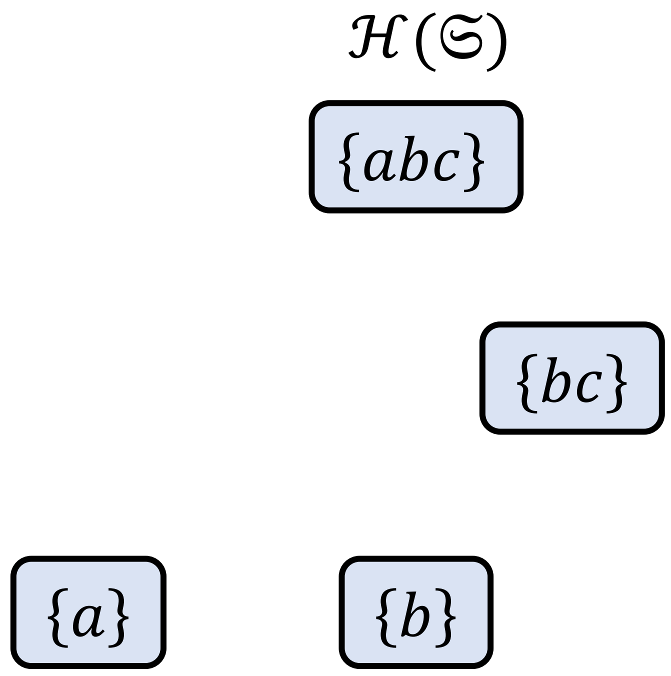

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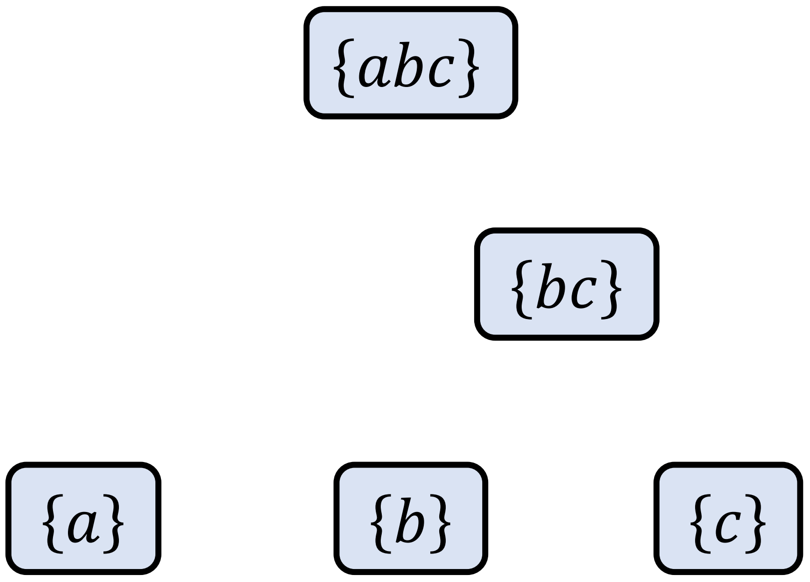

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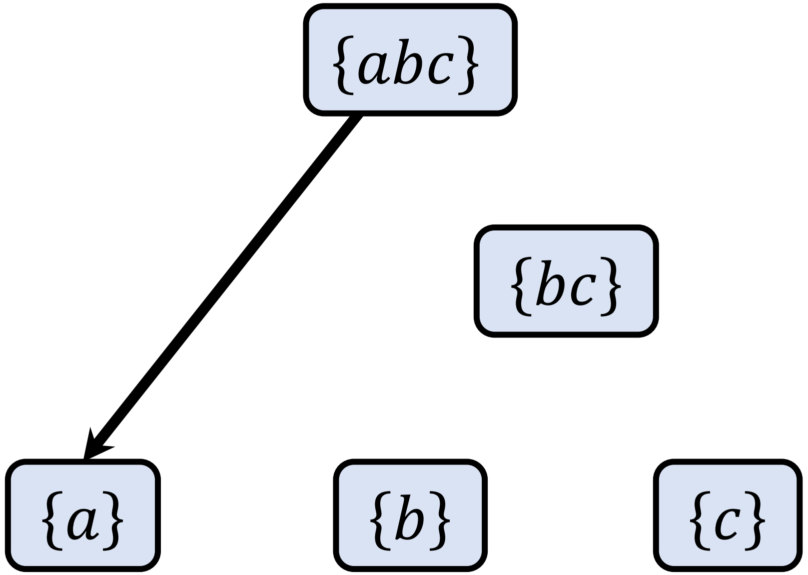

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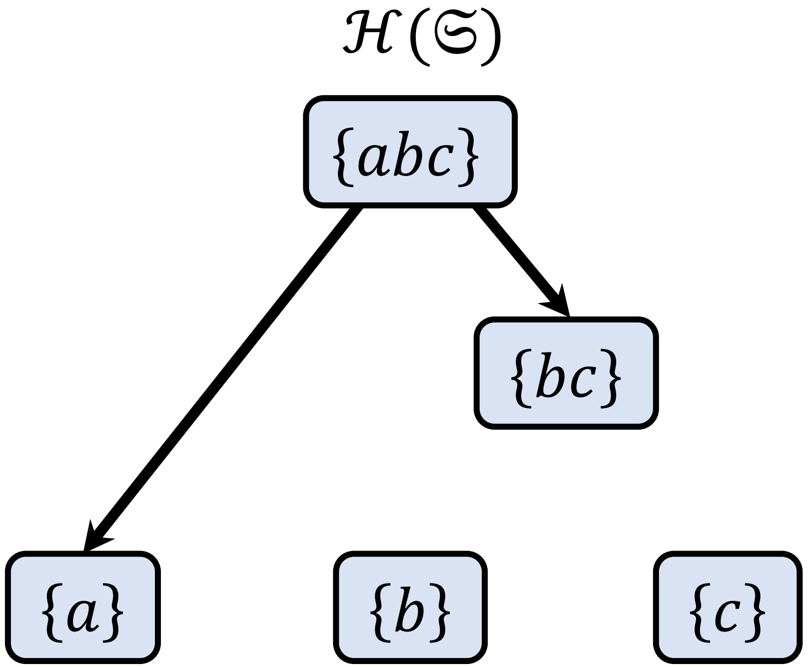

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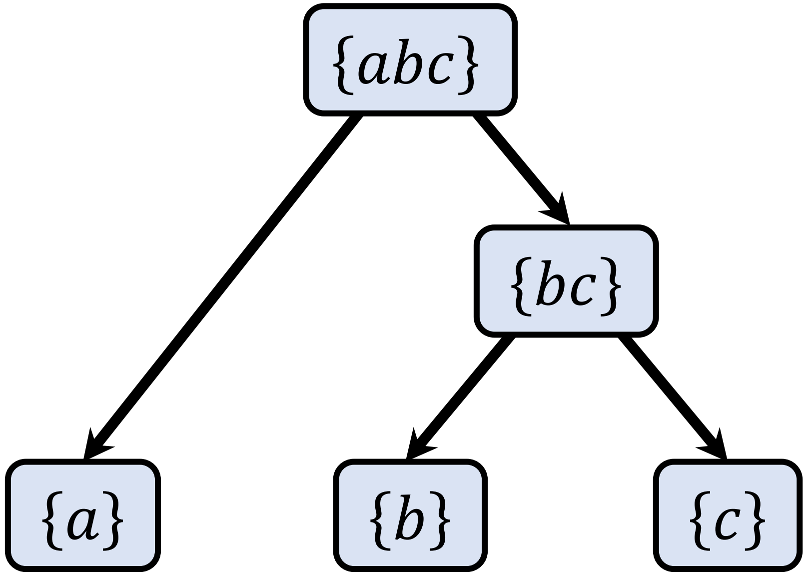

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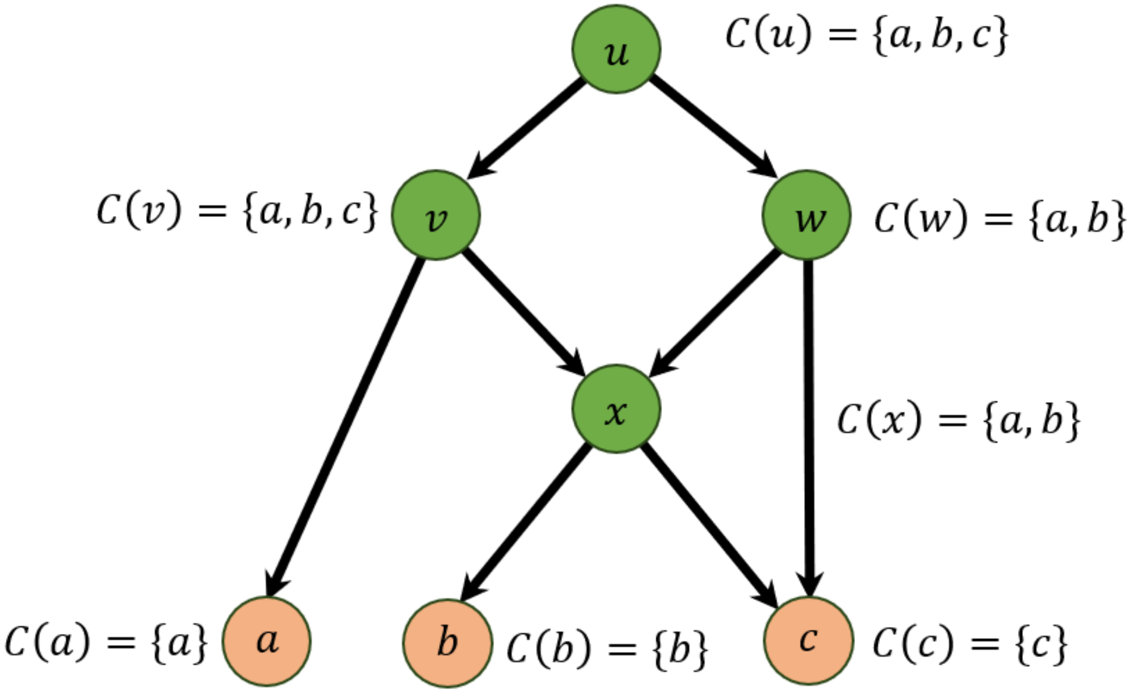

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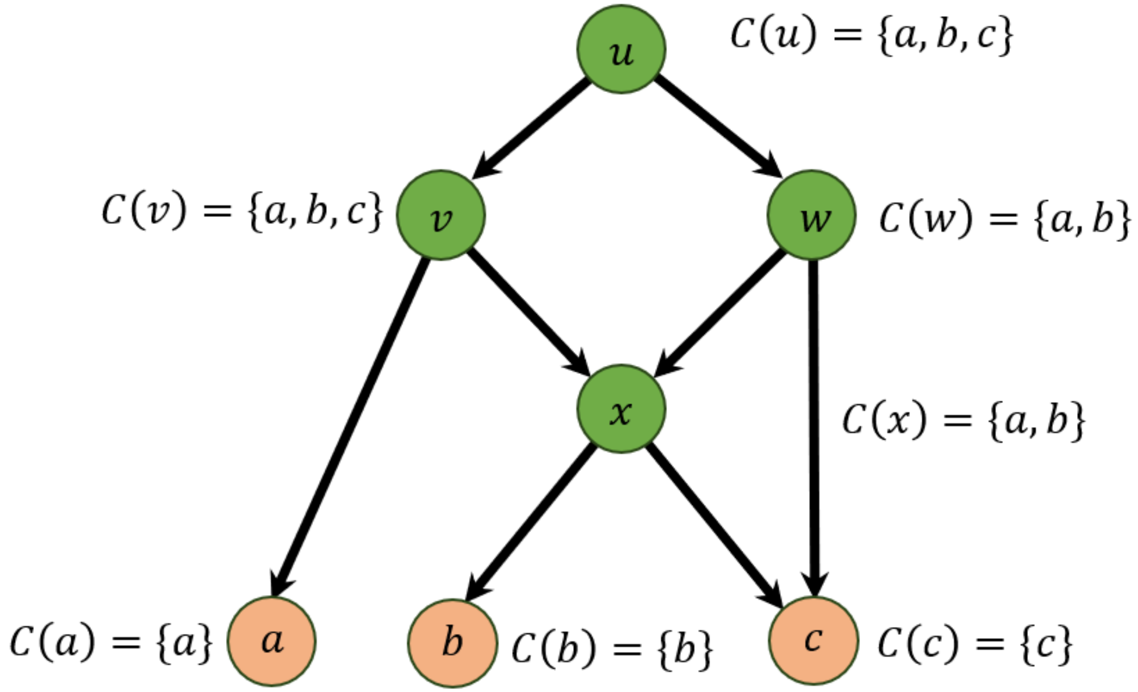

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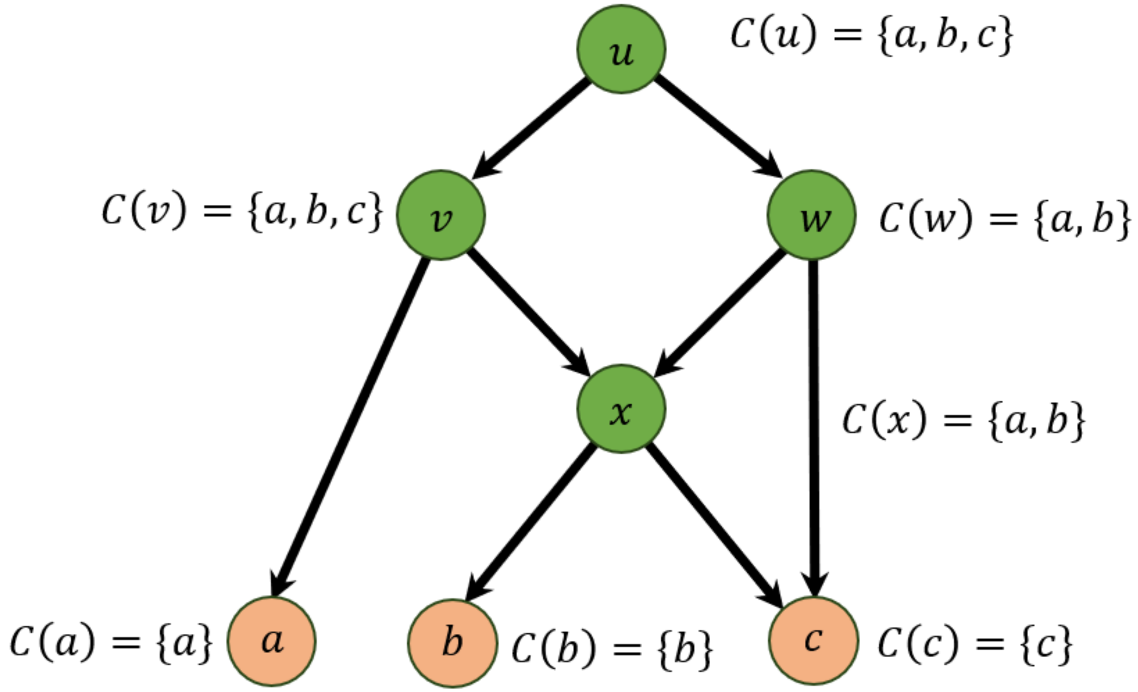

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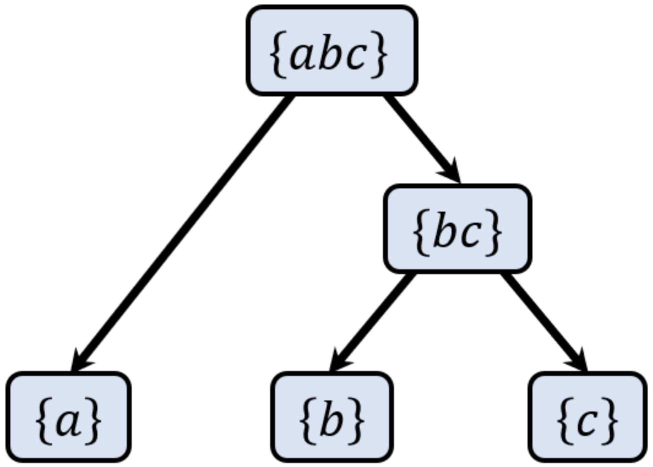

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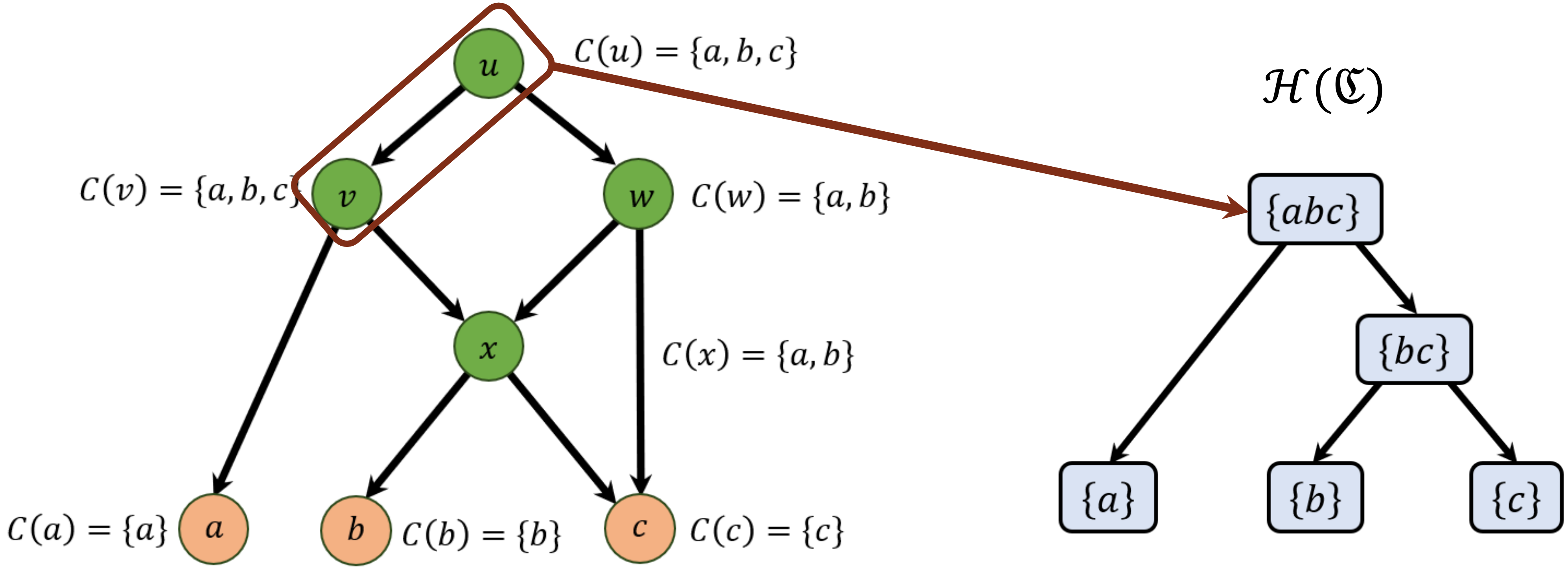

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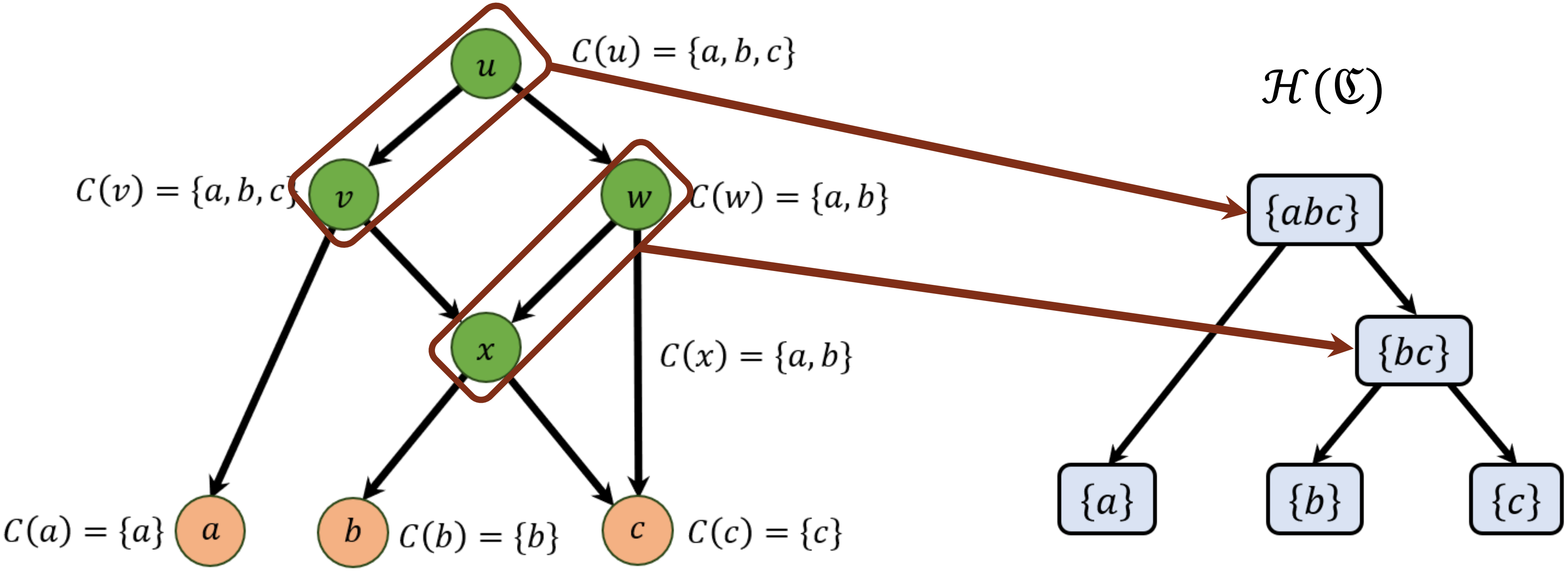

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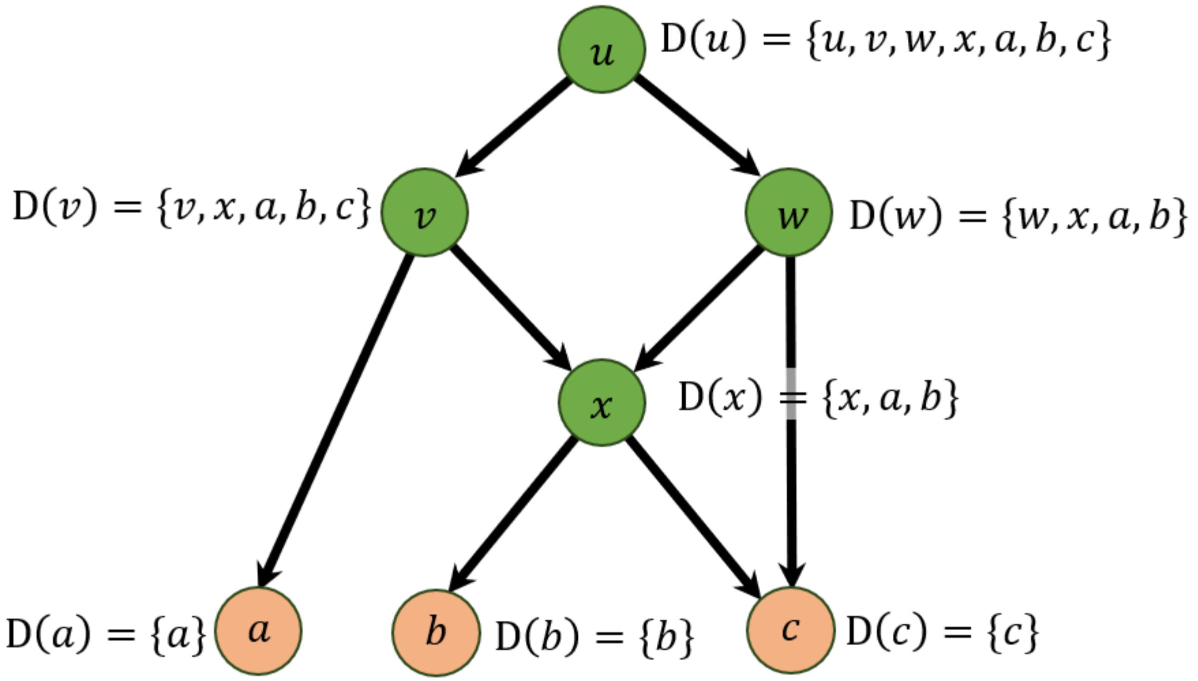

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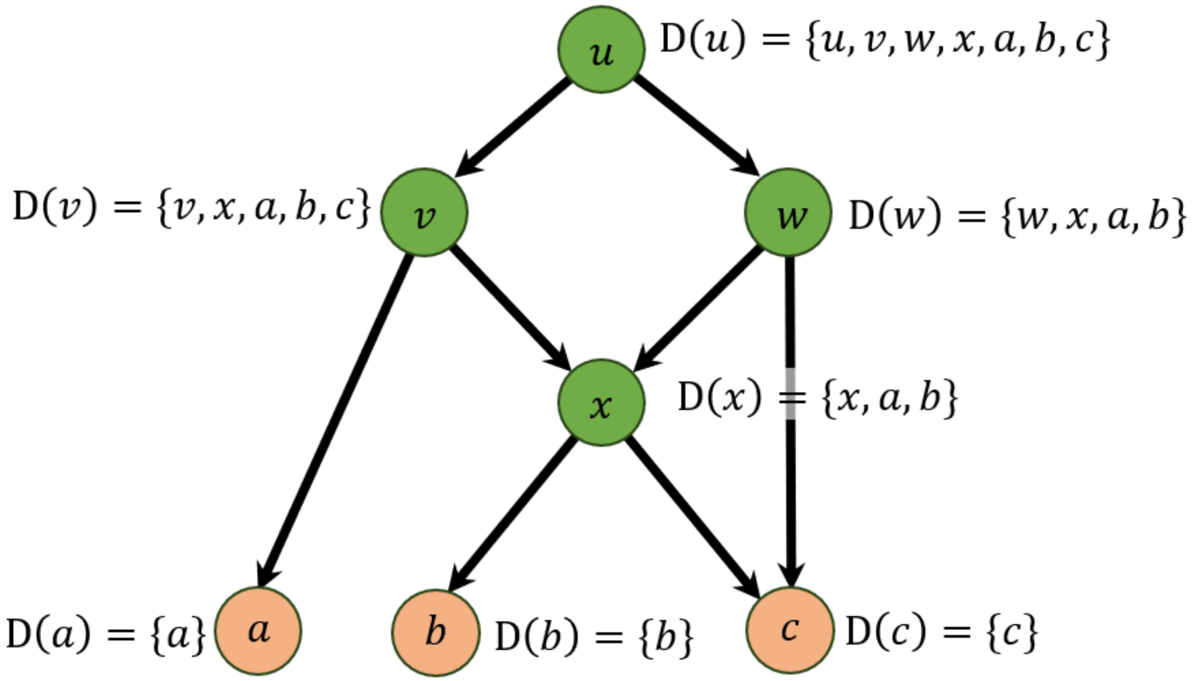

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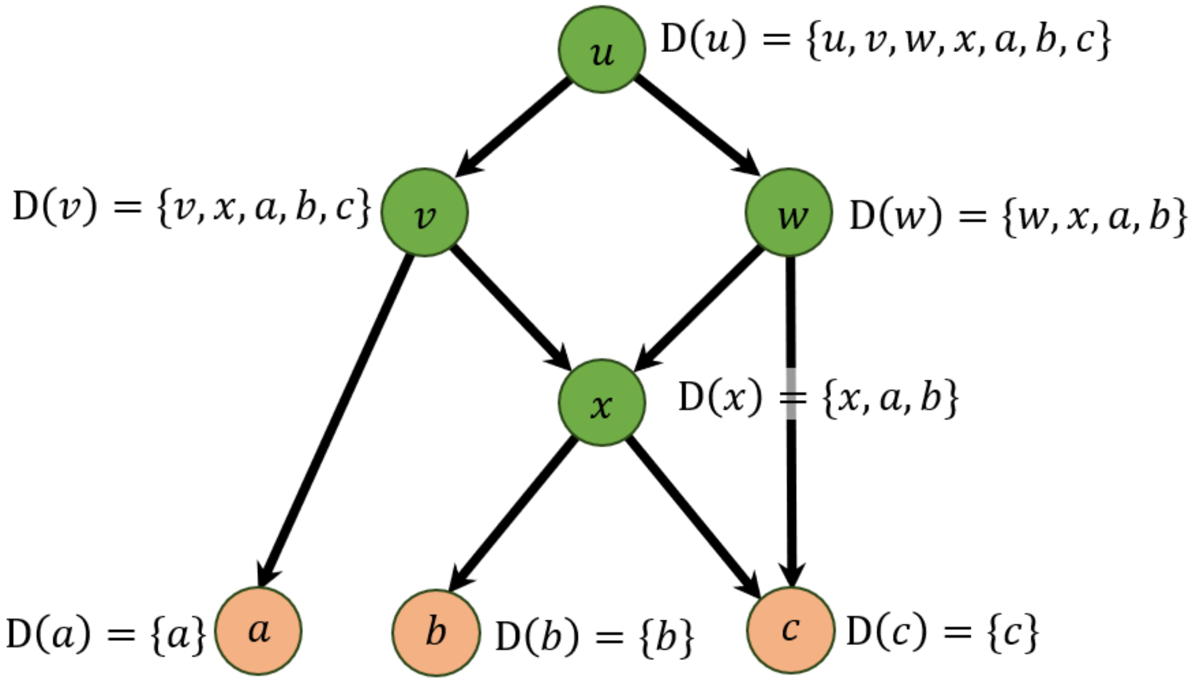

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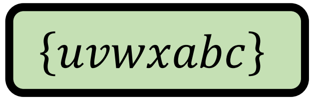

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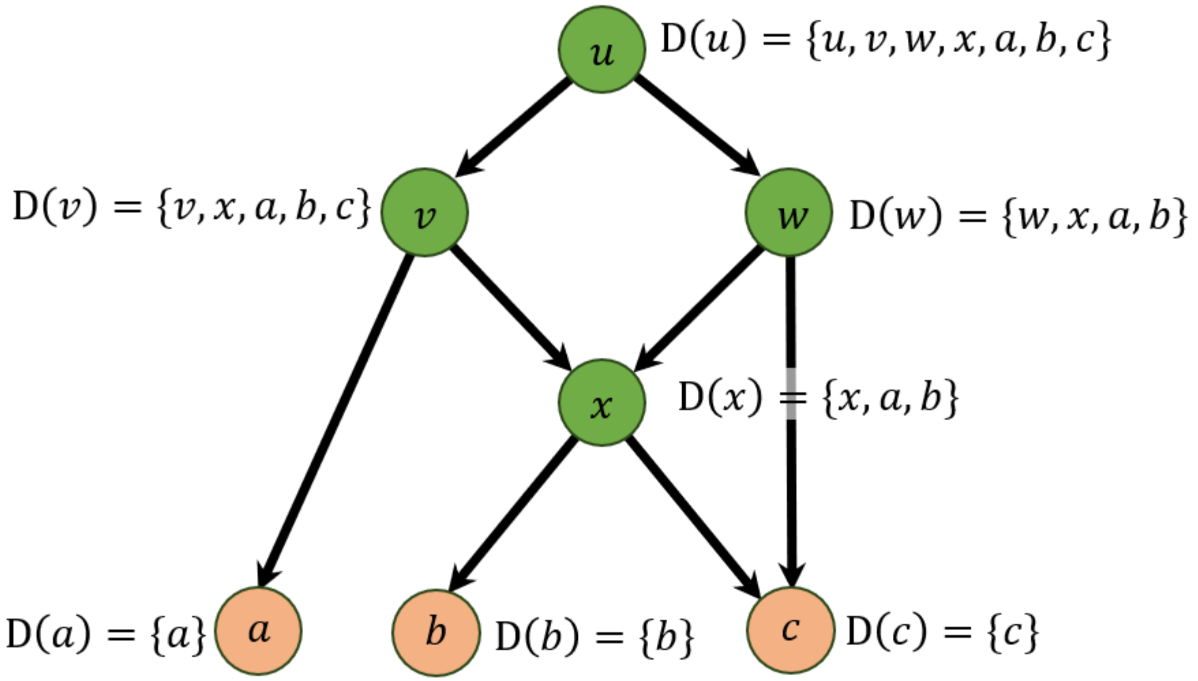

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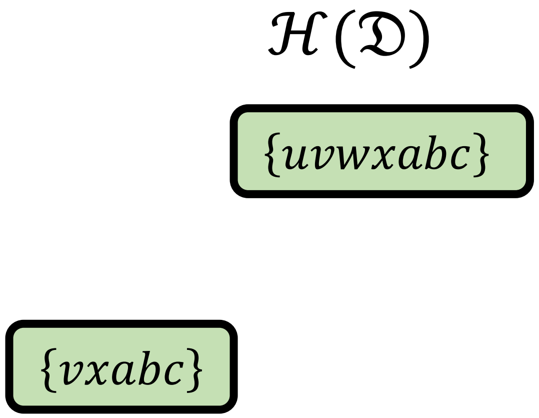

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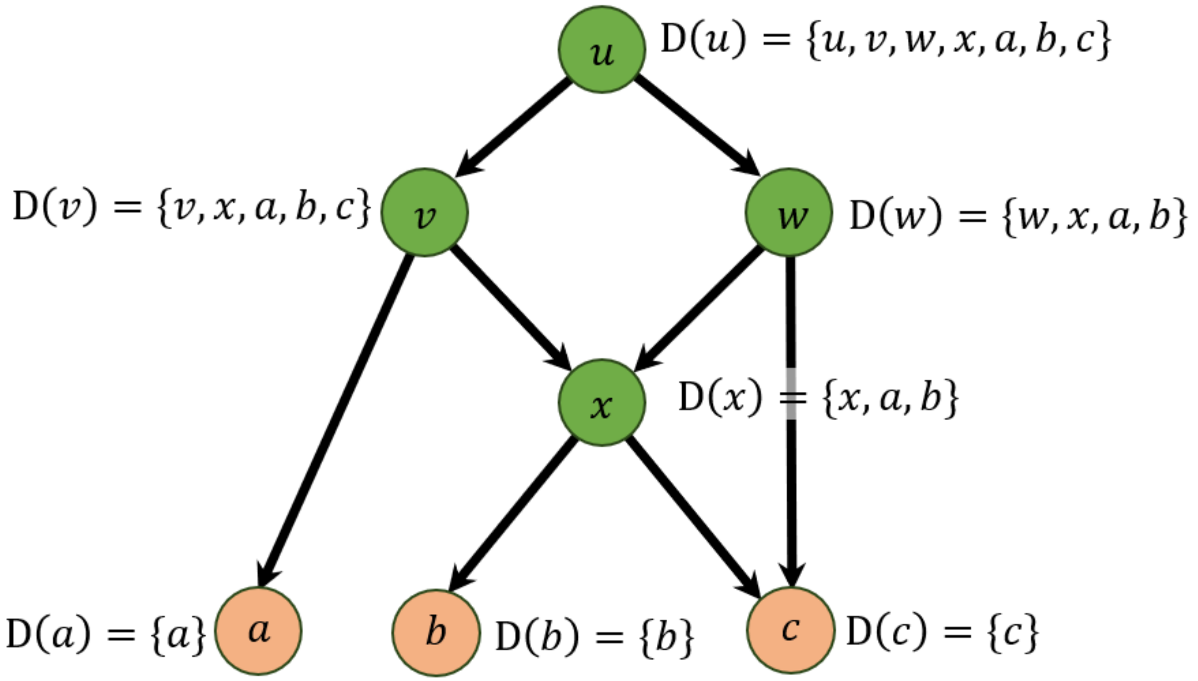

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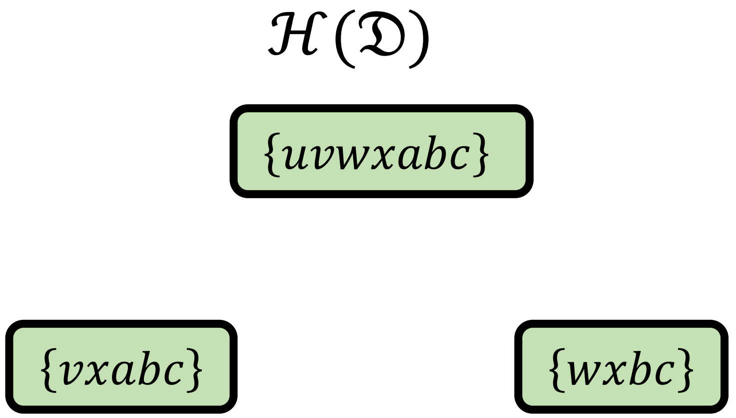

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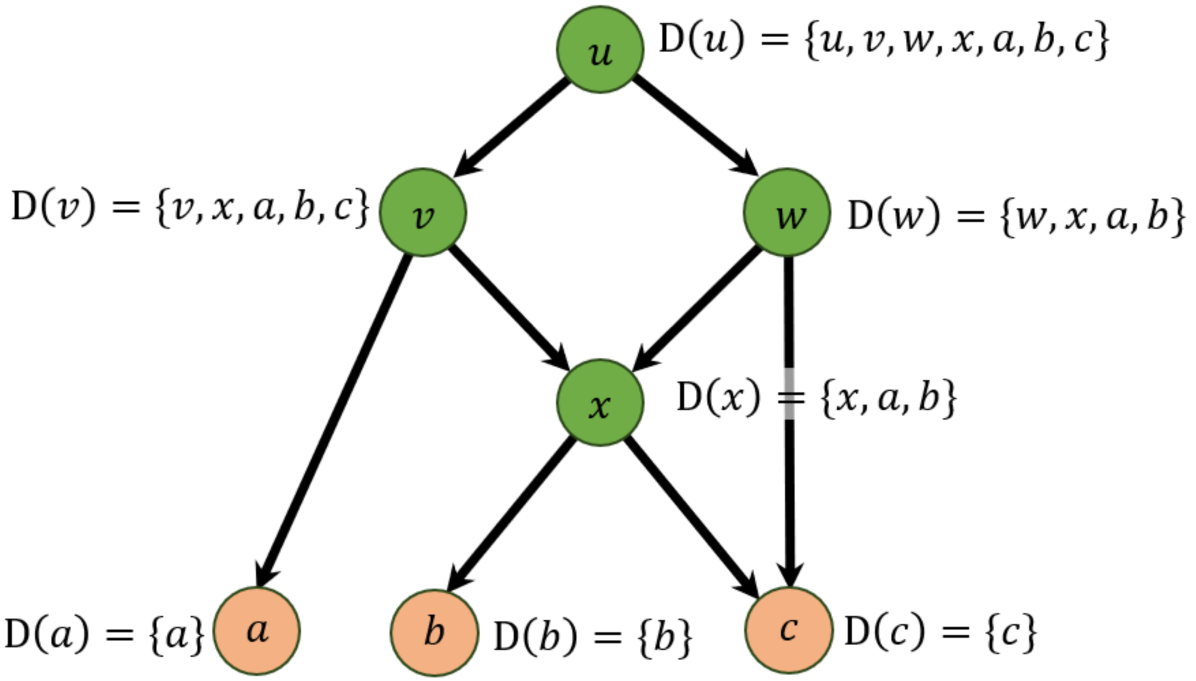

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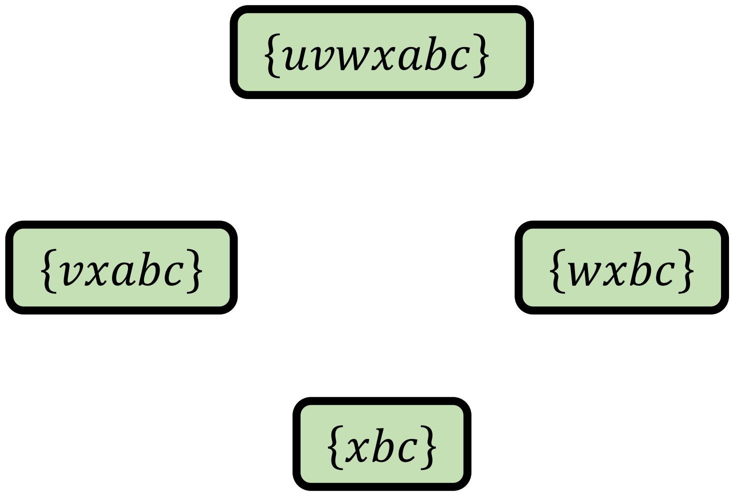
$\mathcal{H}(\mathfrak{D})$



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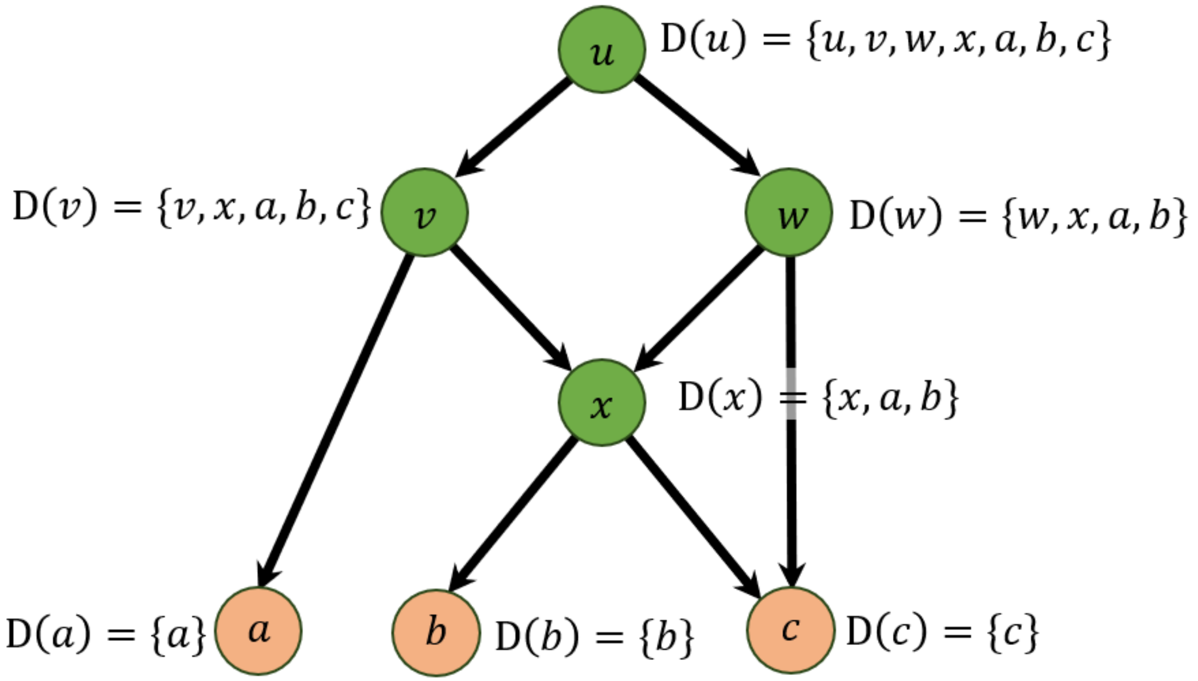

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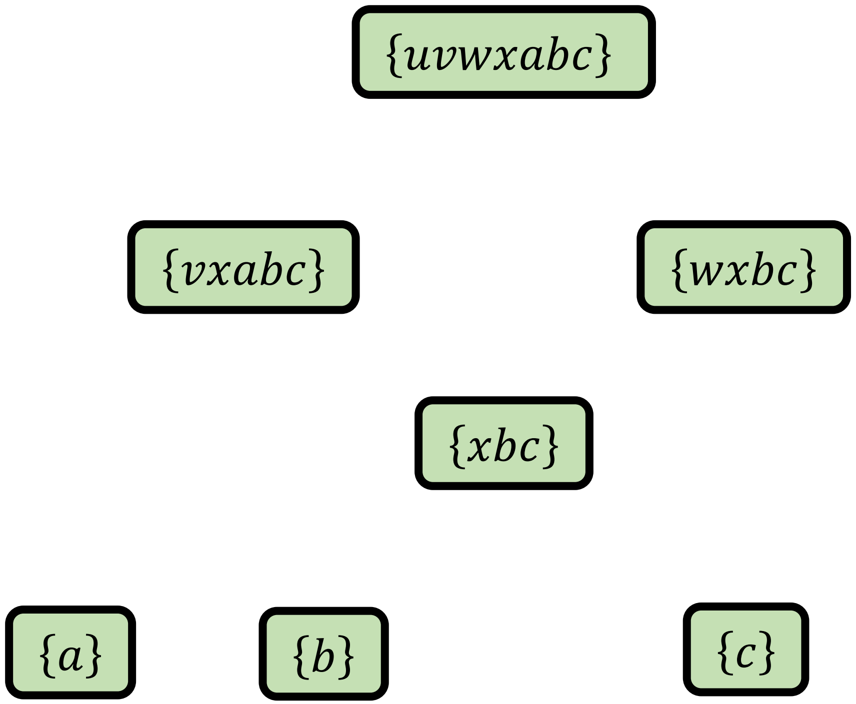

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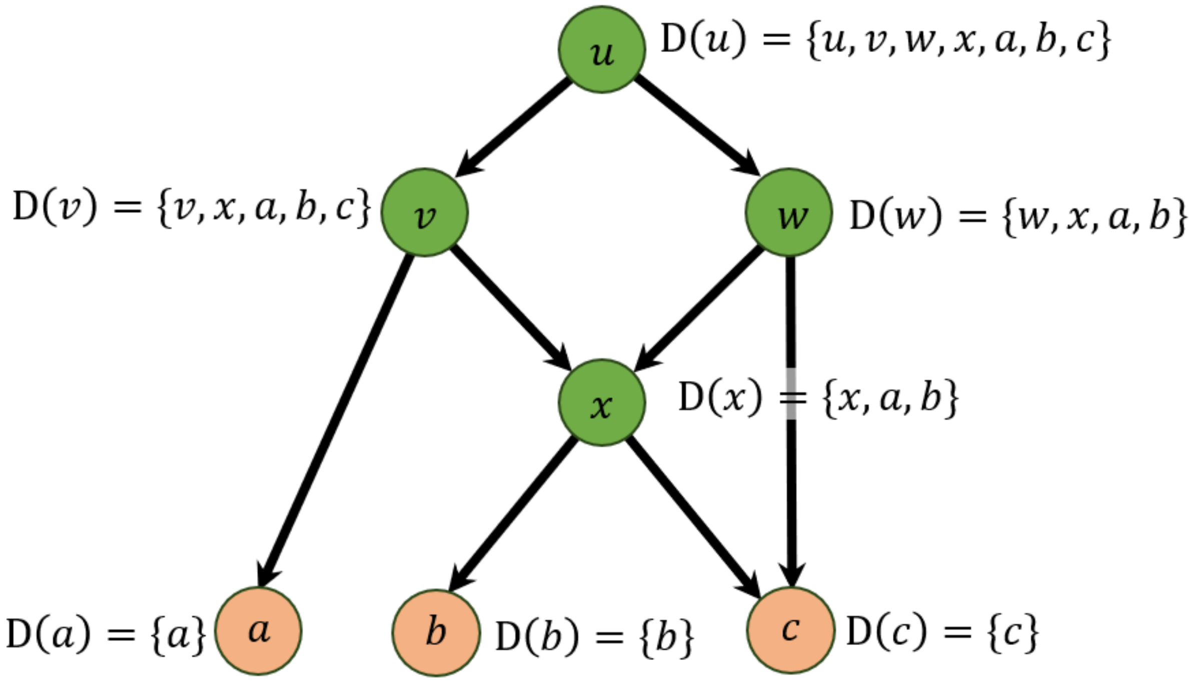

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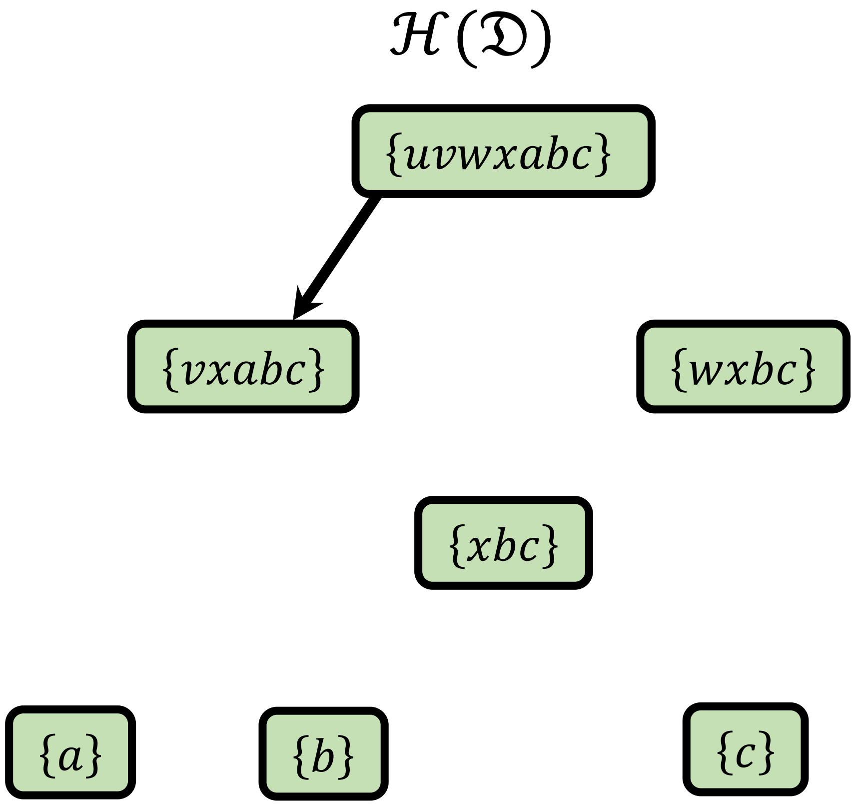

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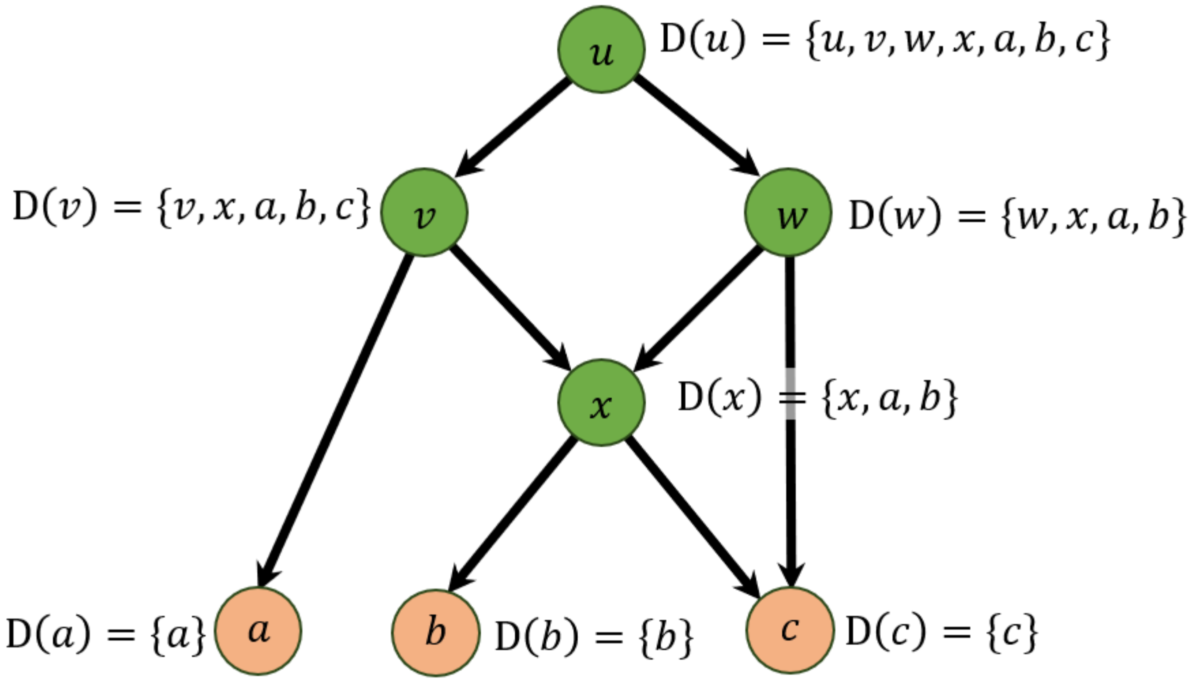

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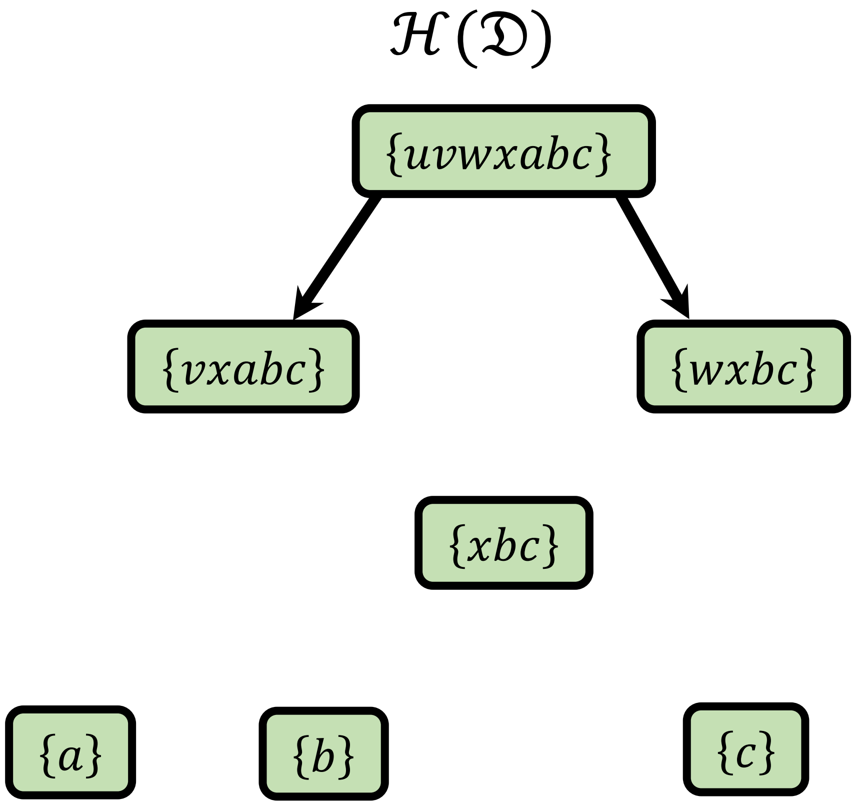

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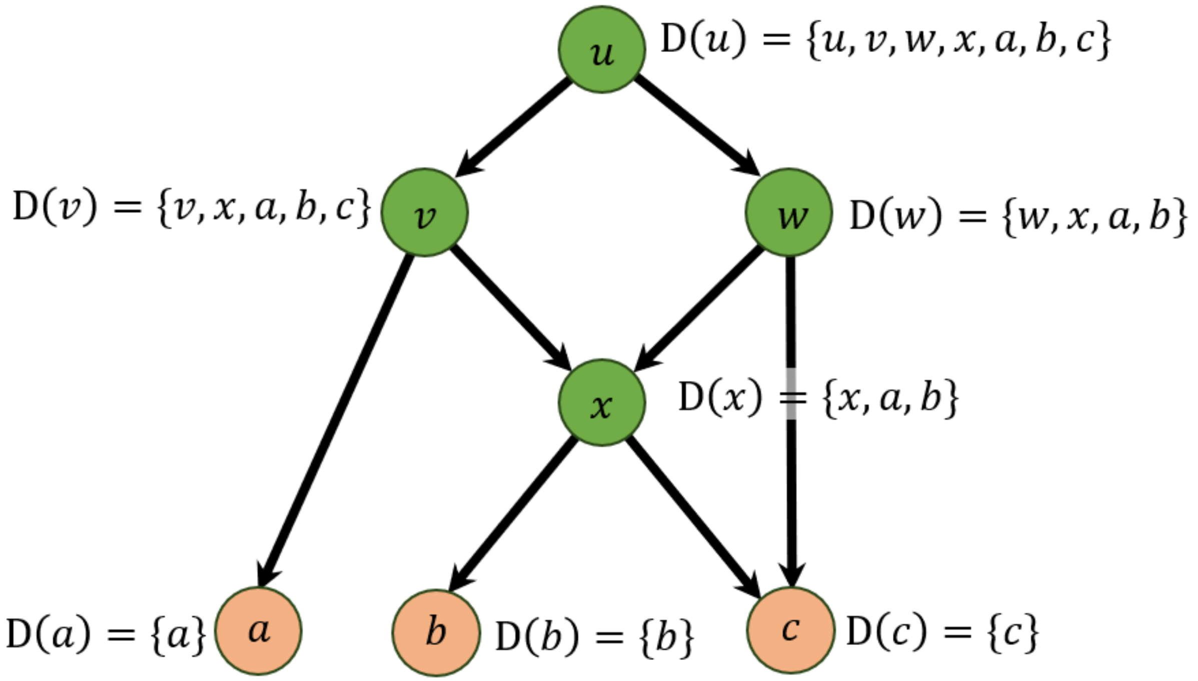

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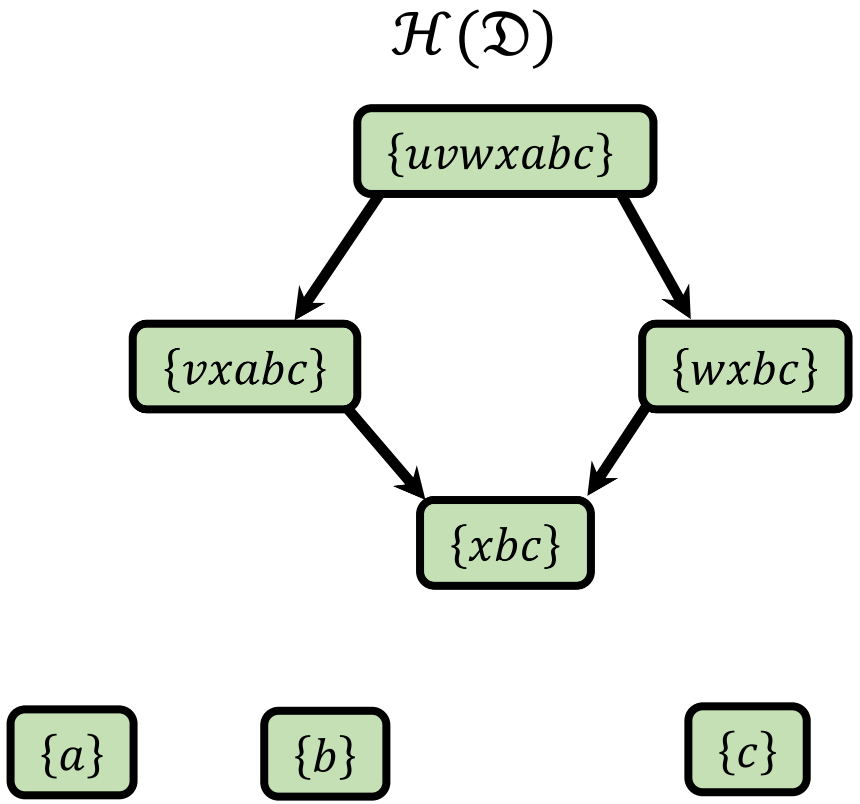

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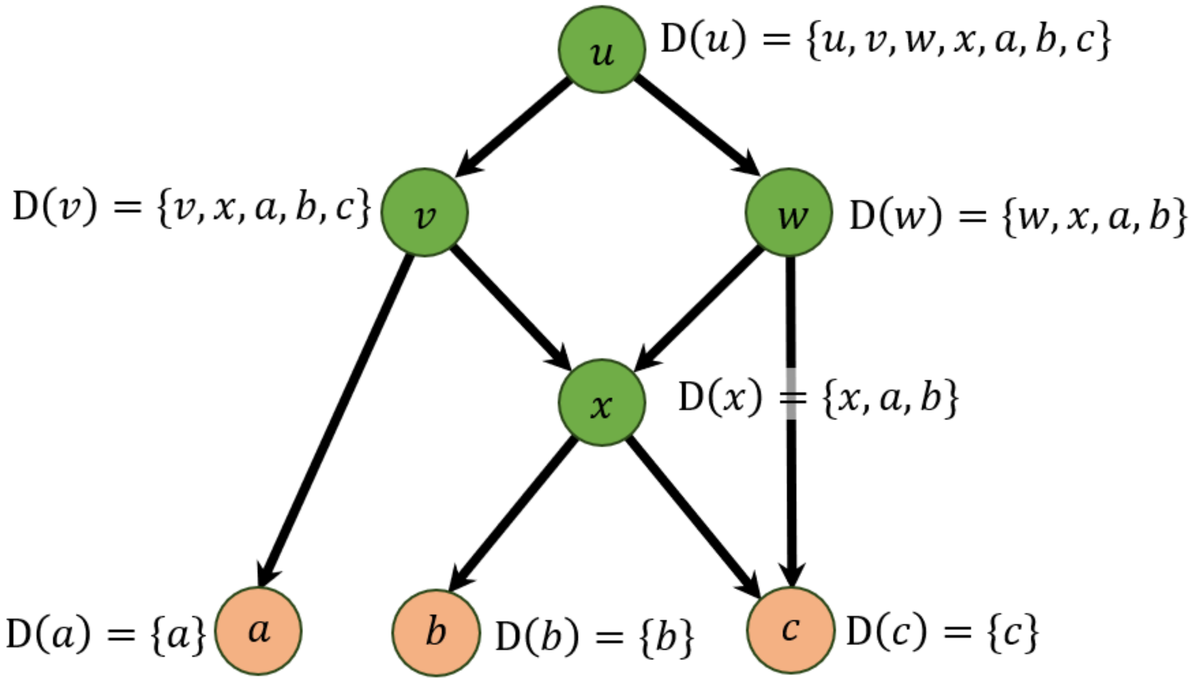

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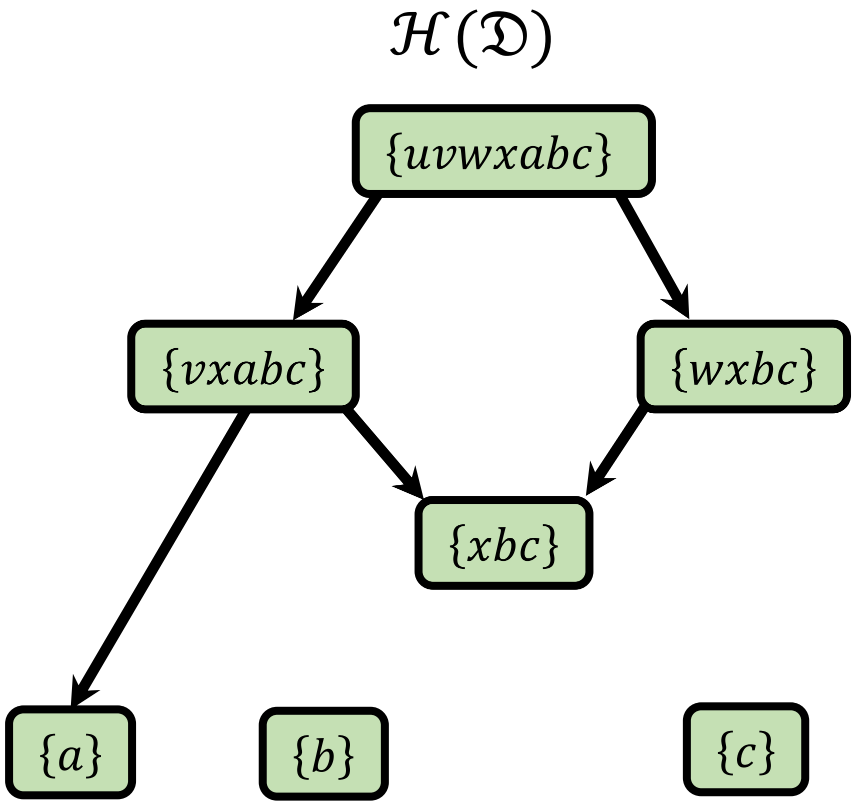

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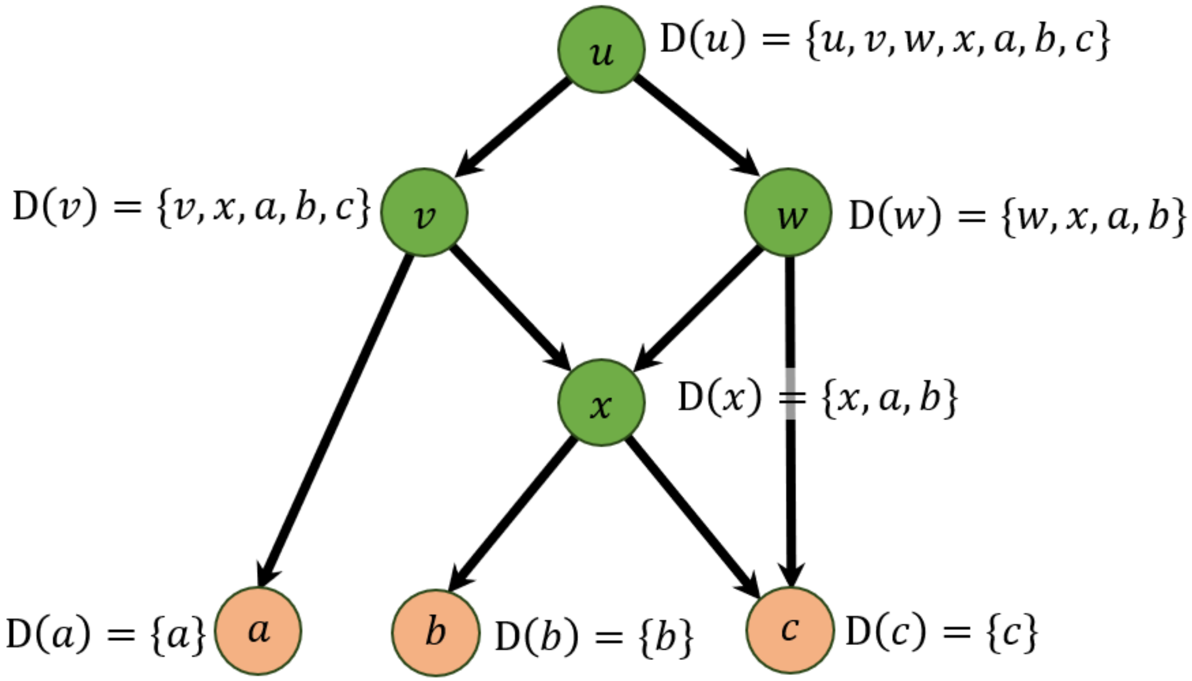

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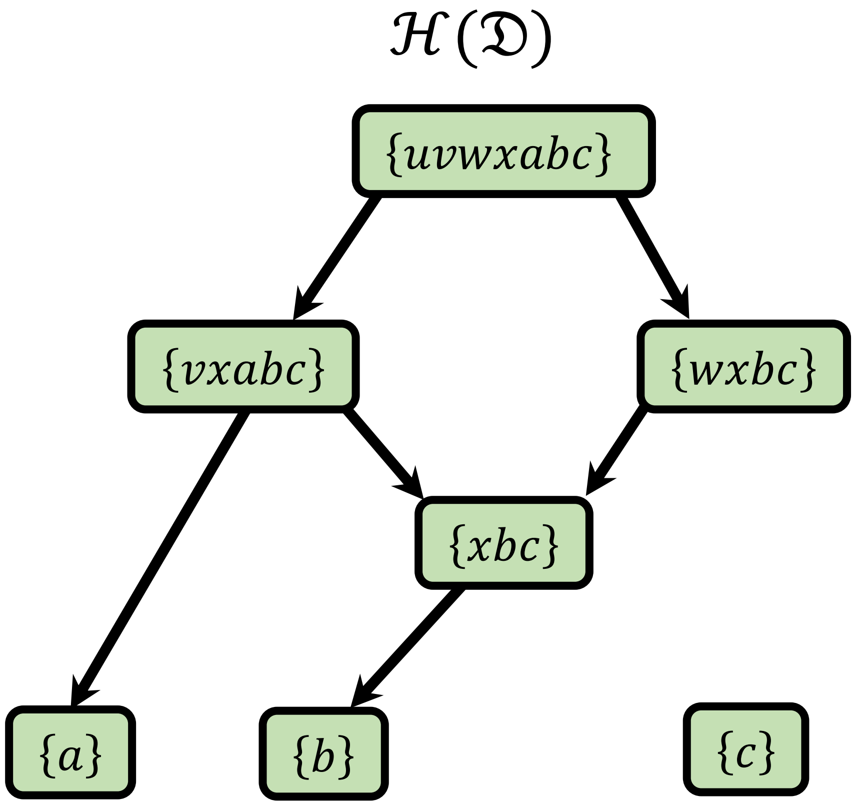

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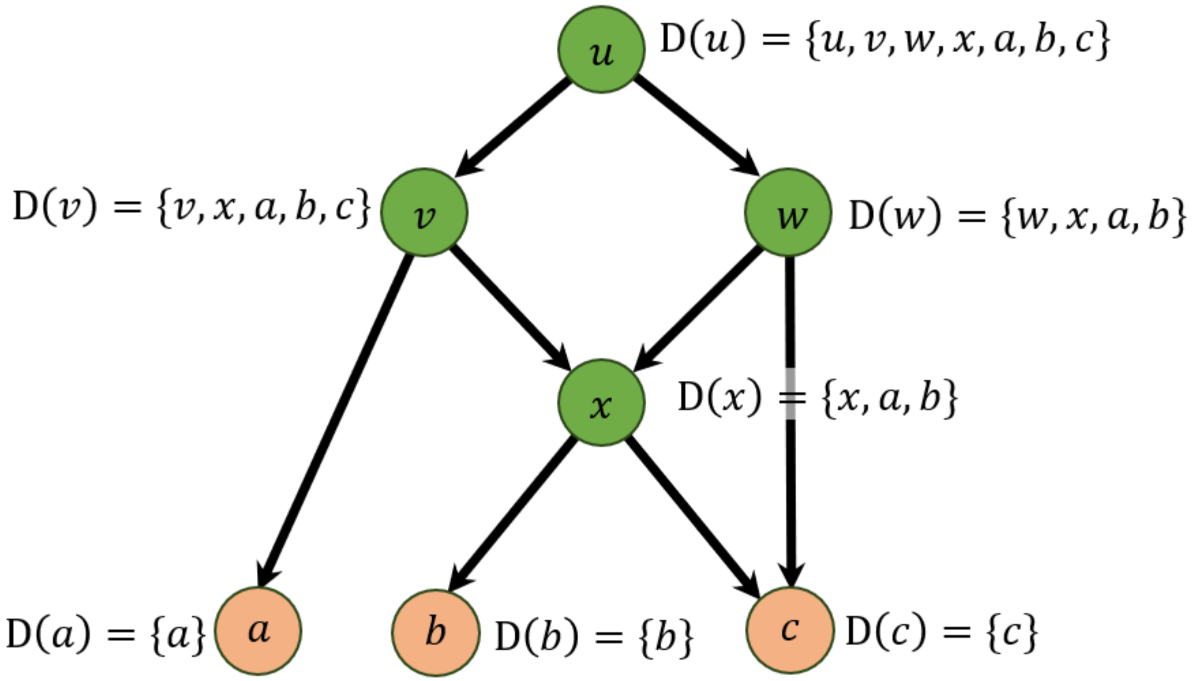

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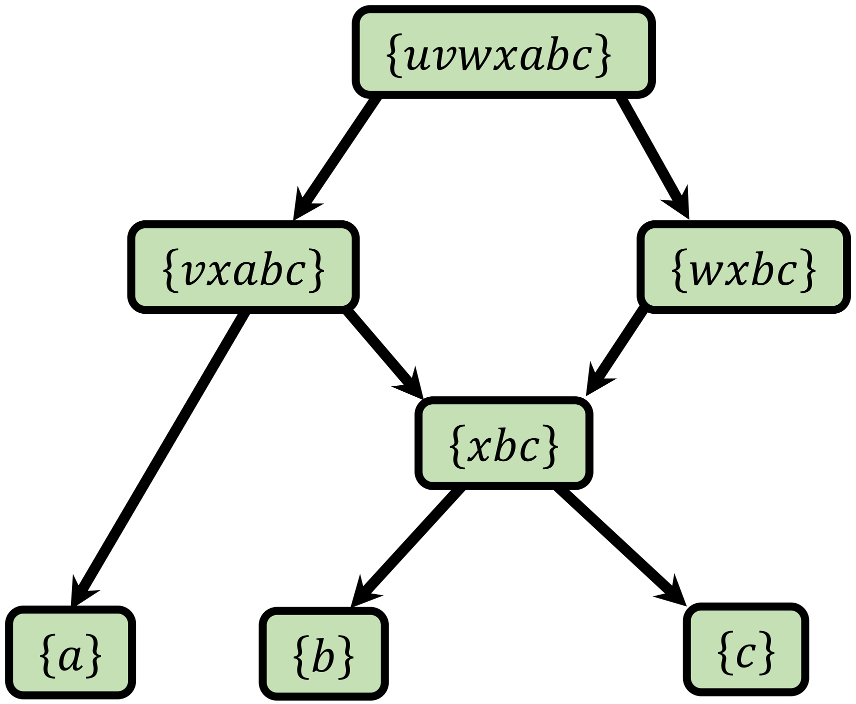

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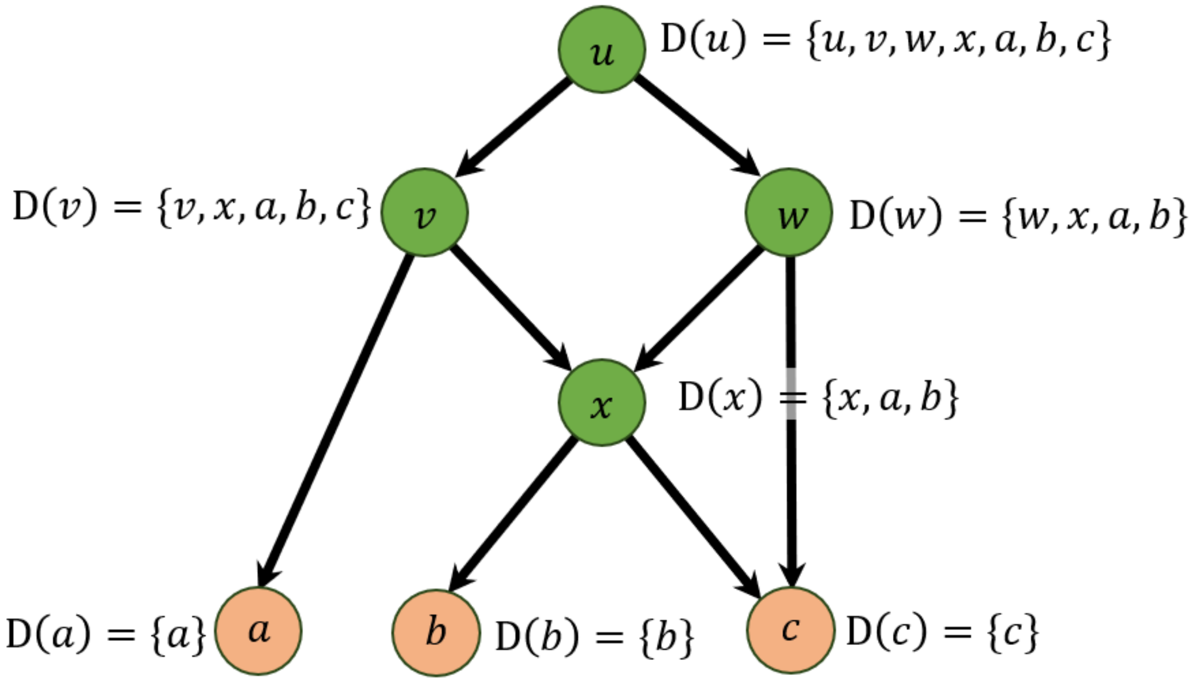

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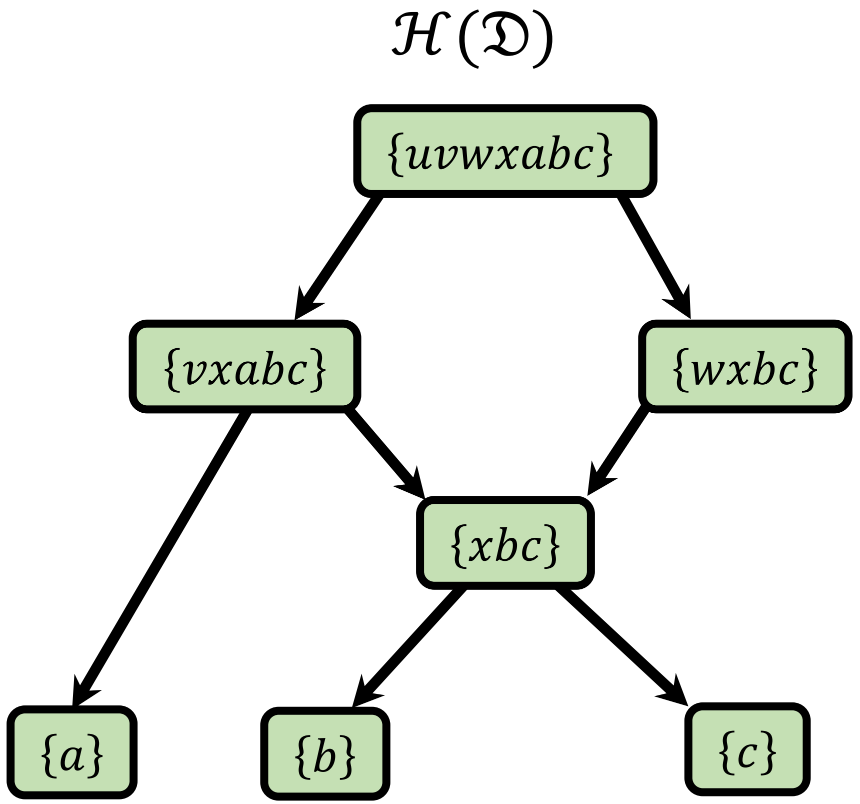

Are G and $\mathcal{H}(\mathfrak{D})$ connected?

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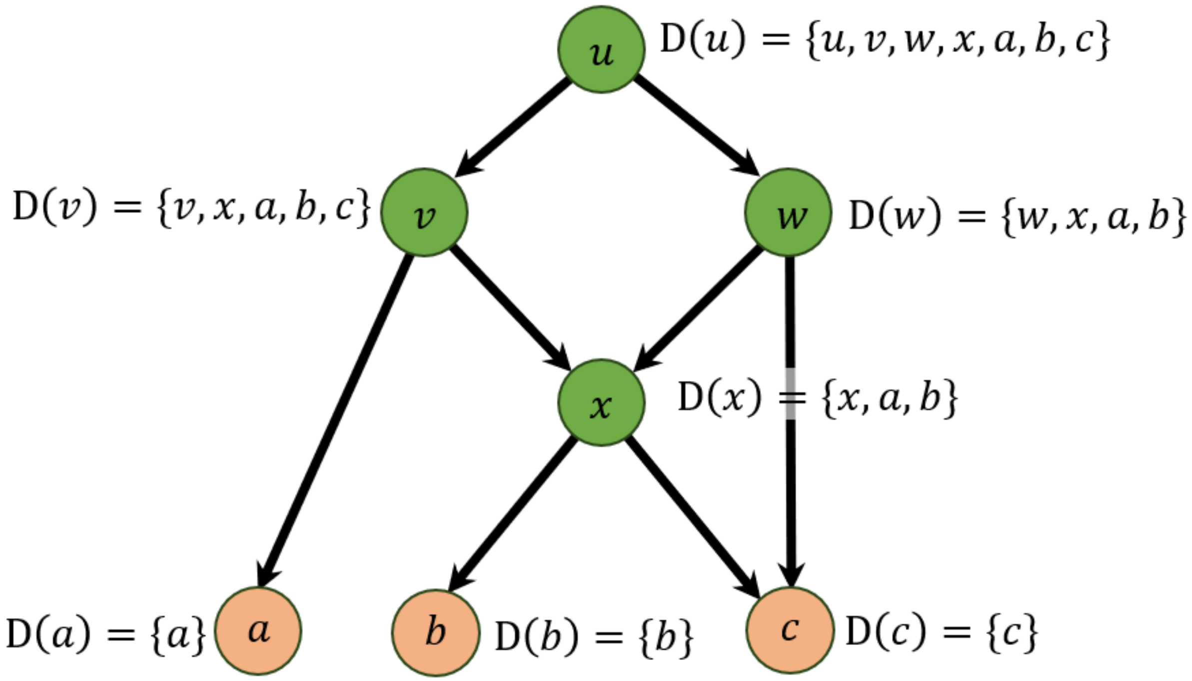

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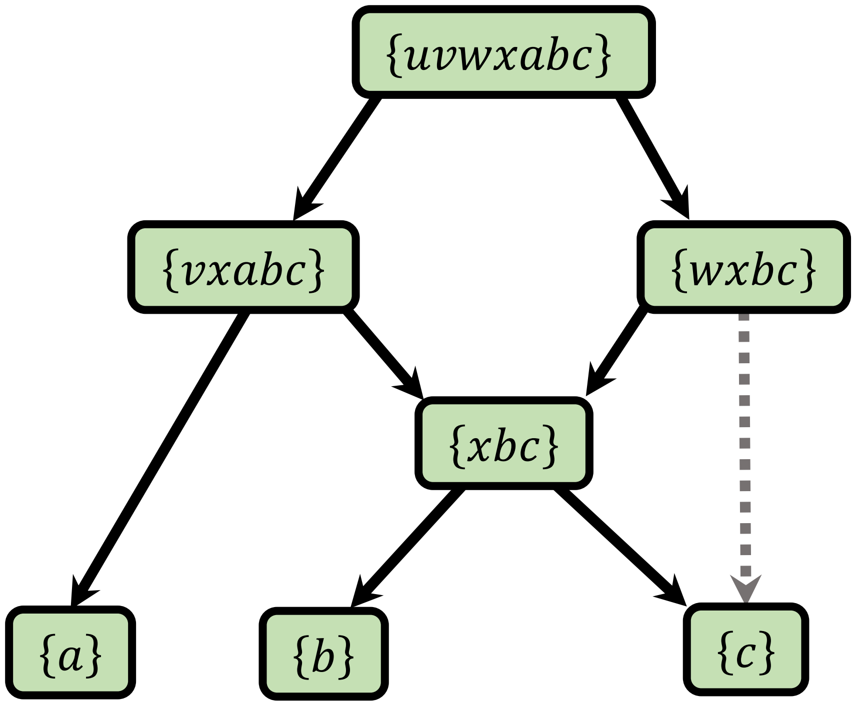

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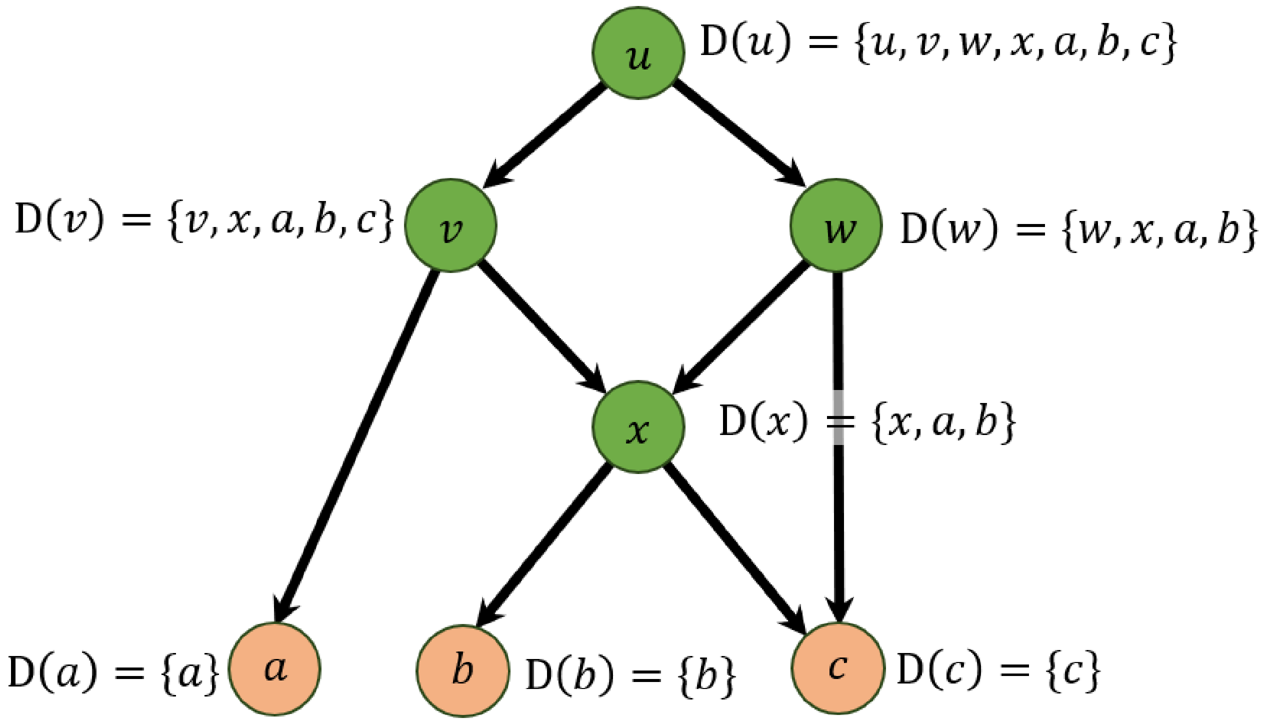
Some Descendant Clusters Properties



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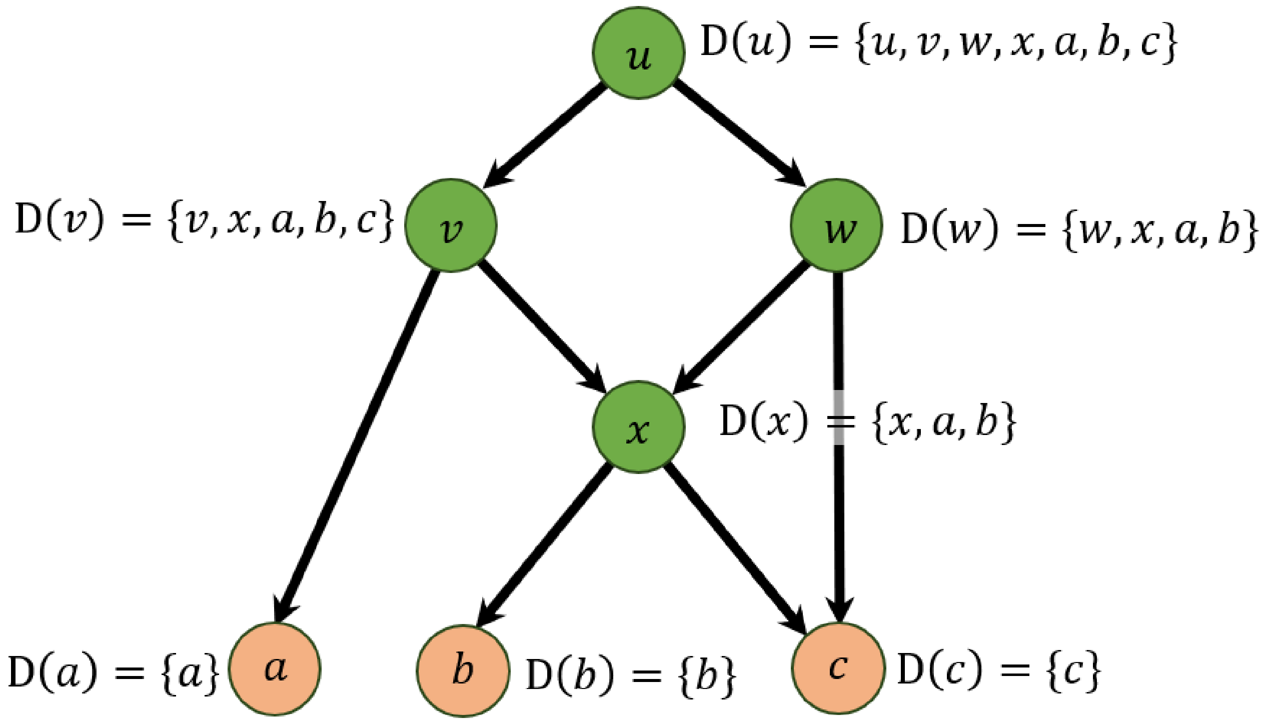

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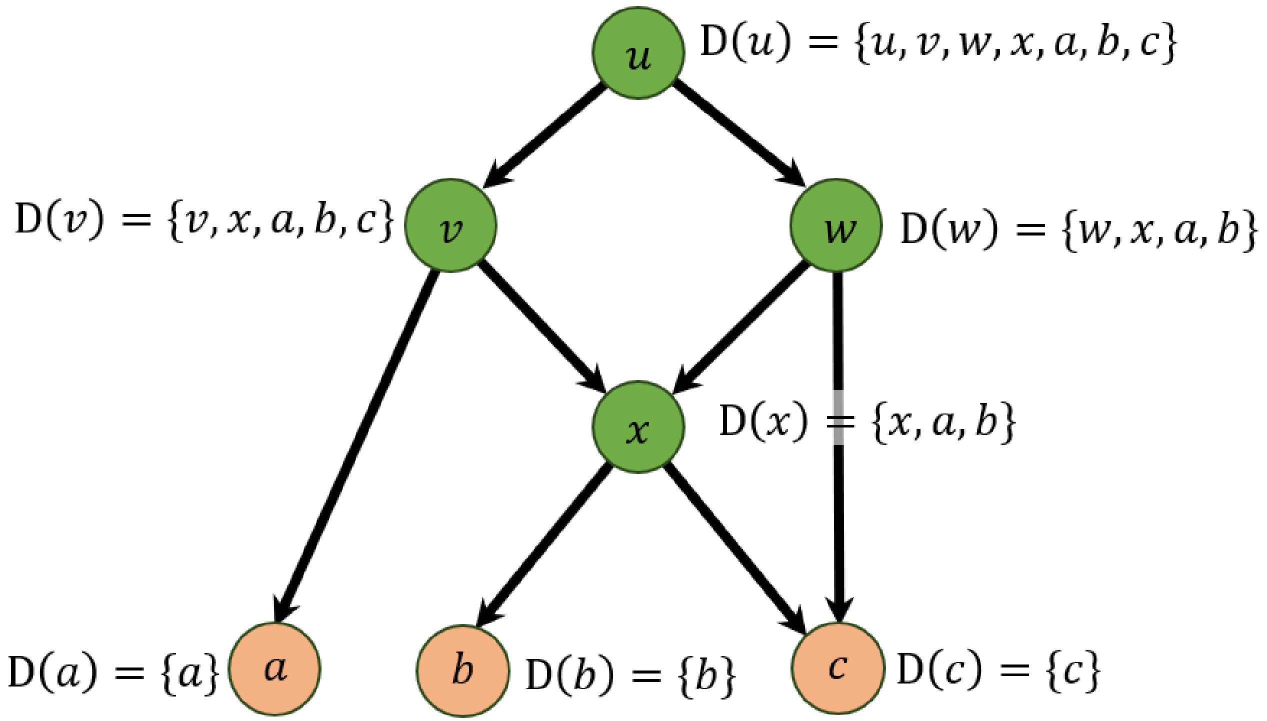
Let G be a DAG, then:

(*i*) $D(u) = D(v)$ if and only if $u = v$

→ Assume $D(u) = D(v), u \neq v$
→ $u \in D(v), v \in D(u)$
→ paths $u \dots v, v \dots u$ in G
→ G contains a cycle ⚡



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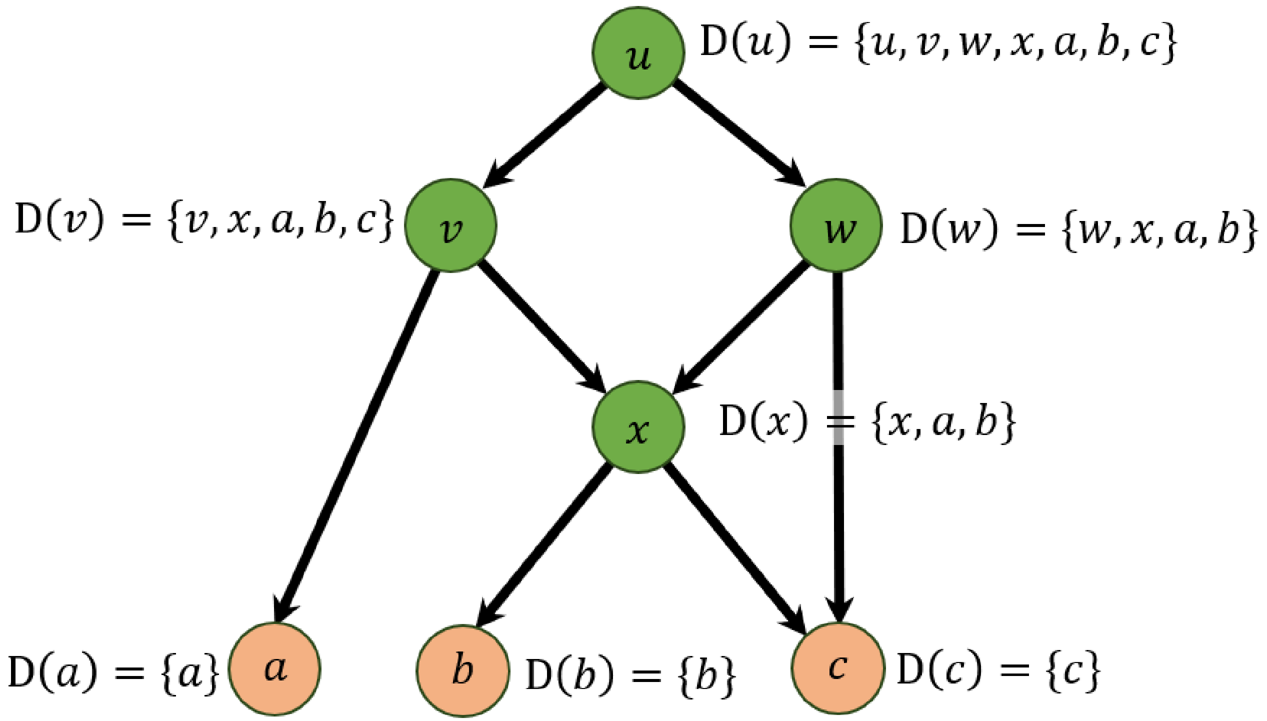

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→ $v \in D(u)$, then there is path $u \dots v$.
  → $\forall x \in D(v)$ there is path $v \dots x$.
  → $\forall x \in D(v)$ there is path $u \dots v \dots x$ (acyclicity)
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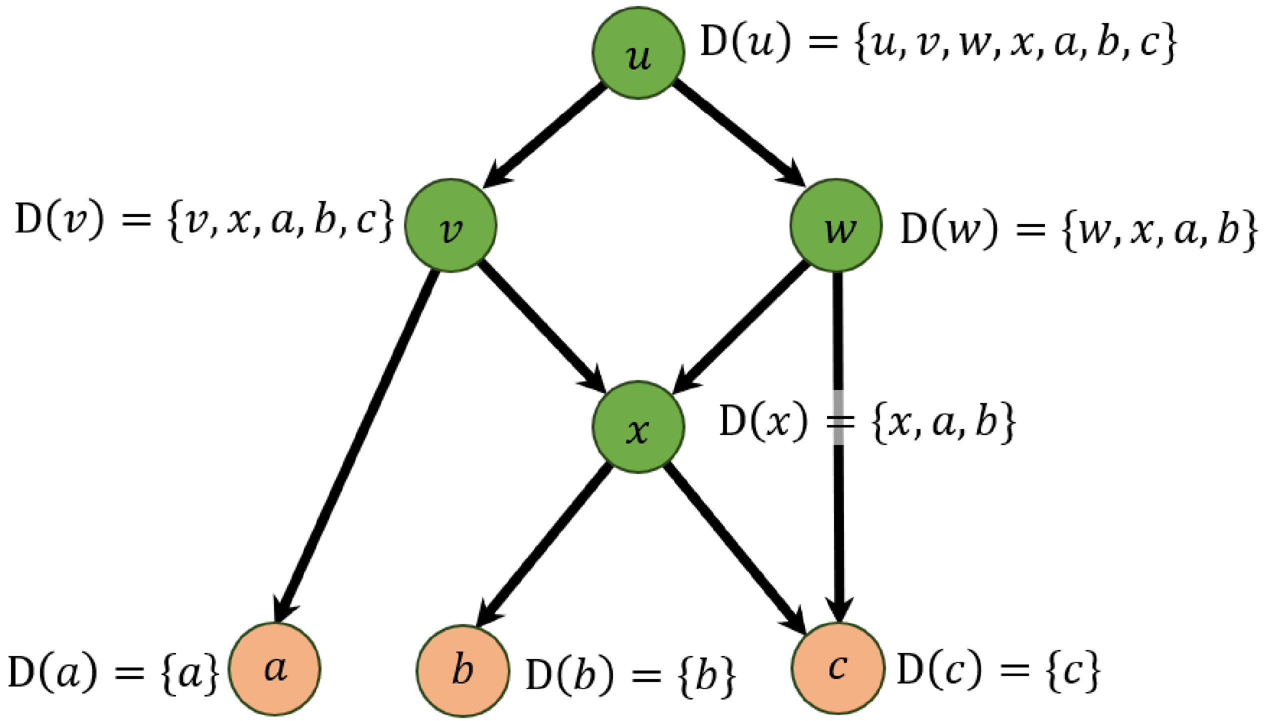

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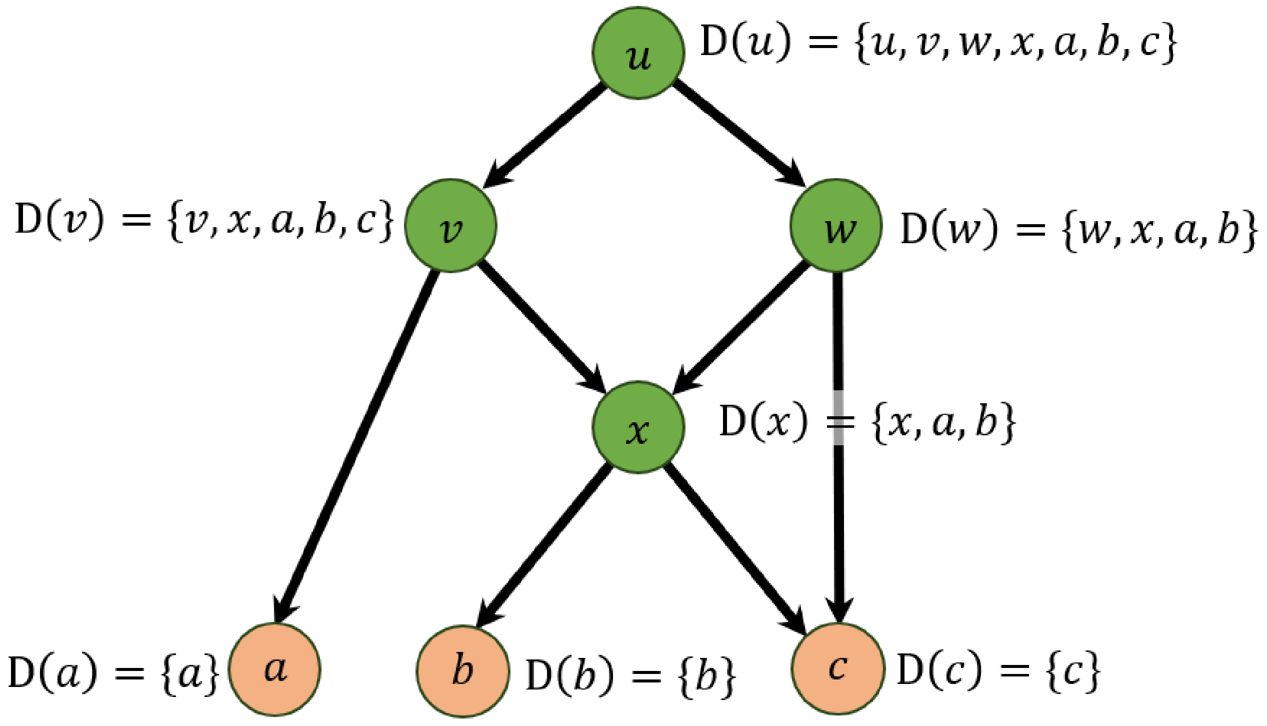
  → $v \in D(v), u \in D(u)$ by definition
  → $D(v) \subseteq D(u)$ implies $v \in D(u)$



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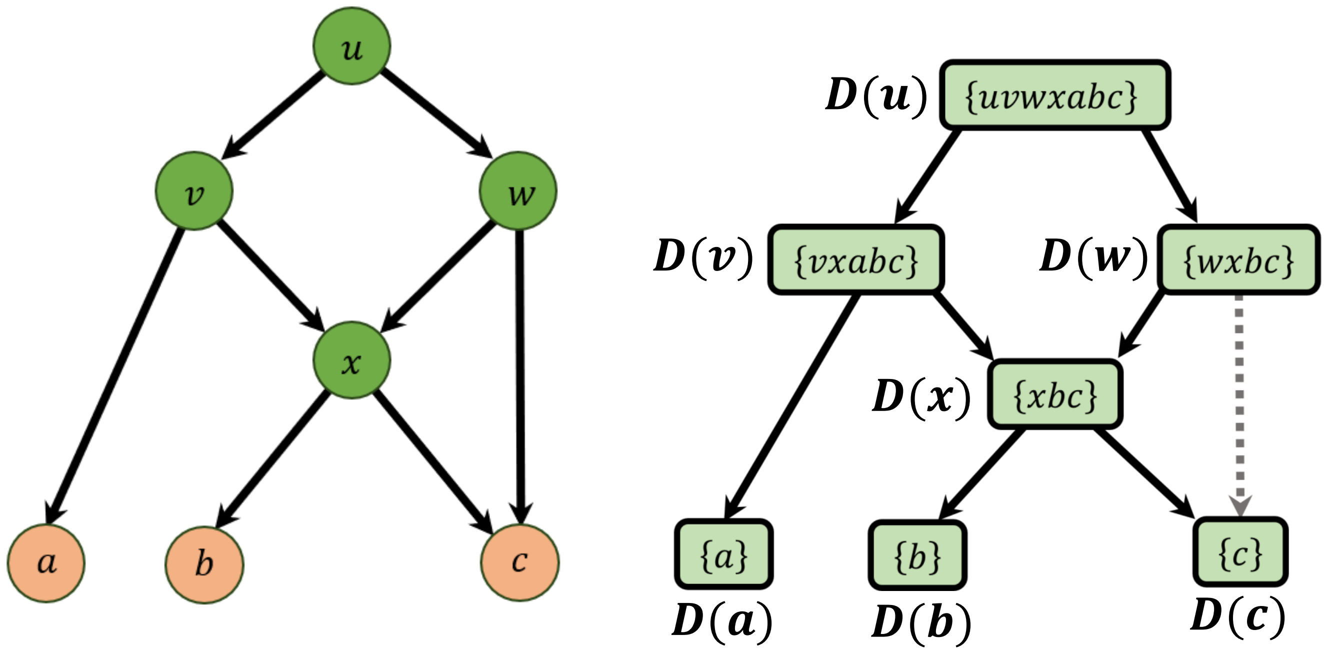

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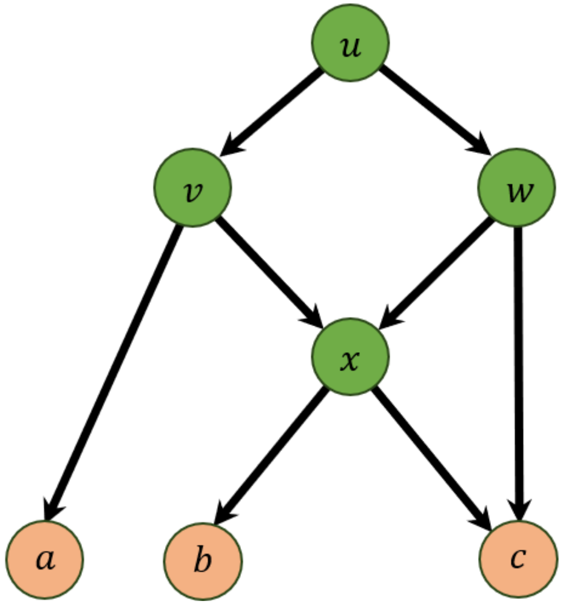

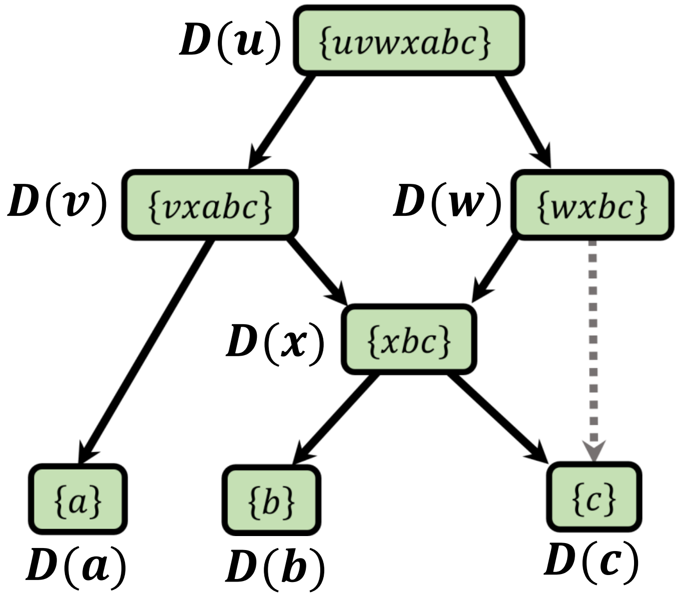

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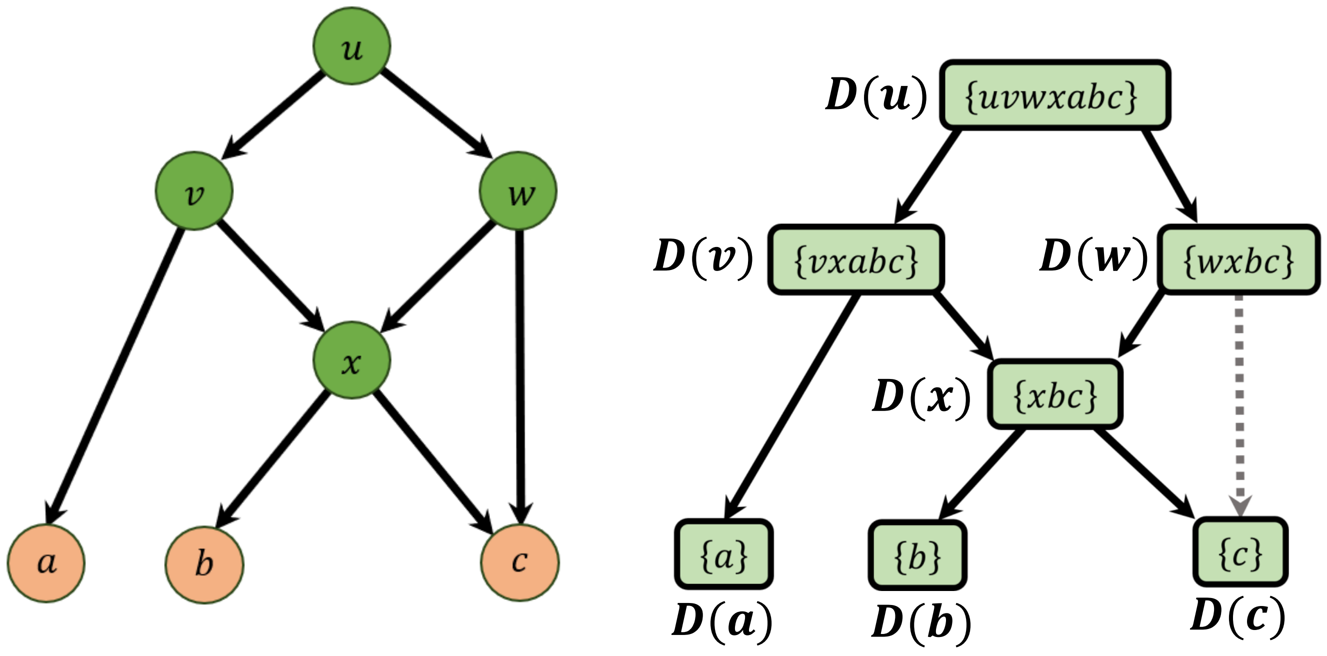

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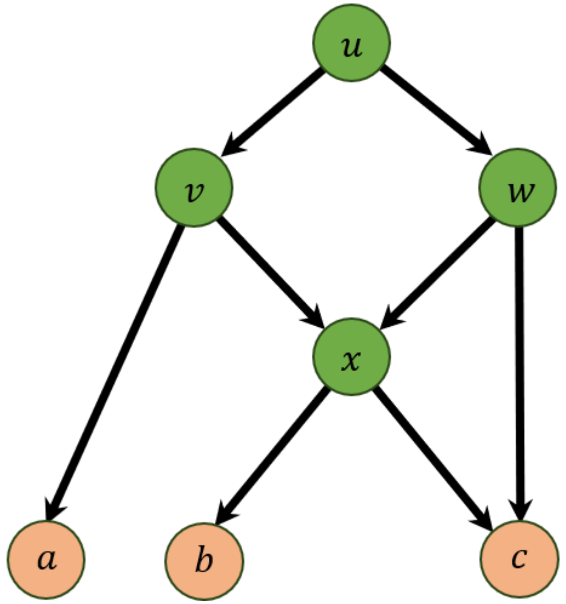

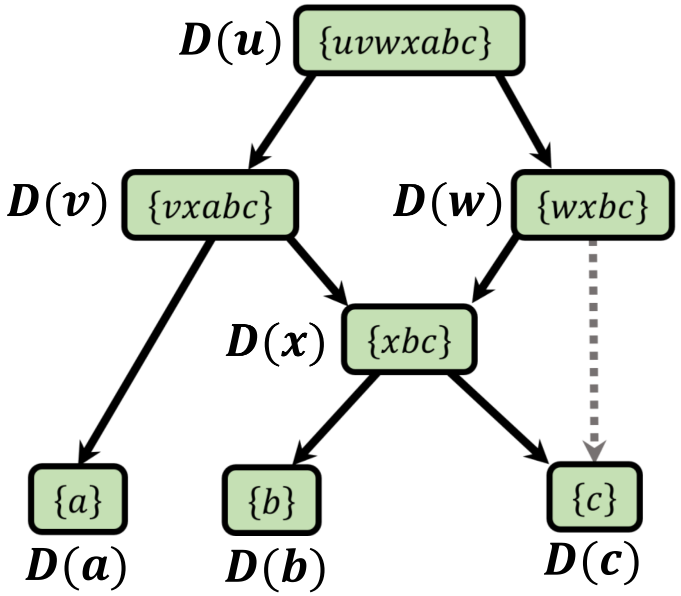

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  → if (u, v) is a shortcut, there is a $w \in D(u)$
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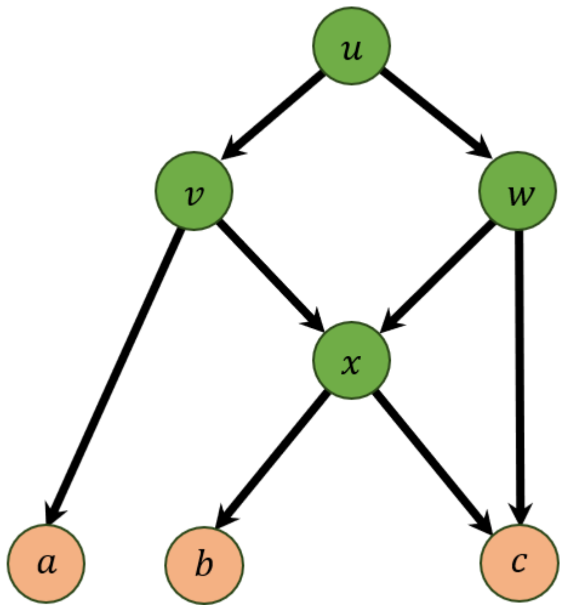

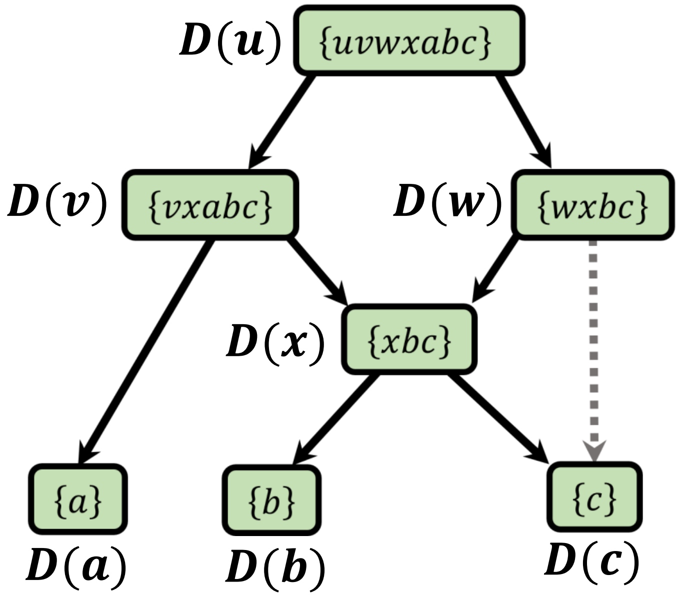

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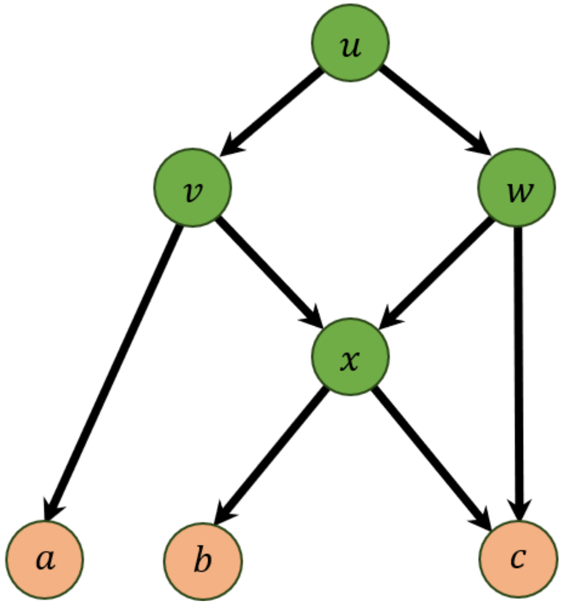

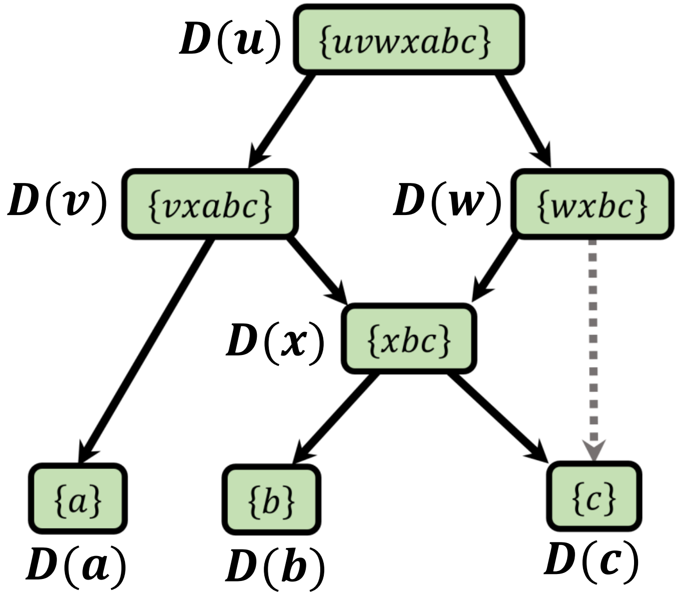

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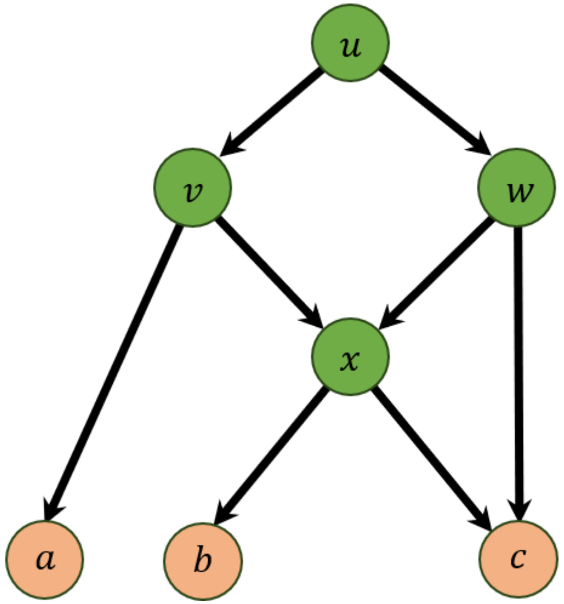

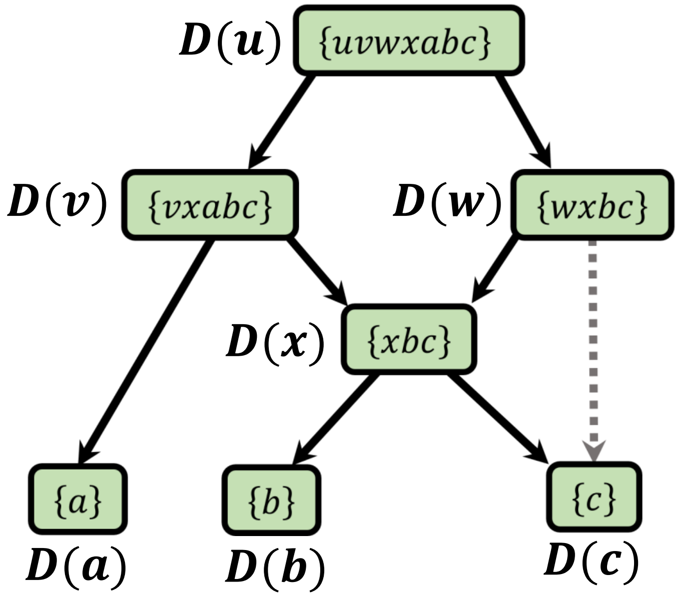

G is isomorphic to $\mathcal{H}(\mathfrak{D})$ **(minus some shortcuts)**
or:
the transitive reduction of G is isomorphic to $\mathcal{H}(\mathfrak{D})$.

Descendant Clusters

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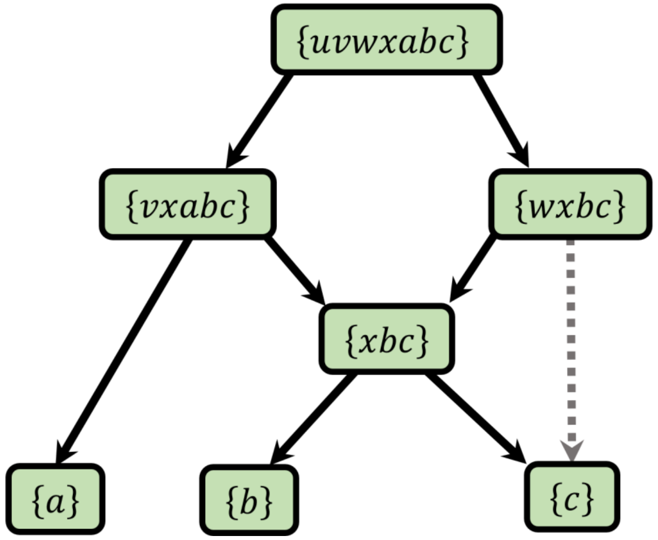

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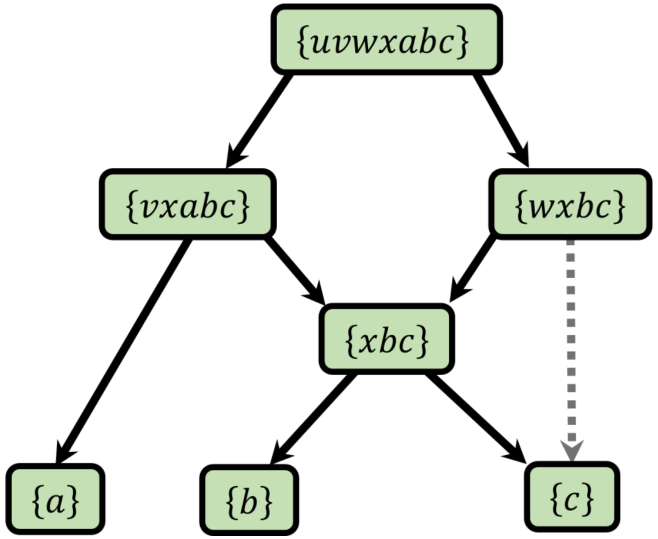

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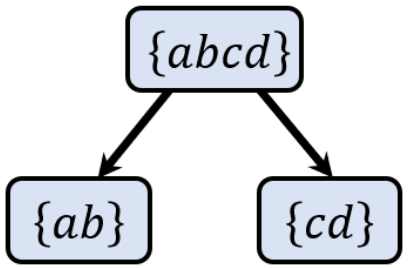

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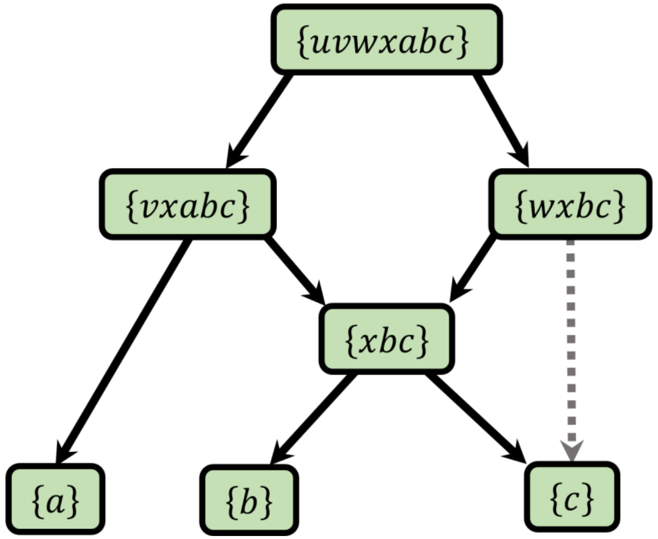

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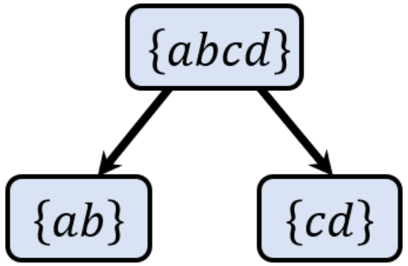

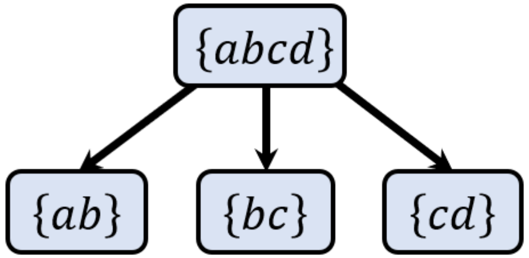

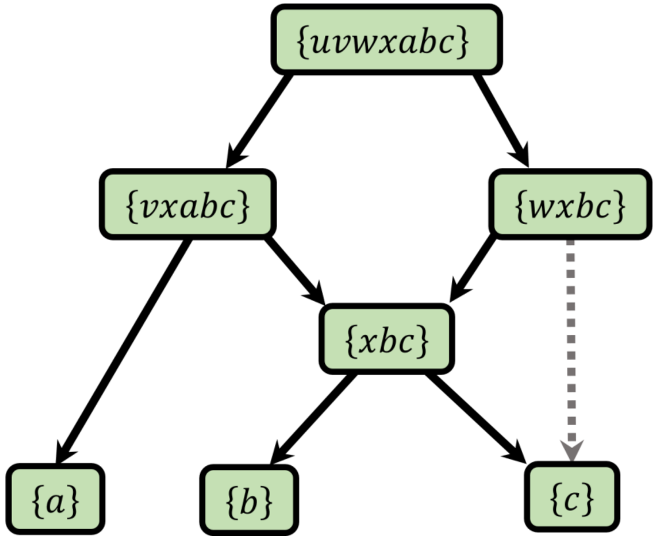

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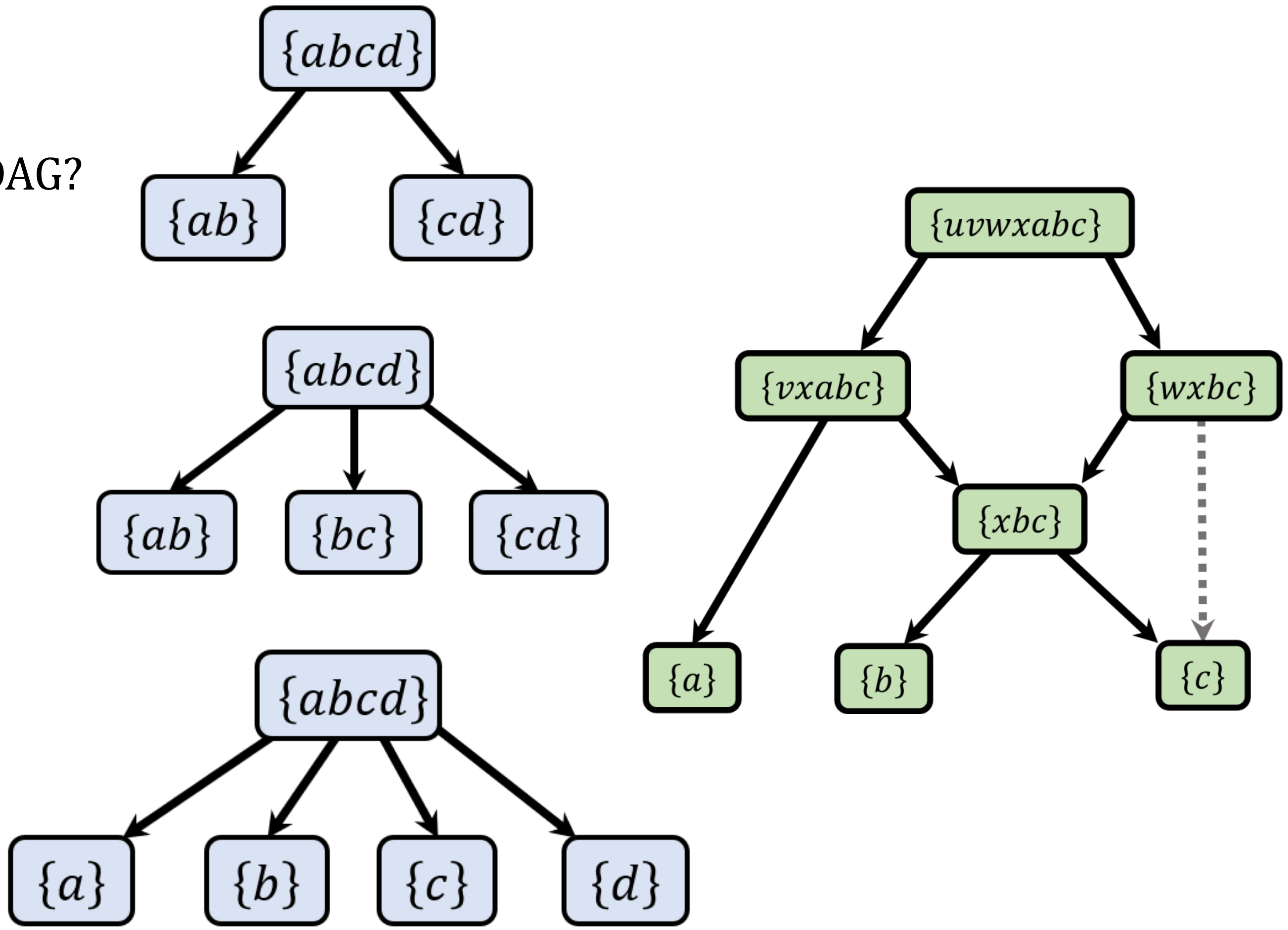

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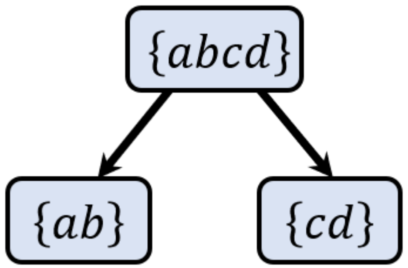
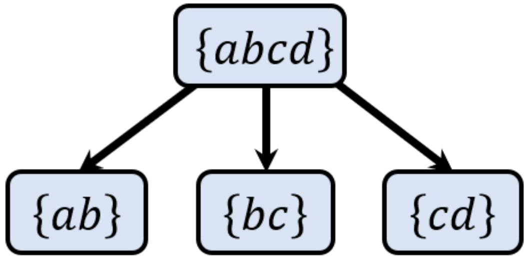
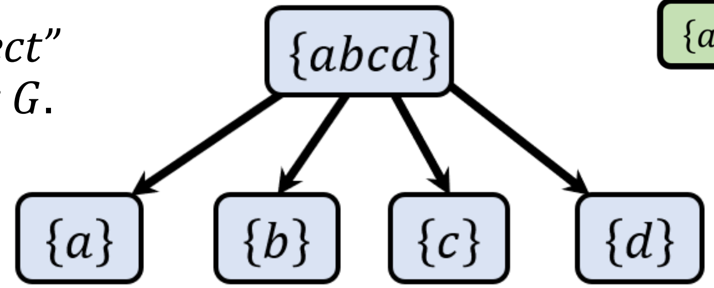
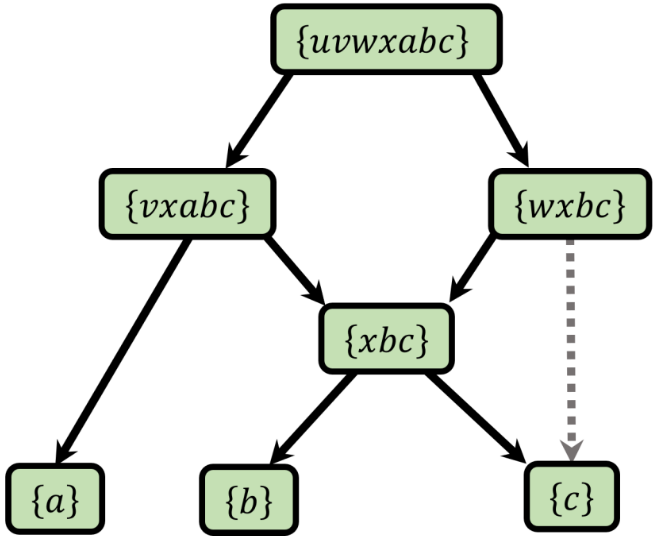
- Maybe we start with a function that finds the *"correct"* descendant set in $\mathfrak{D}$ for a vertex without consulting G.









D-Snake! $\widetilde{D}$

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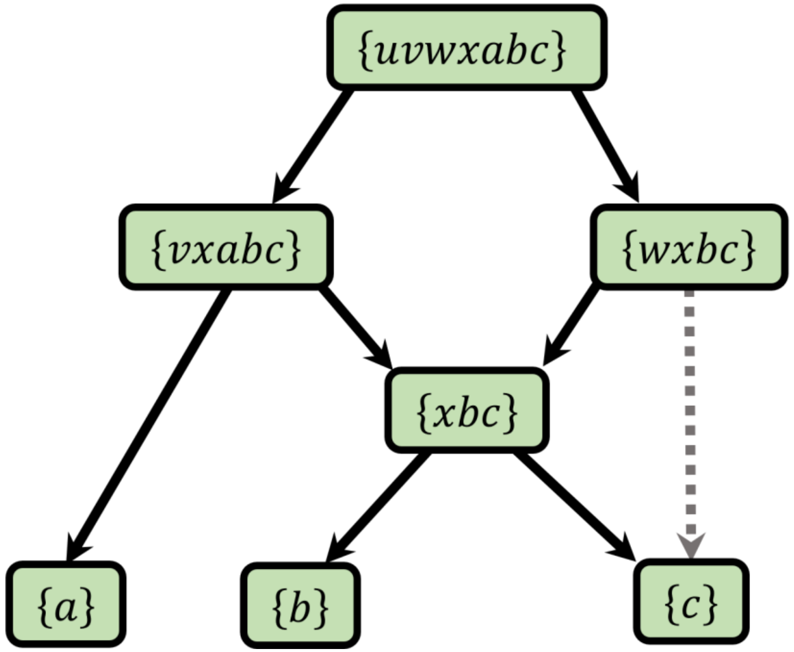

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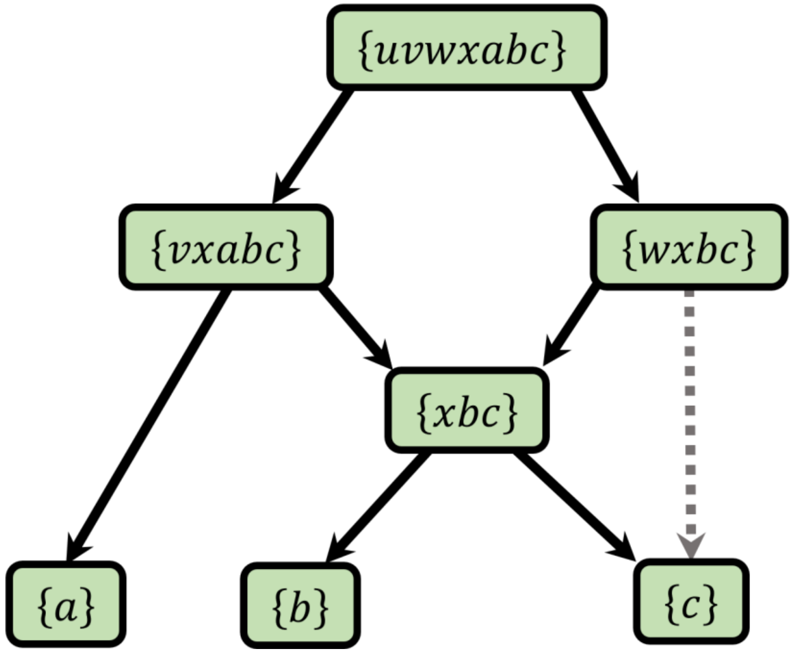

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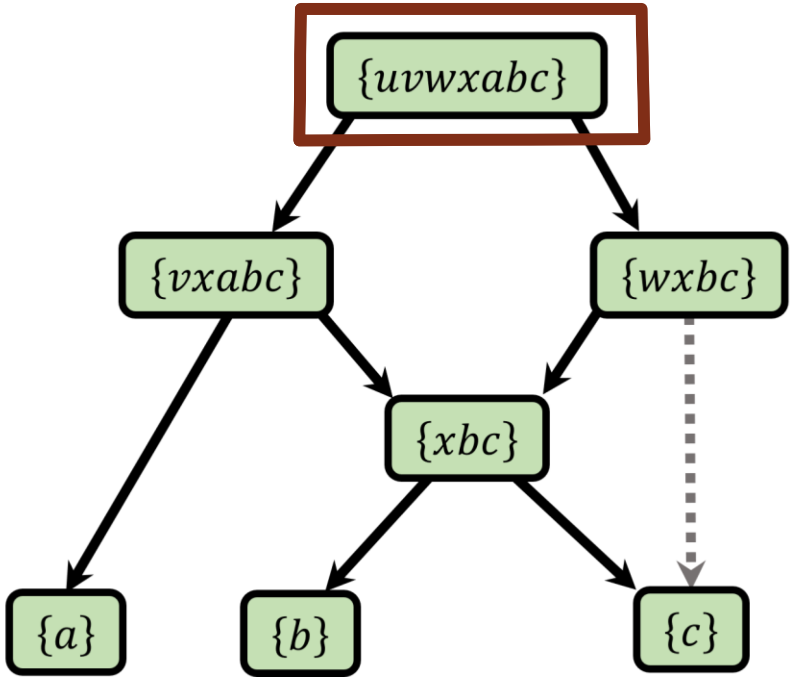

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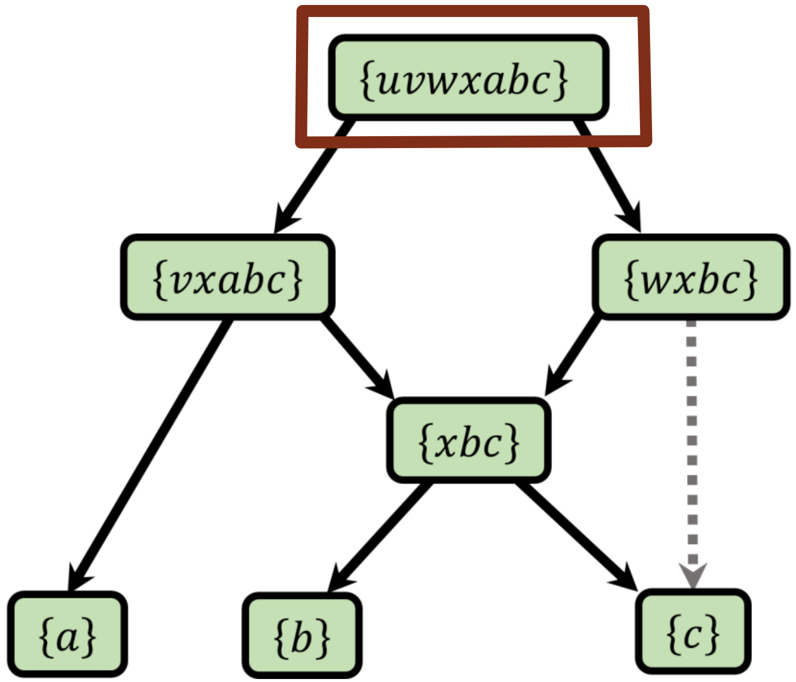

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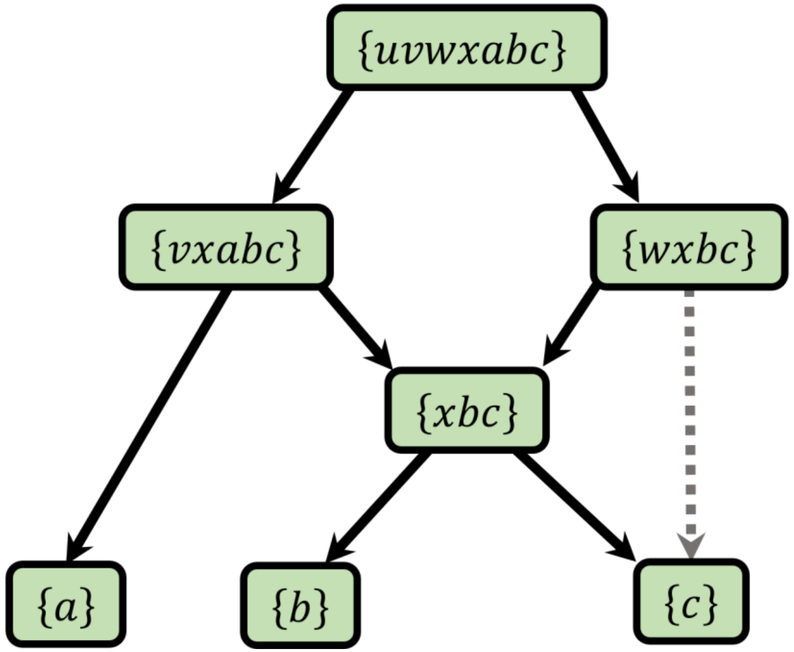

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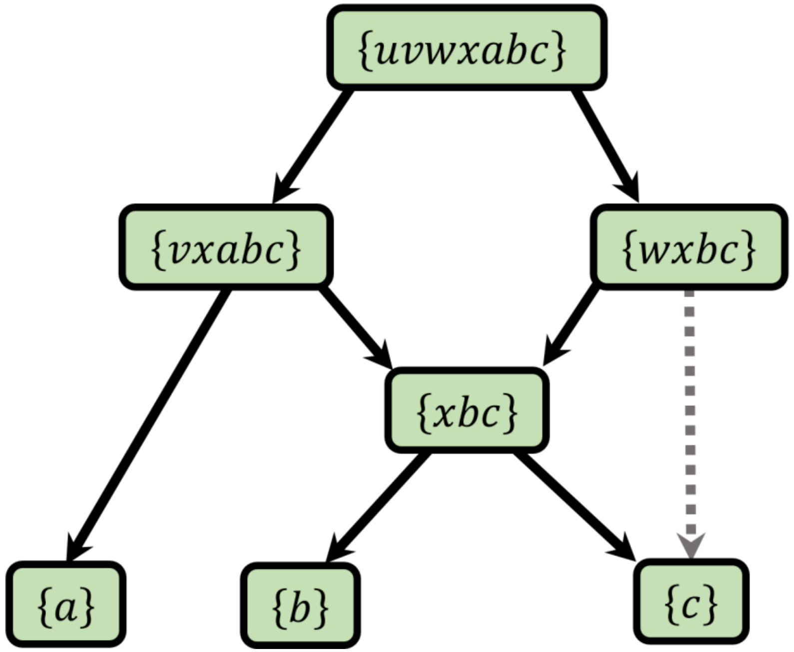

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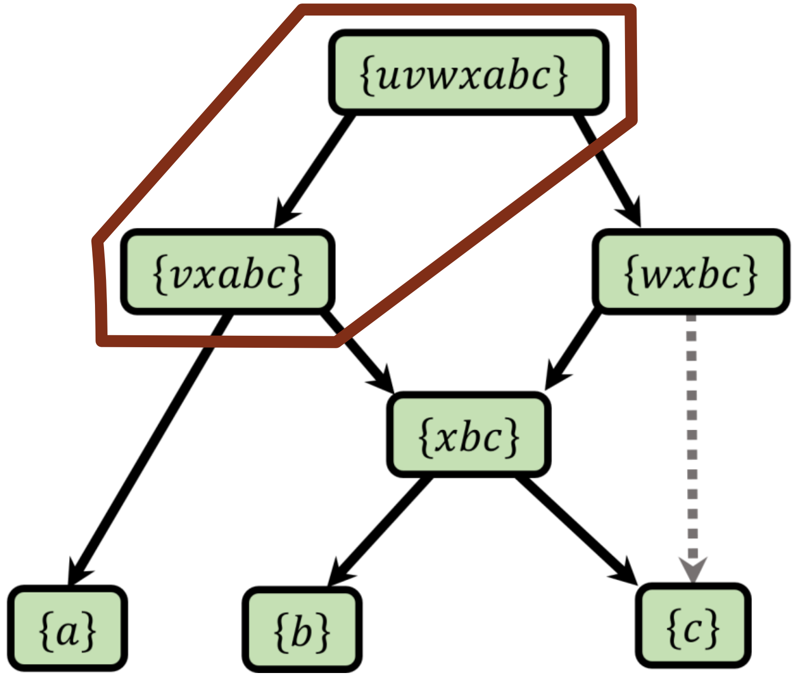

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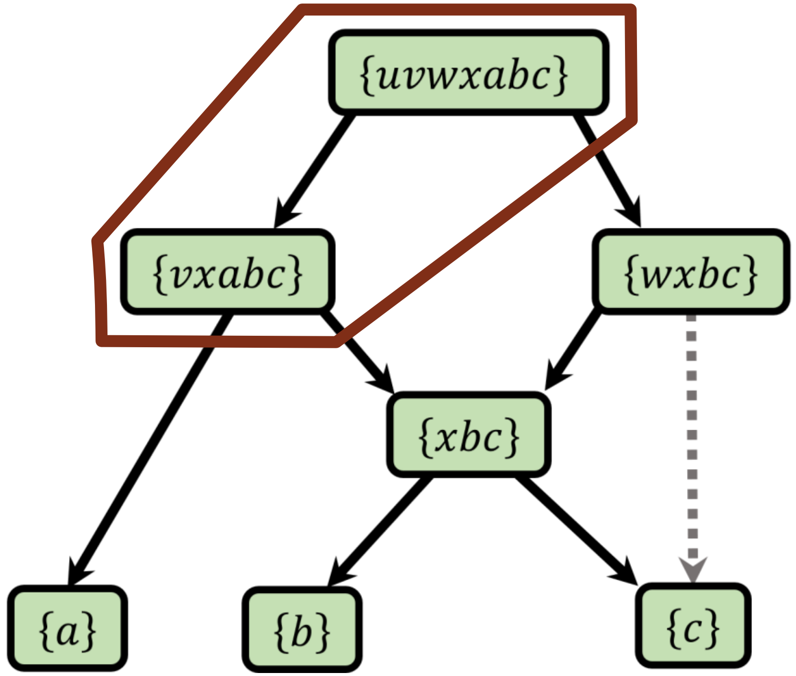

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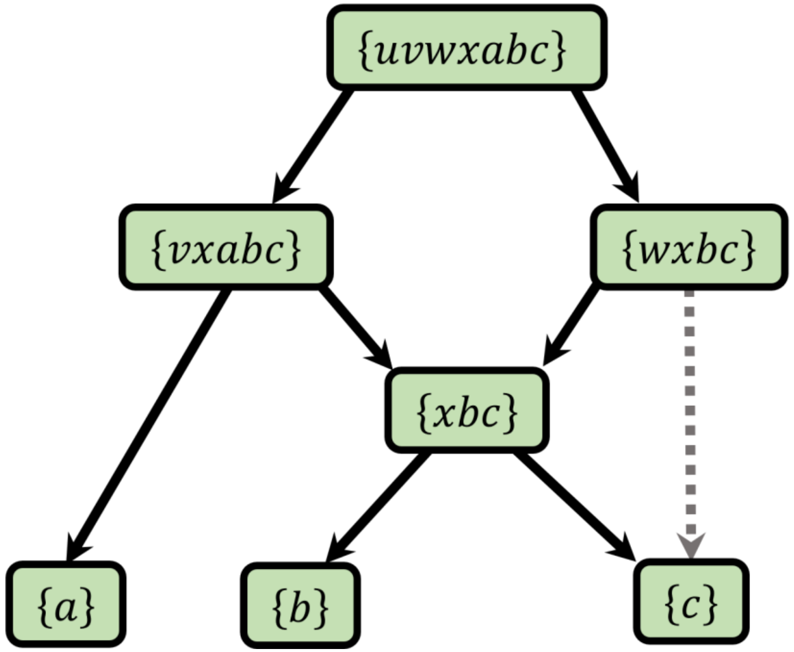

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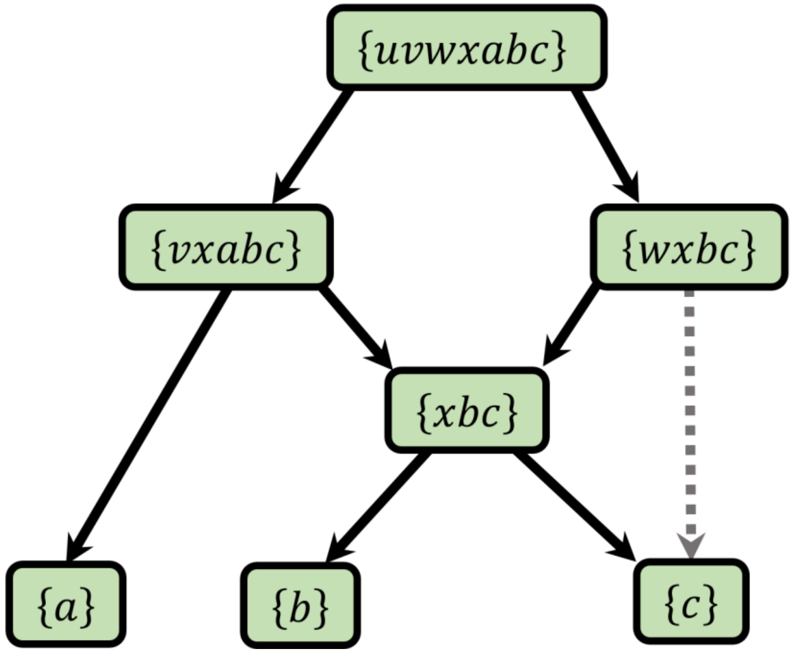

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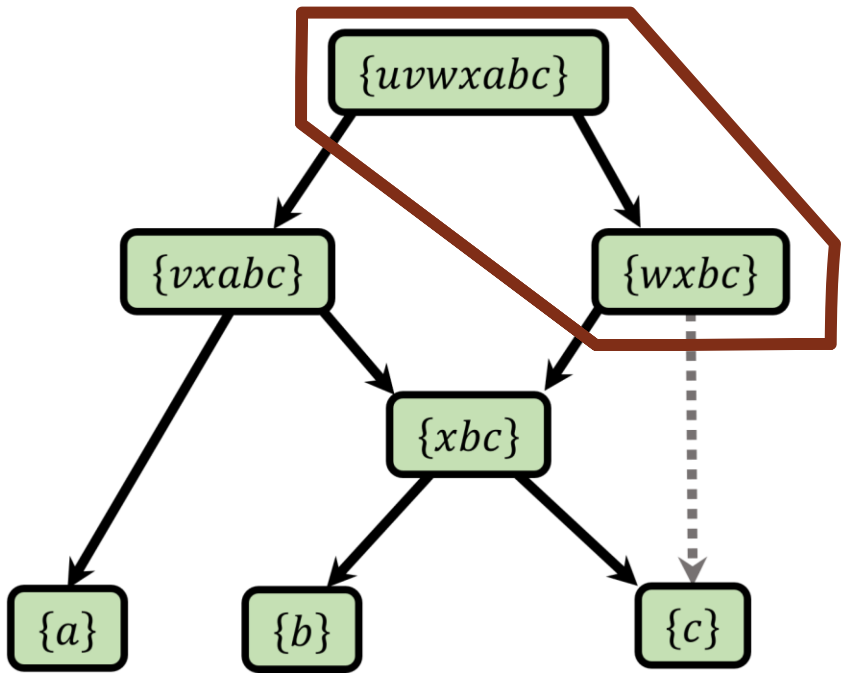

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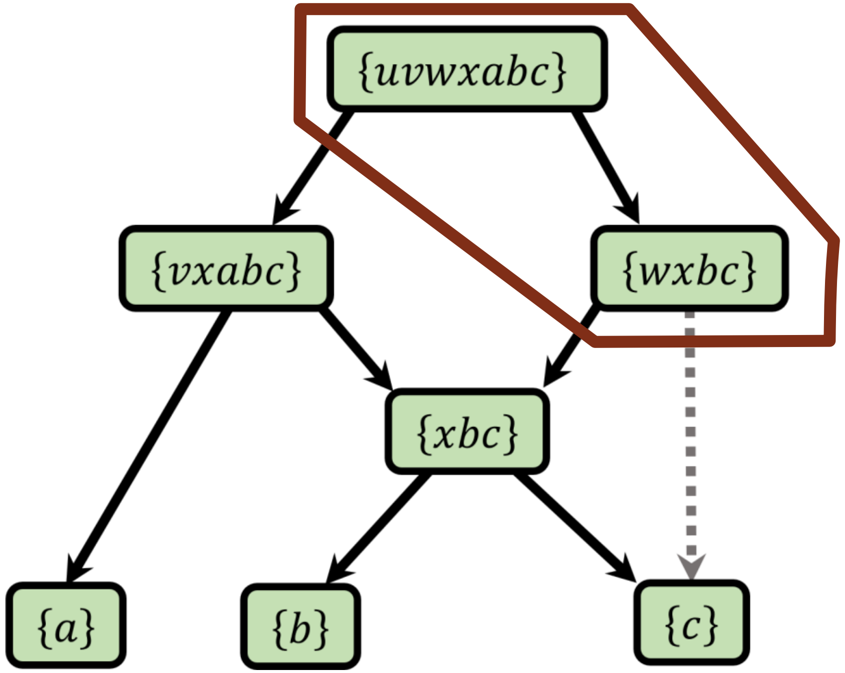

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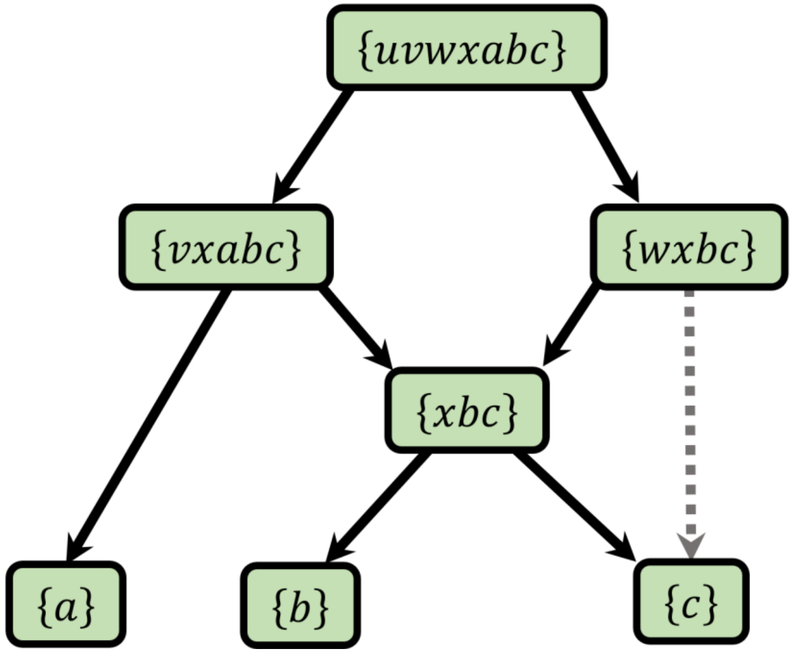

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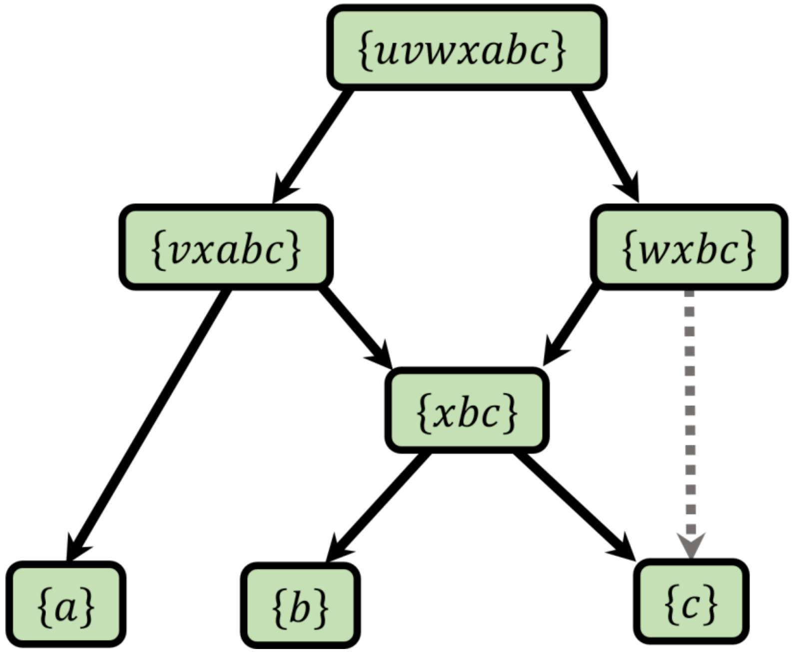

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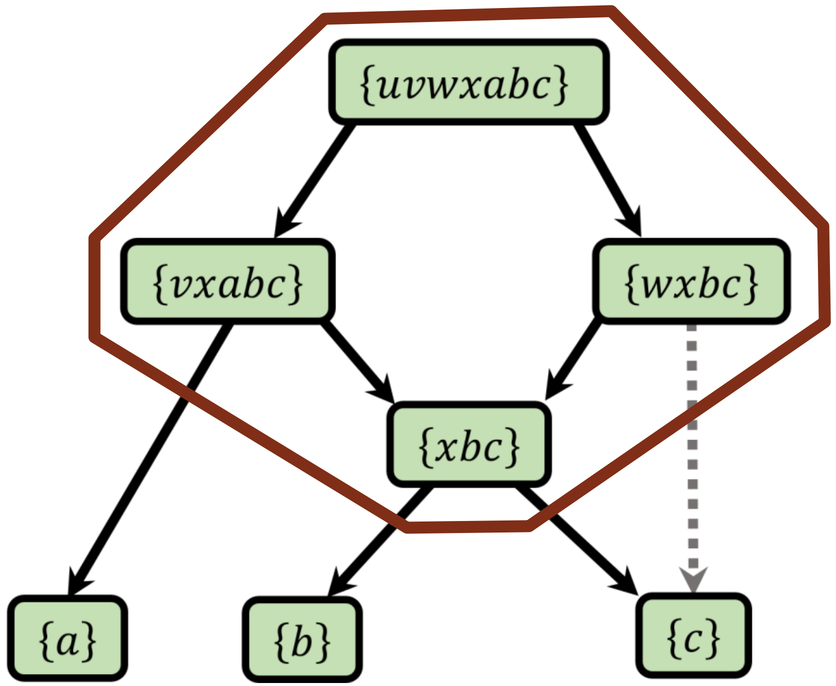

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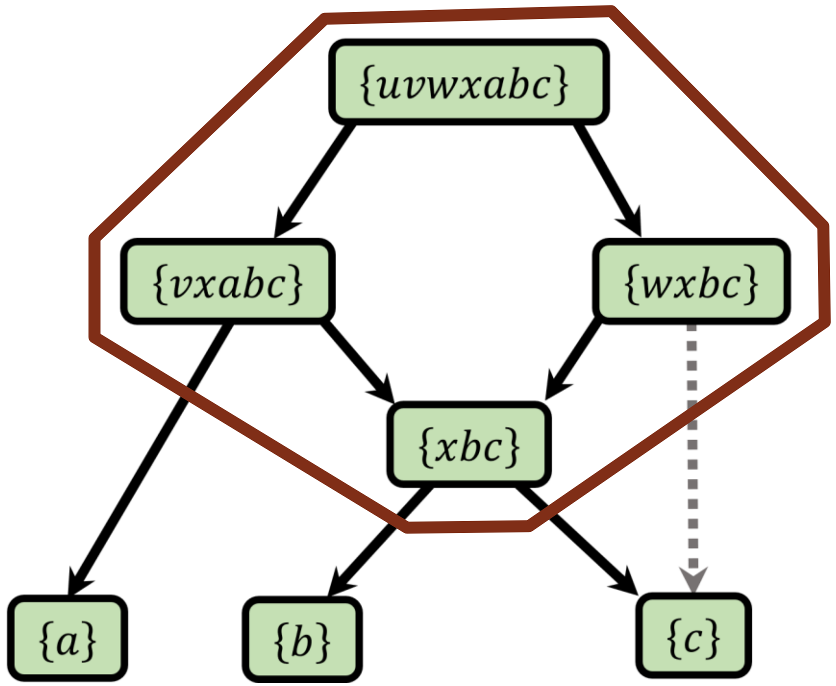

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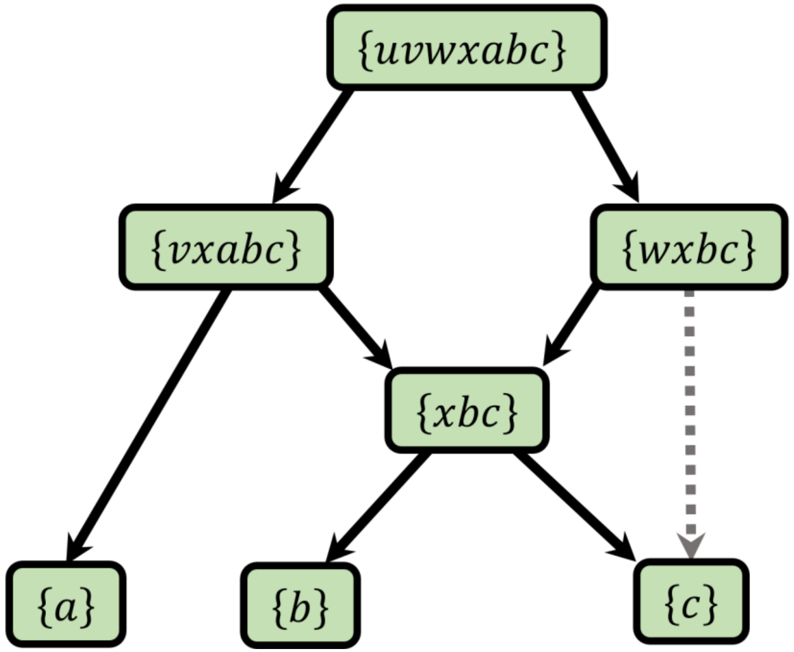

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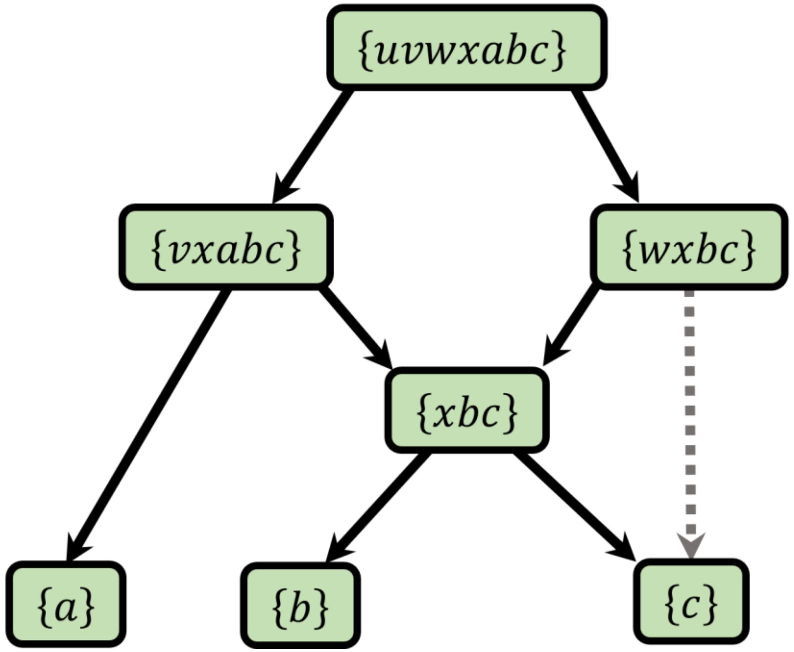

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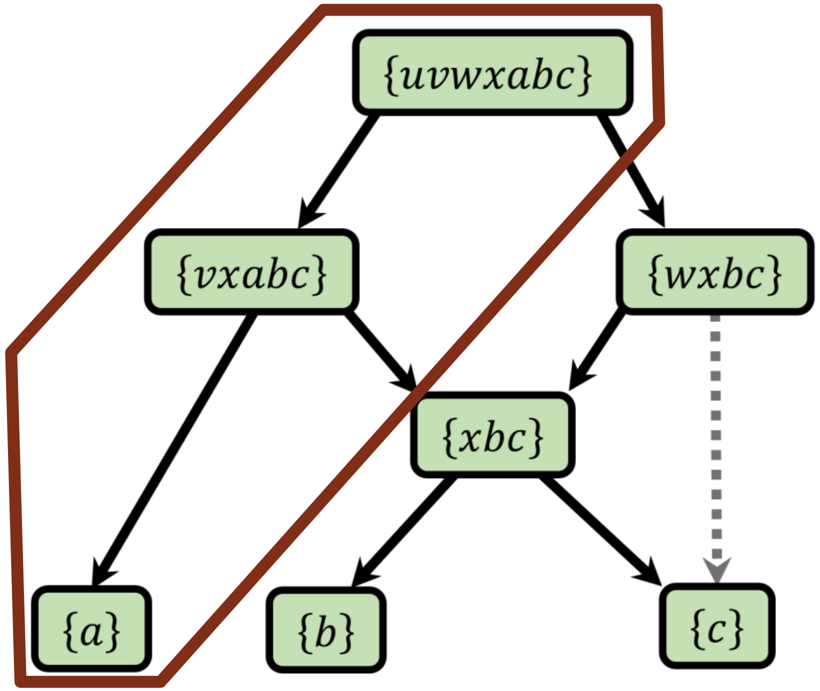

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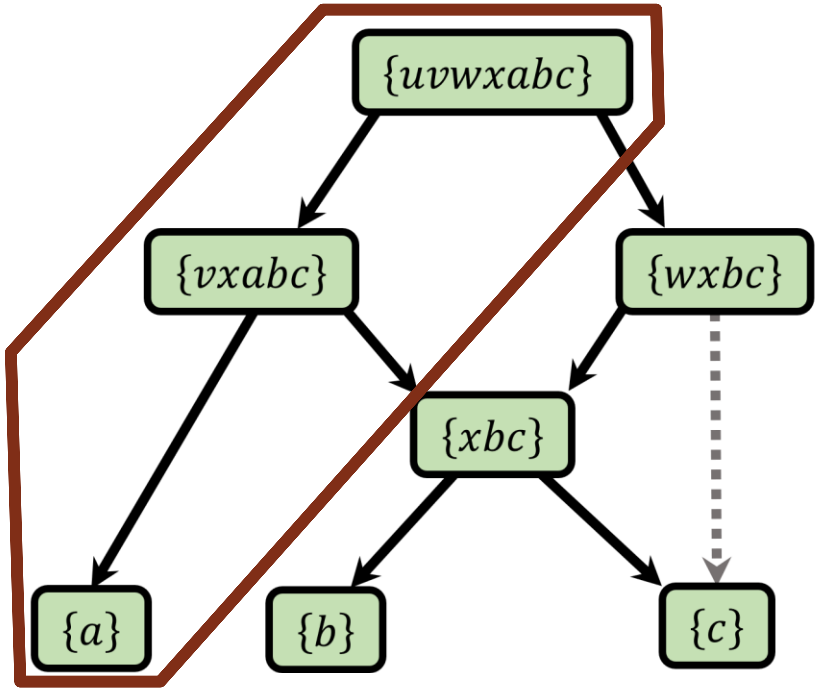

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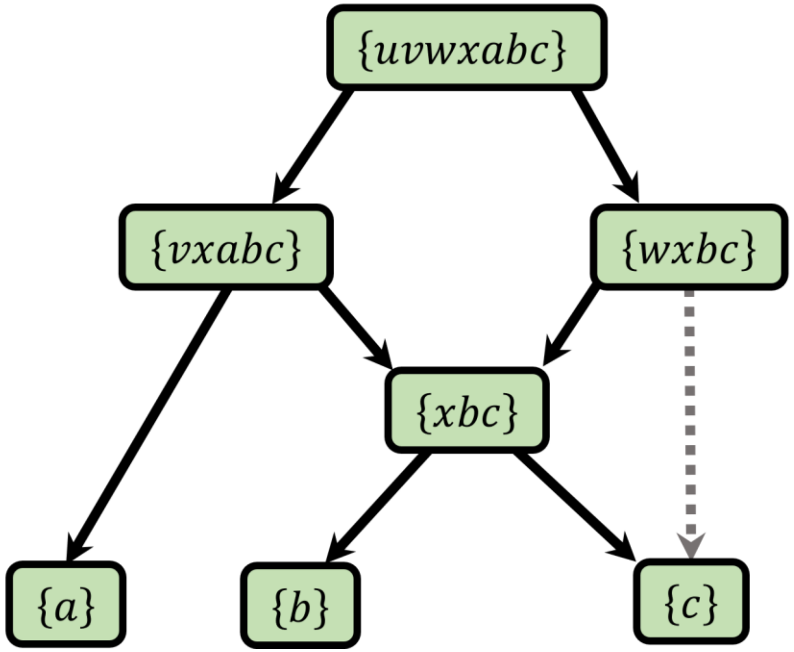

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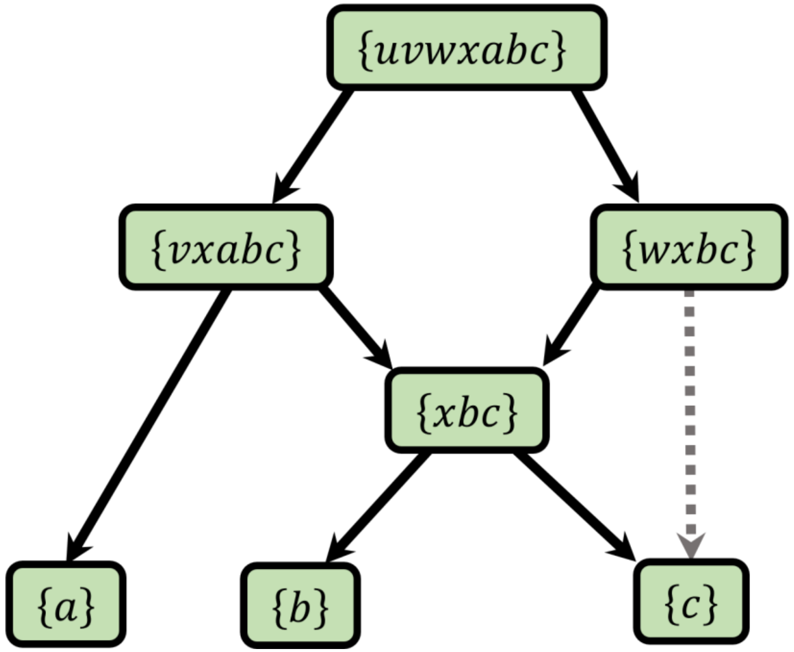

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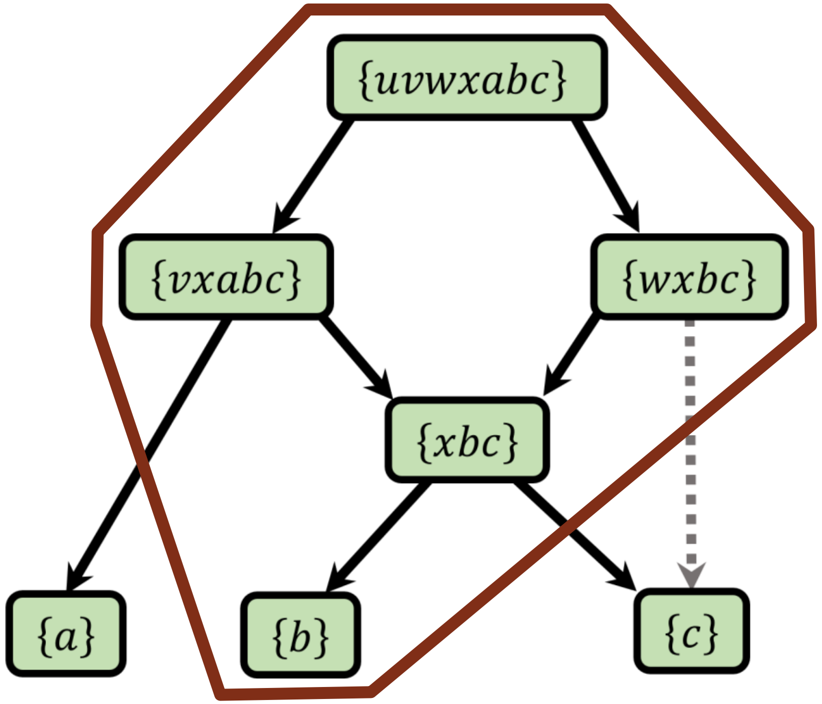

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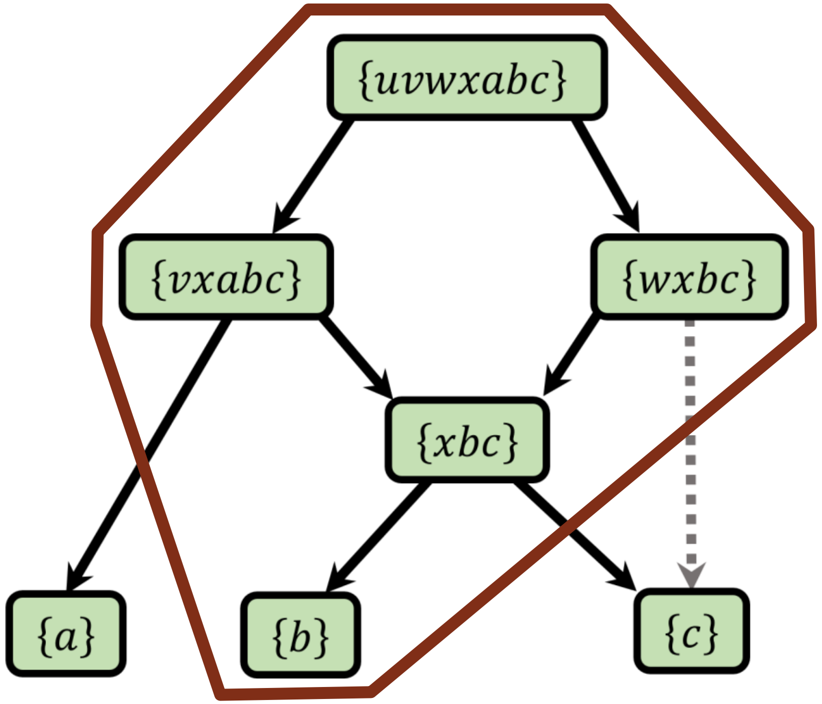

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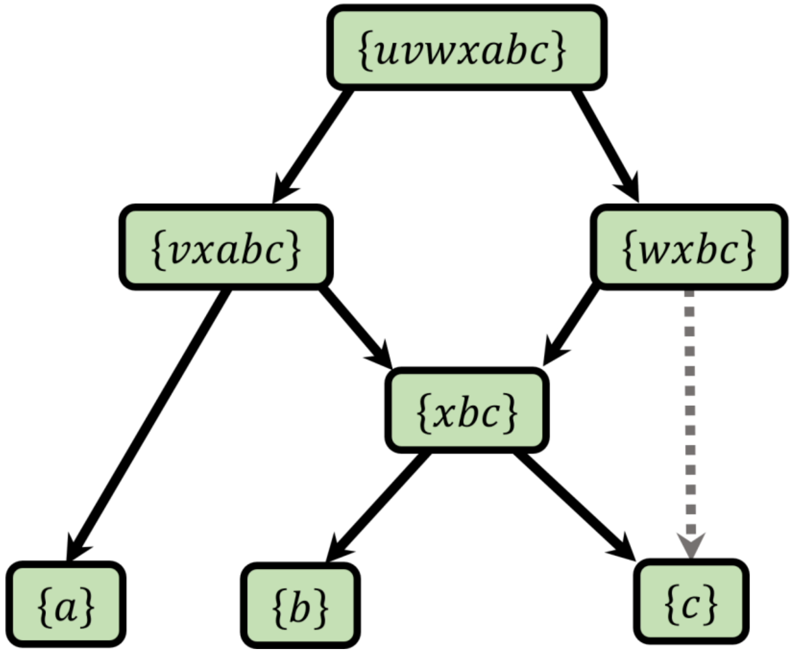

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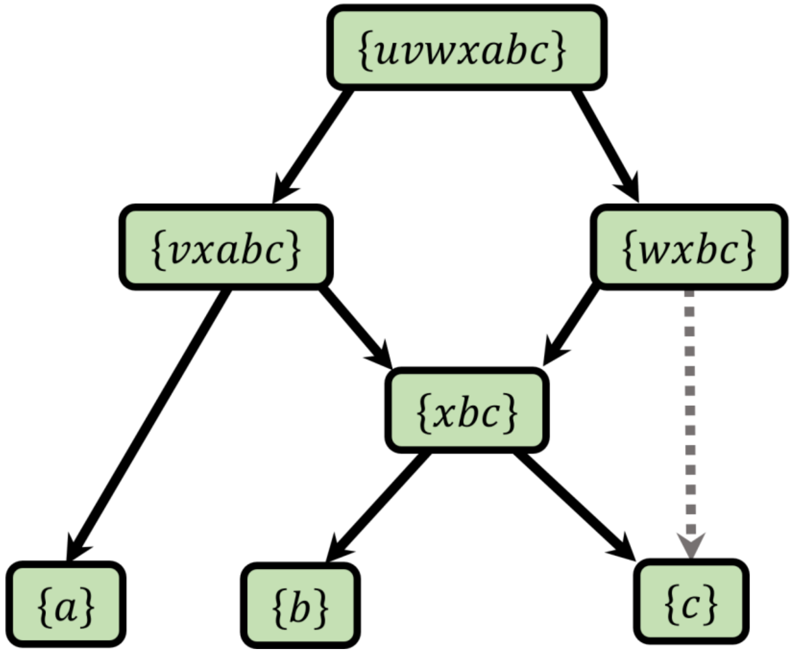

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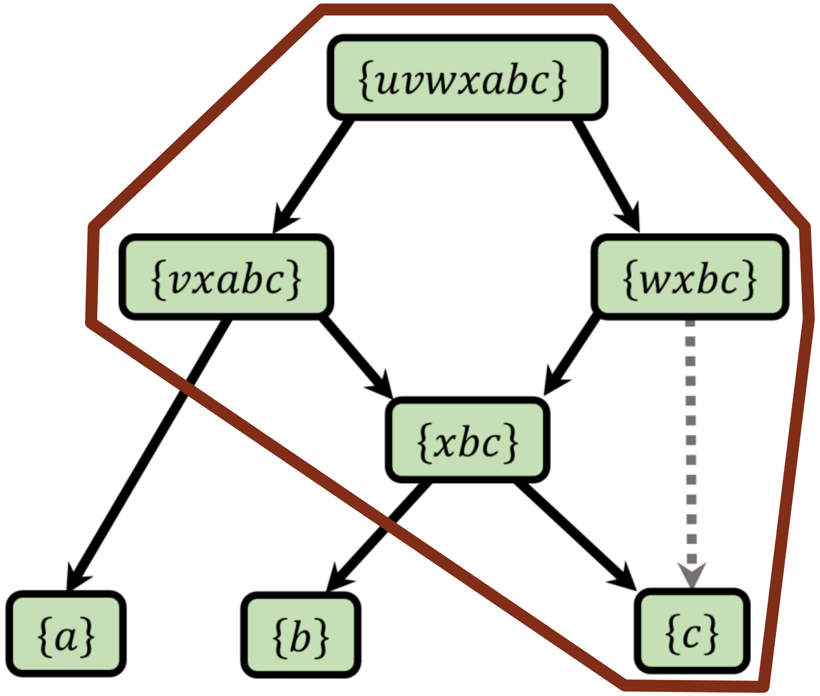

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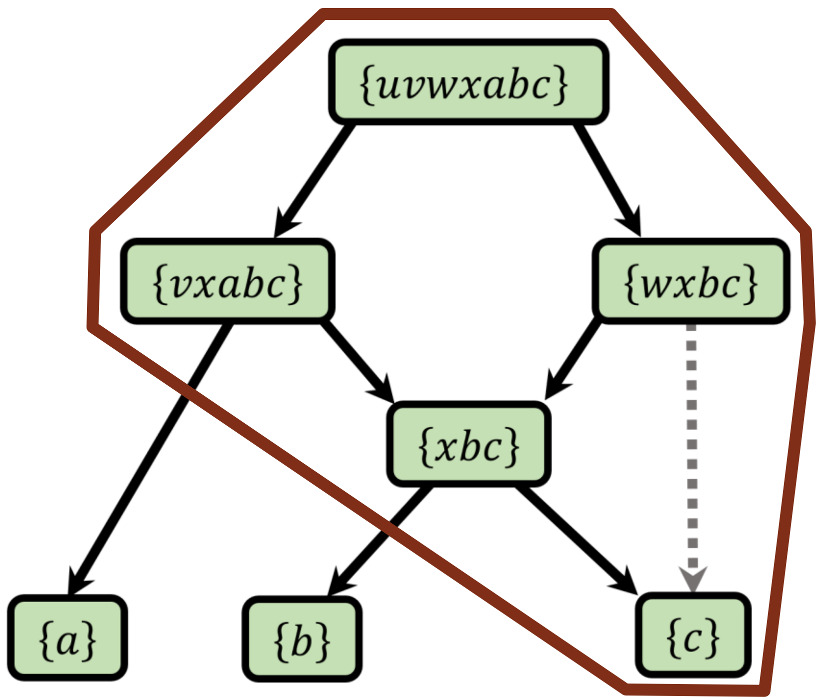

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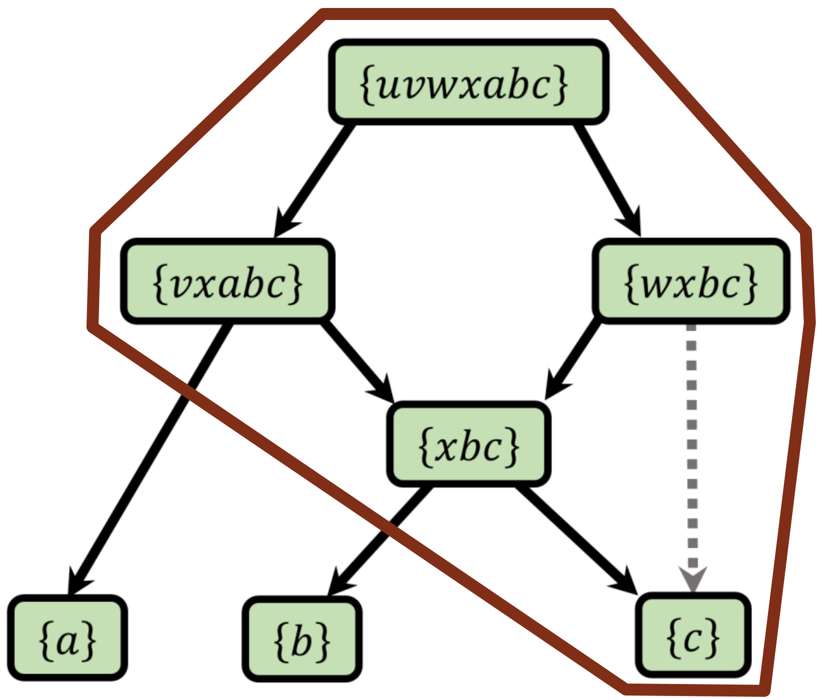

Let $\boldsymbol{G}$ be a DAG and $\mathfrak{D}$ be its descendant cluster. Then, $\boldsymbol{D}(\boldsymbol{u}) = \widetilde{\boldsymbol{D}}(\boldsymbol{u})$ and $\boldsymbol{f}: \boldsymbol{V}(\boldsymbol{G}) \to \mathfrak{D}, \boldsymbol{f}(\boldsymbol{u}) \coloneqq \widetilde{\boldsymbol{D}}(\boldsymbol{u})$ is bijective.
(Has also been proved with non-anecdotal arguments)

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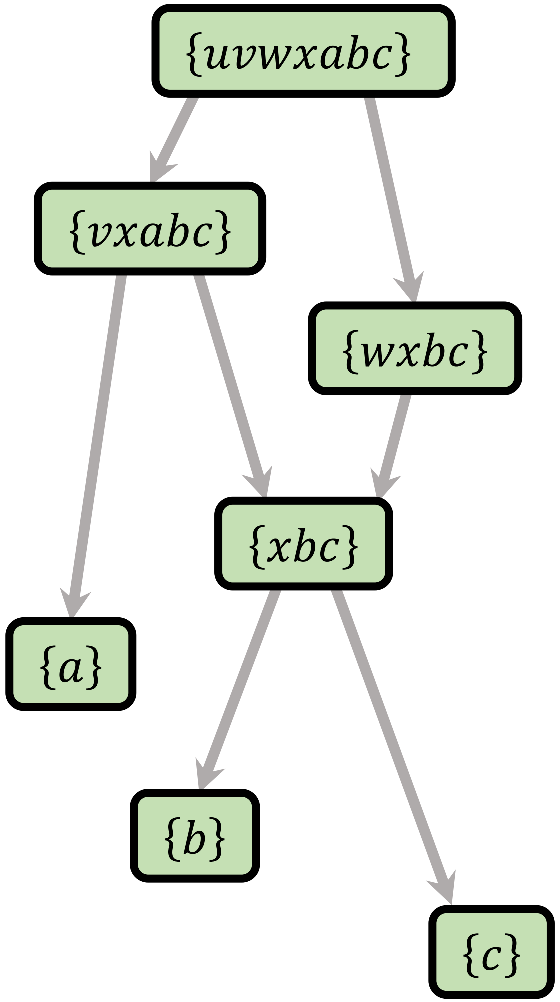

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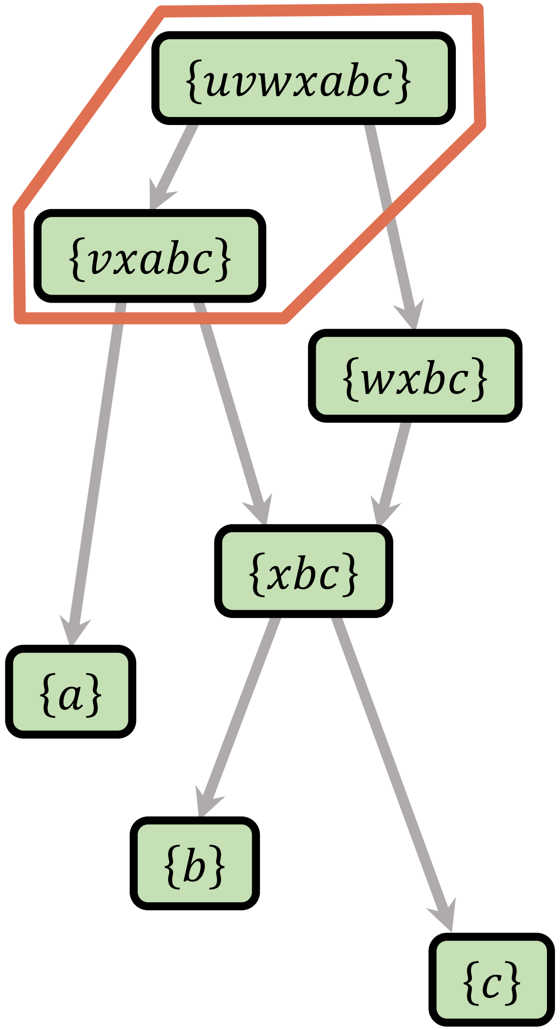

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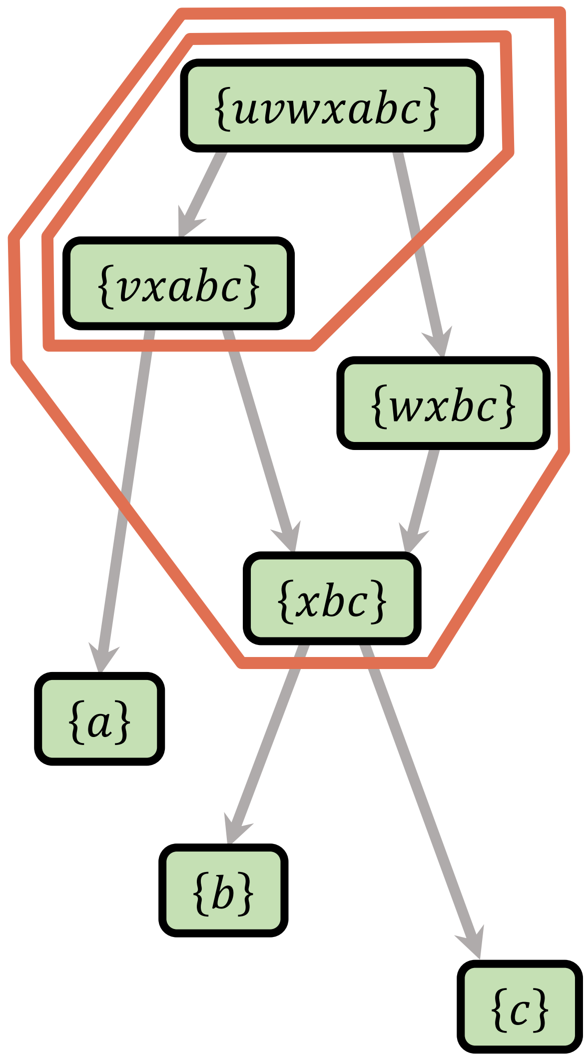

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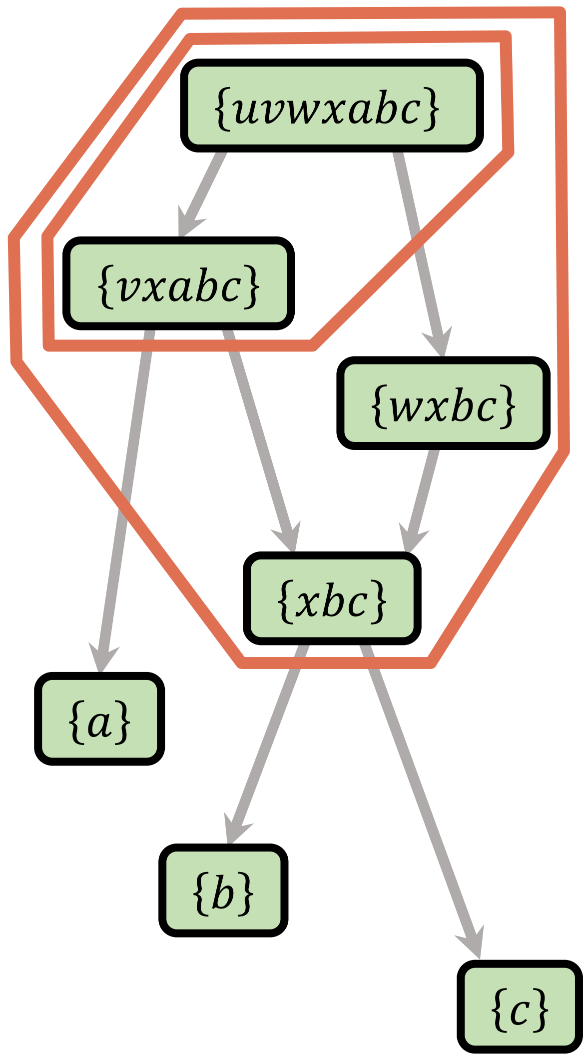

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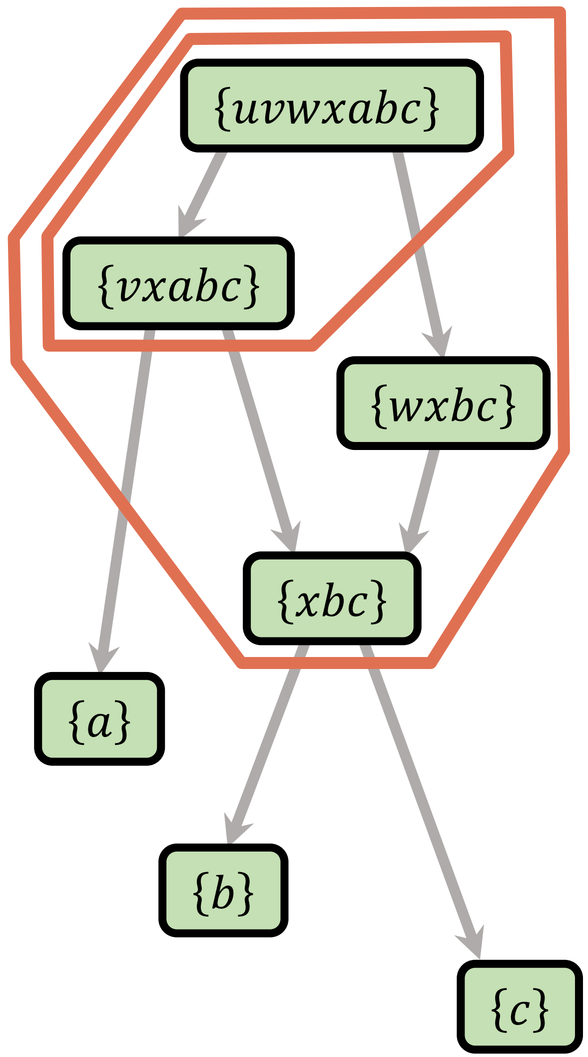

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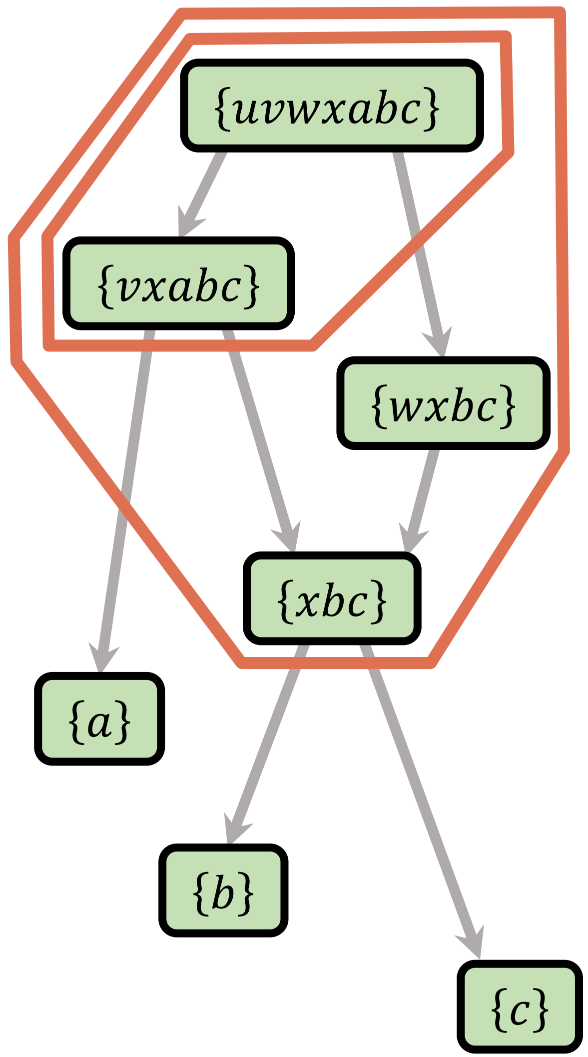

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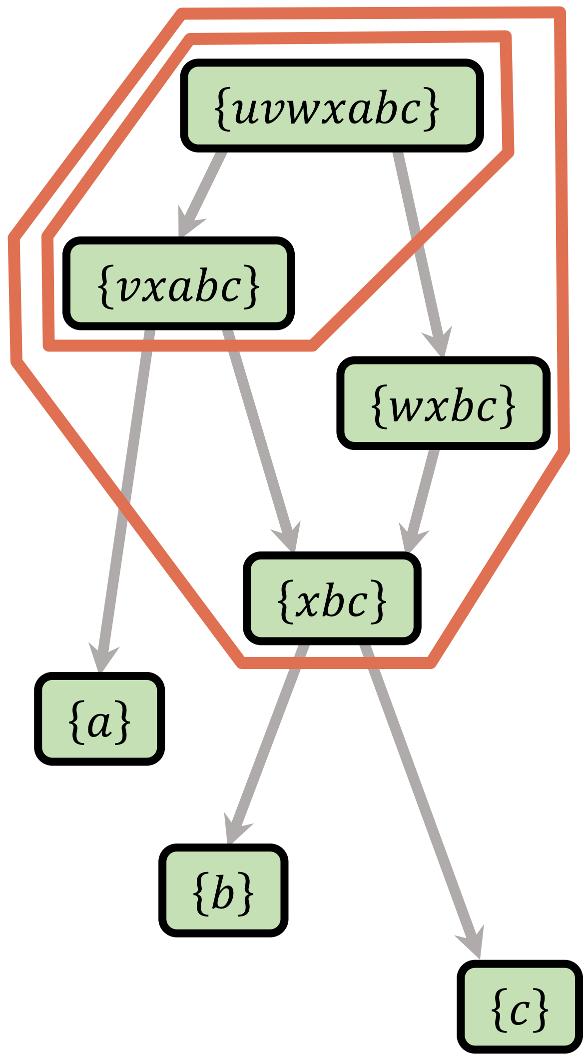

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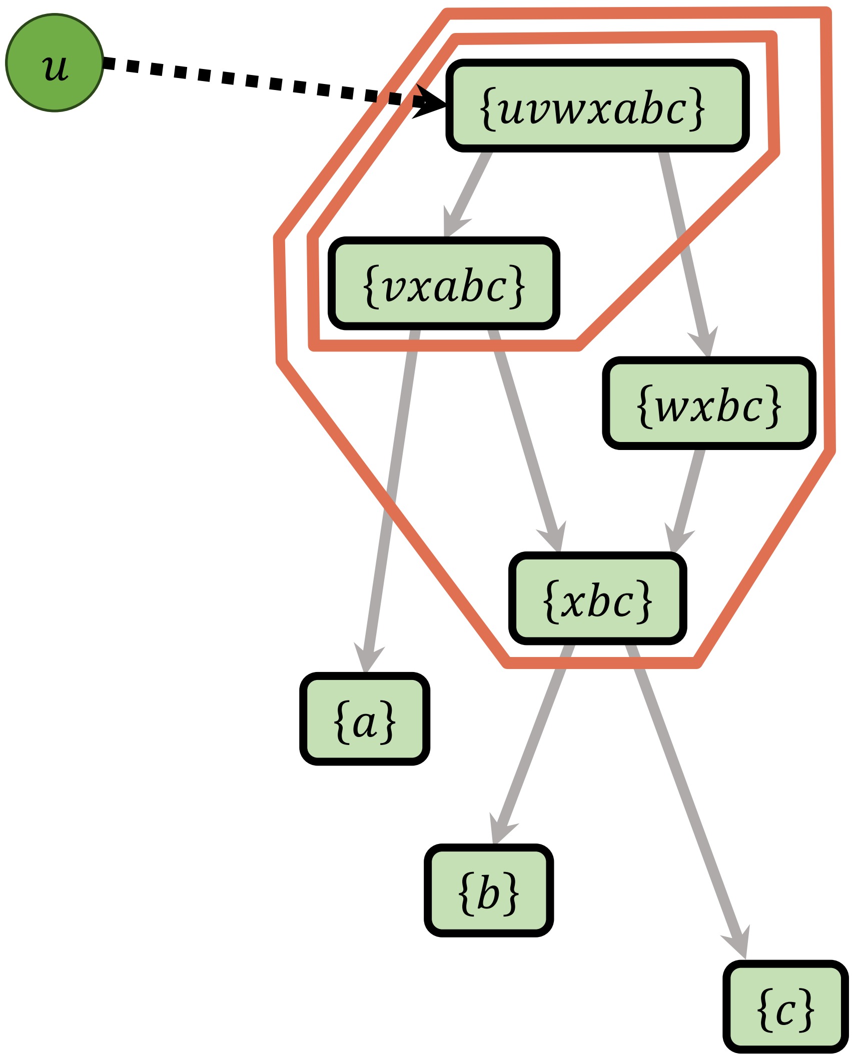

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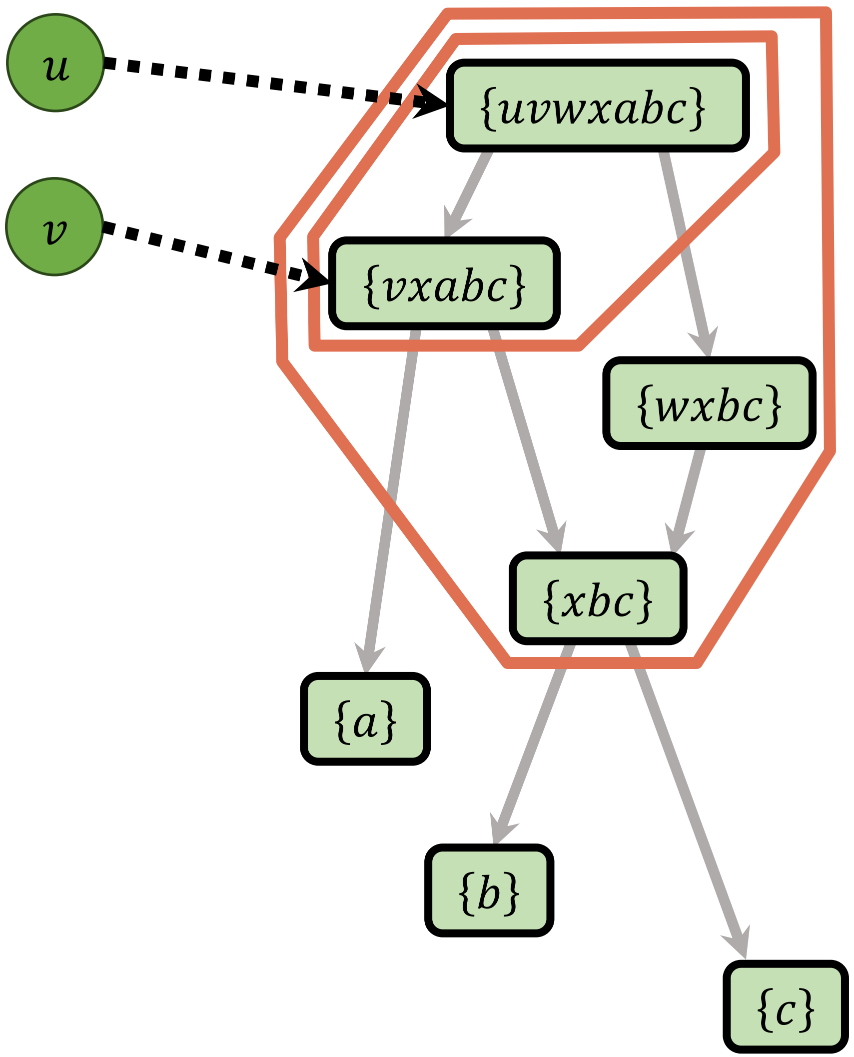

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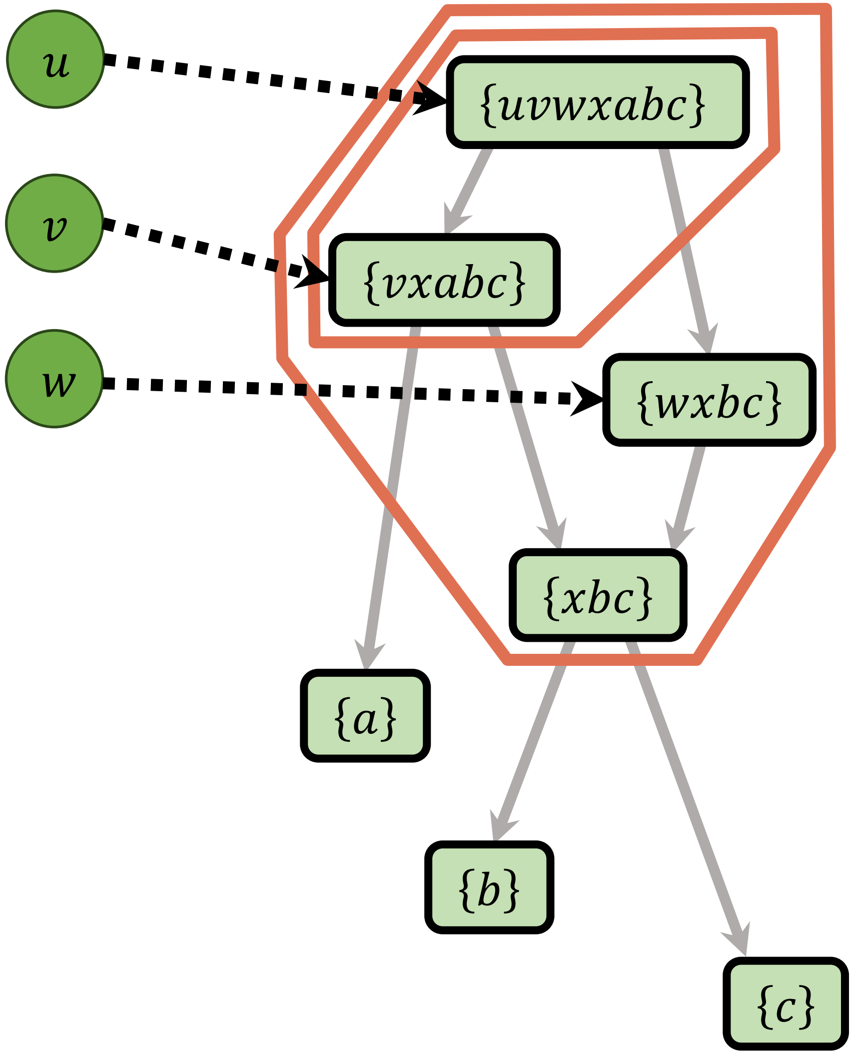

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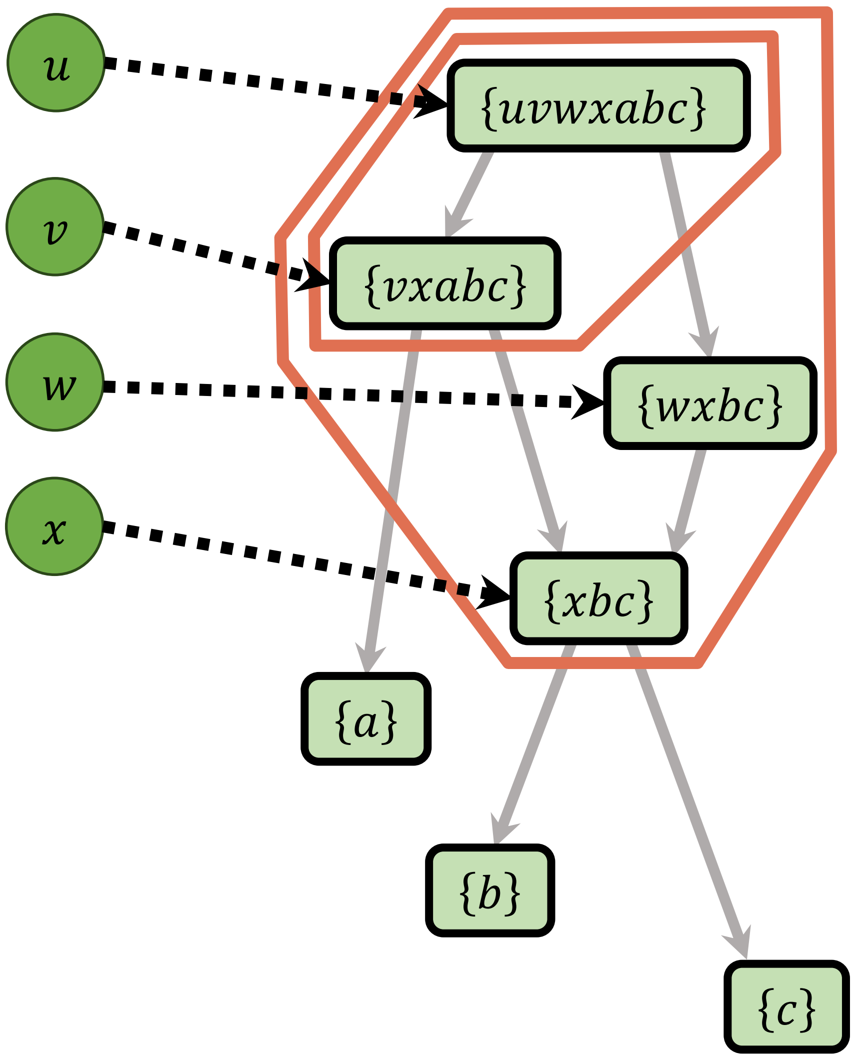

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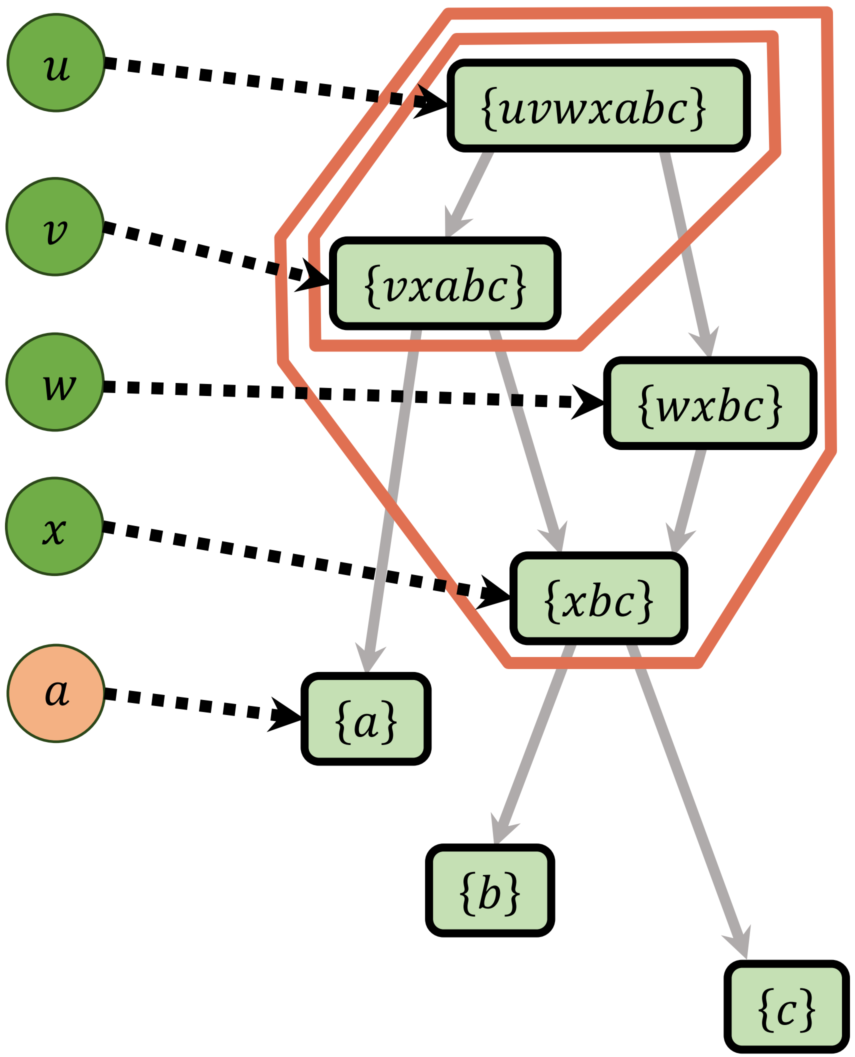

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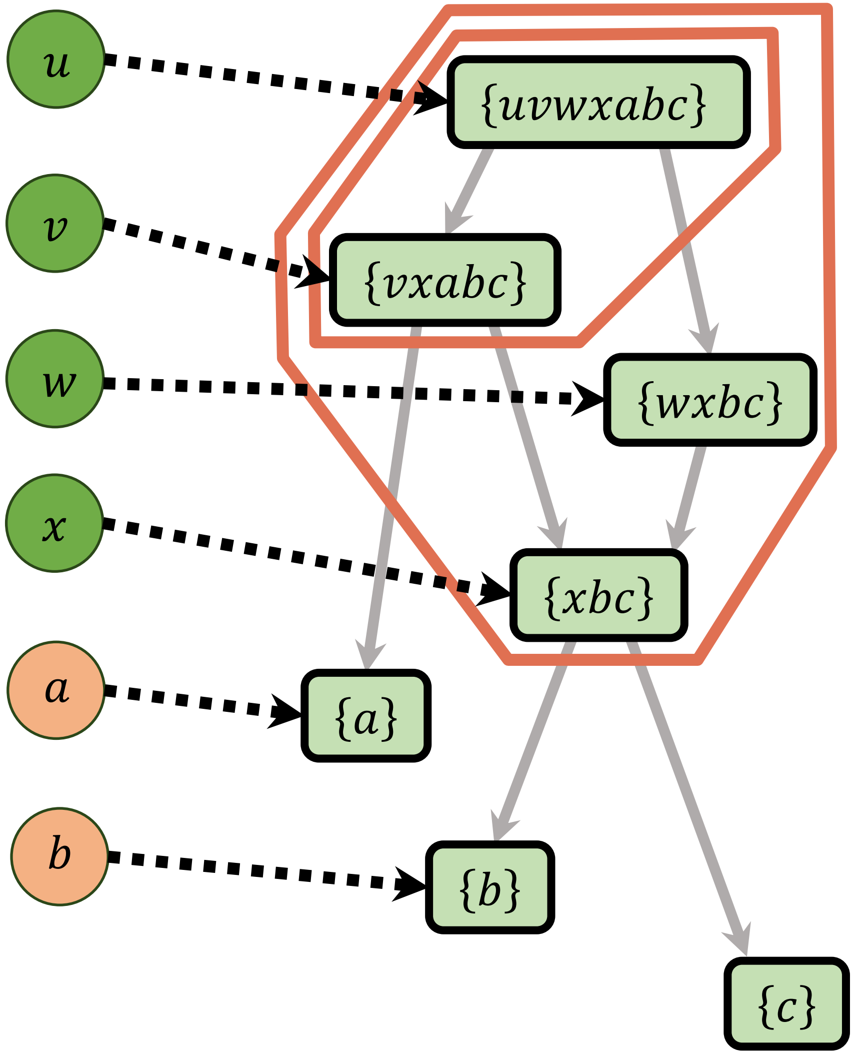

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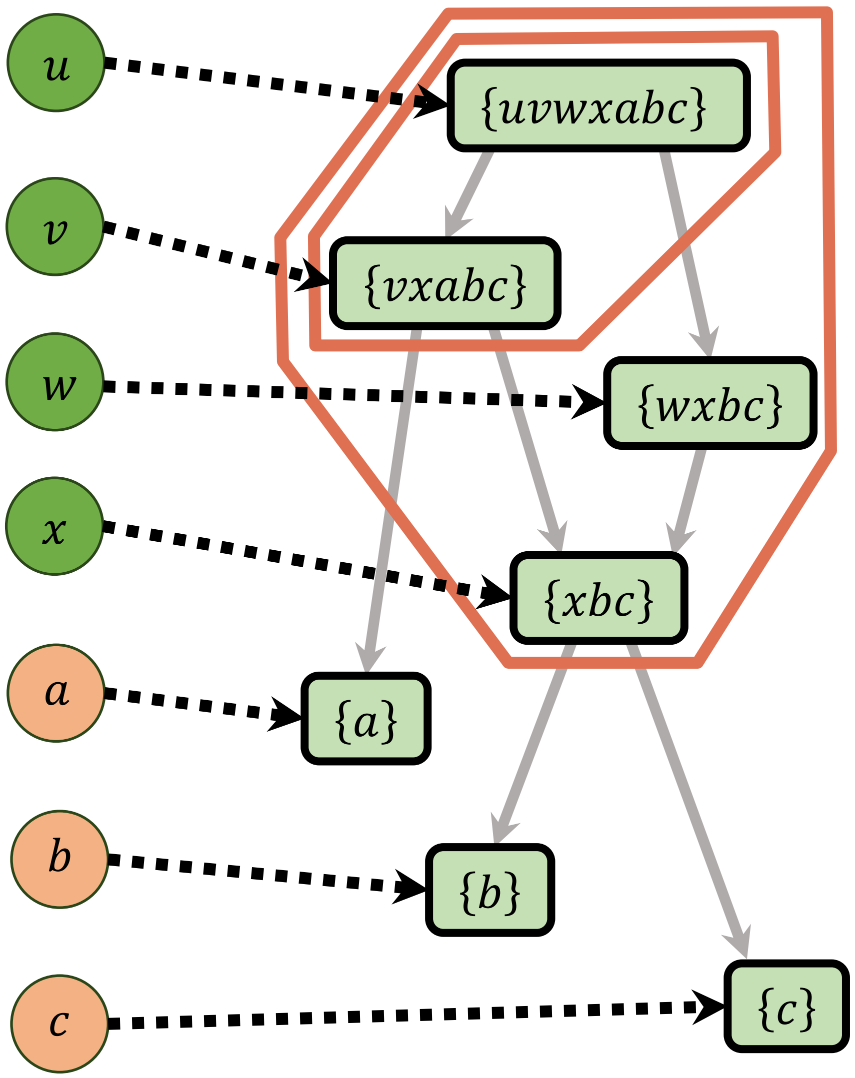

Outlook

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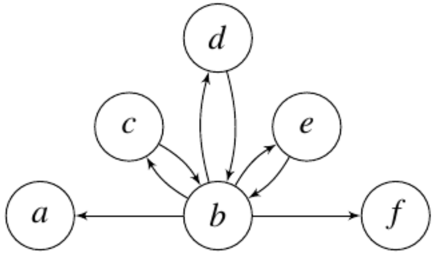

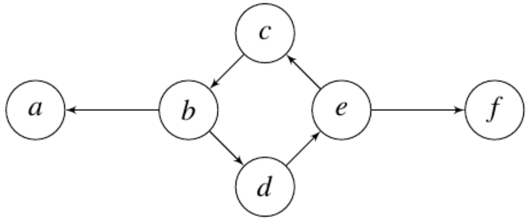

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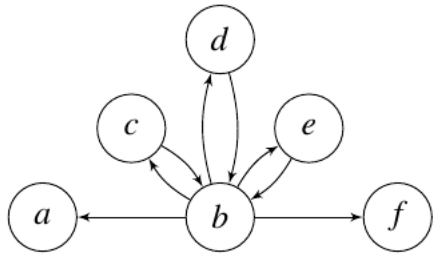

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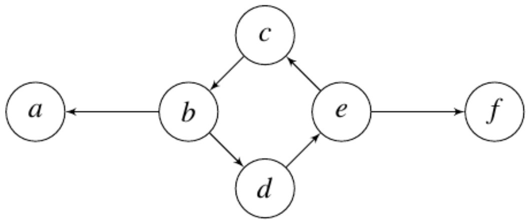


- Somewhat coincides with the all-path transit function for DAGs:
  $A(u, v) = D(u) \cap P(v)$ (where P denotes all predecessors of v)

Thank you!

Marc Hellmuth

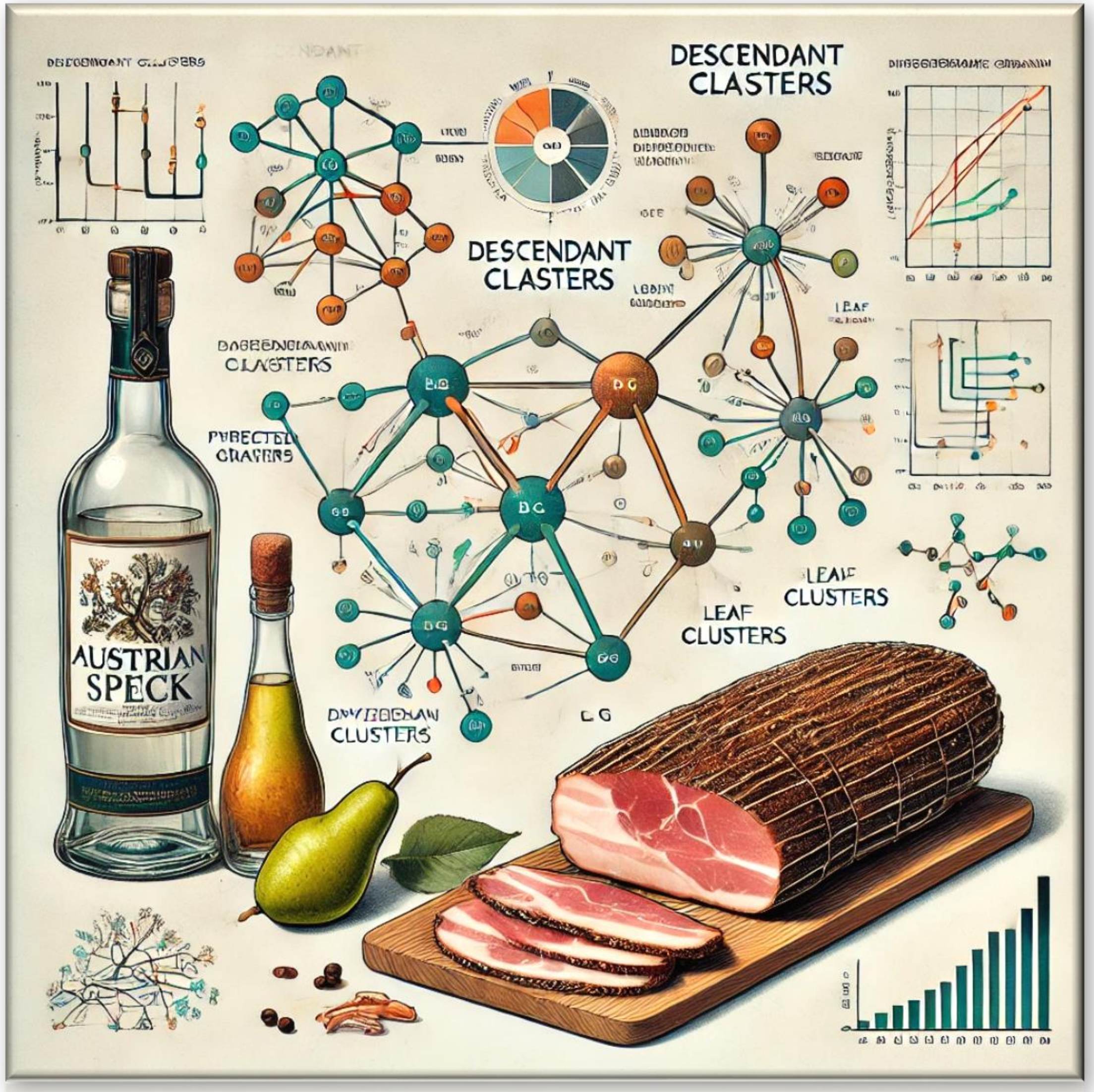
Peter F. Stadler

&

Everyone attending!

“Here is your **corrected graphical abstract**, now with **Austrian Speck accurately depicted as a large piece of cured bacon** on the right side and the **bottle of clear pear liquor on the left**. The scientific focus remains intact while subtly and carefully integrating the conference setting.”

~ChatGPT & Dall-E



Leaf extended DAG