Complexity of a Reaction Network

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February 13, 2025

40th TBI Winterseminar in Bled

How do we measure the complexity of a reaction network?

- based on the number of reactants

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How do we measure the complexity of a reaction network?

- based on the number of reactants
- based on the number of unique reaction centers

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How do we measure the complexity of a reaction network?

- based on the number of reactants
- based on the number of unique reaction centers
- based on the number of parameters involved

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the maximum number of persistent subspaces over all possible inflows under a general dilution flow

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Outline

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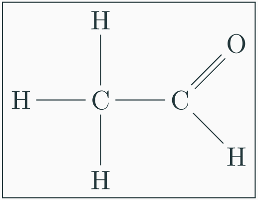
Preliminaries

Reaction Network

Set vs. Multiset

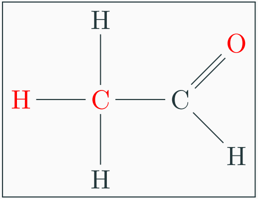


Set -

Multiset -

Reaction Network

Set vs. Multiset

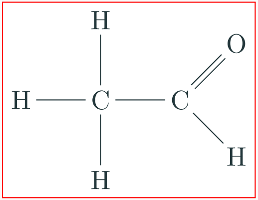


Set - $\{H, C, O\}$

Multiset -

Reaction Network

Set vs. Multiset



Set - $\{C, H, O\}$

Multiset - $\{H, H, H, H, C, C, O\}$

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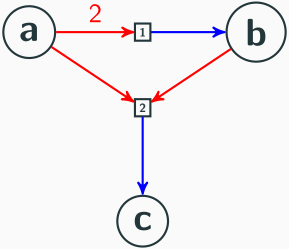
Reaction Network

A reaction network is a pair (M, R) where M is a set of molecules and R is a set of reactions $R \subseteq Pmult(M) \times Pmult(M)$.



$$M : \{a, b, c\}$$

$$R : [Reactands : \{a, a\} Product : \{b\}$$

$$Reactands : \{a, b\} Product : \{c\}]$$

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Organization Theory

A set of molecules is said to be an organization if:

1. **Closed** – subset A of M is closed if by applying reactions from R_A to multisets over A we **cannot produce molecules not in** A.
2. **Self-maintaining** – applying reactions from R_A to a multiset over M **does not reduce the number of molecules of any species of** A.

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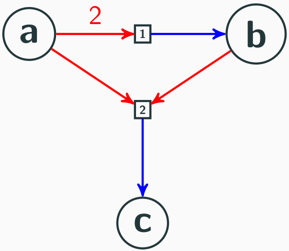
Organization Theory



- $A = \{a\}$: not an organization

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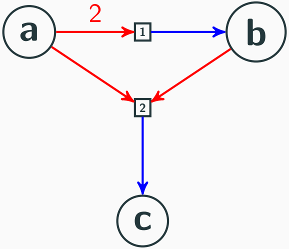
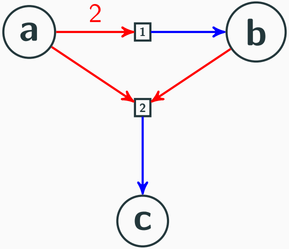
Organization Theory



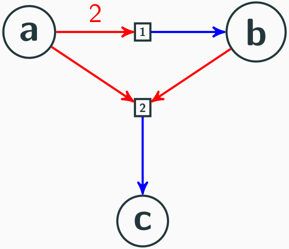
- $A = \{a\}$: not an organization
- $A = \{b\}$: an organization

Organization Theory



- $A = \{a\}$: not an organization
- $A = \{b\}$: an organization
- $A = \{c\}$: an organization

Organization Theory



- $A = \{a\}$: not an organization
- $A = \{b\}$: an organization
- $A = \{c\}$: an organization
- $A = \{a, b\}$: not an organization

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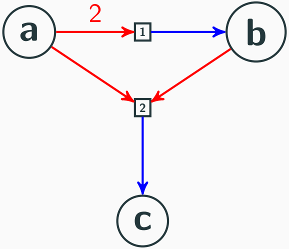
Organization Theory



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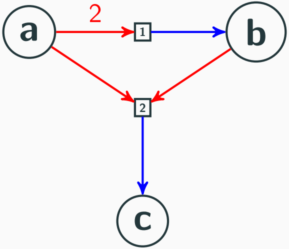
Organization Theory



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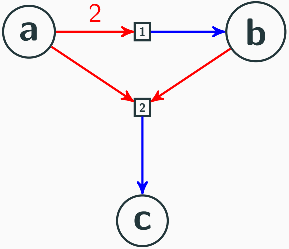
Organization Theory



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- $A = \{a, c\}$: not an organization
- $A = \{a, b, c\}$: not an organization

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Organization Theory

only molecular species that forms an organization can be persistent

Method

Given (M, R), its **persistent subspace complexity** $C(M, R)$ is defined as:

$$C(M, R) = \max_{A \subseteq M} |O((M, R''_A))|,$$

where $O((M, R''_A))$ is the set of organizations of the reaction network (M, R''_A) and R''_A denotes the reactions of the original network plus reactions for inflow of A and outflow of all molecules.

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Method

In the absence of initial compounds, is there an **inflow set that can generate the different states** of the system?

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Method

Why are networks with high subspace complexity interesting?

- they can go into different states with different composition of species

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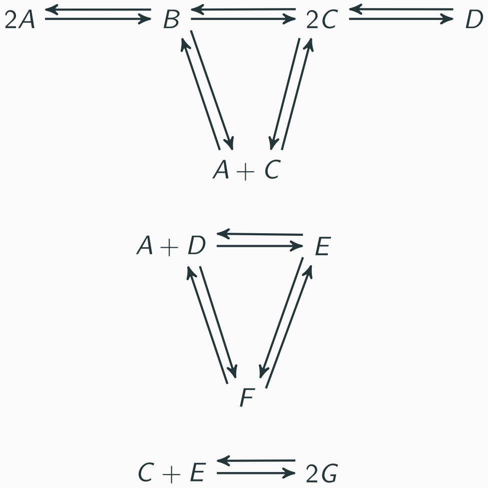
Application

Minimal Subspace Complexity

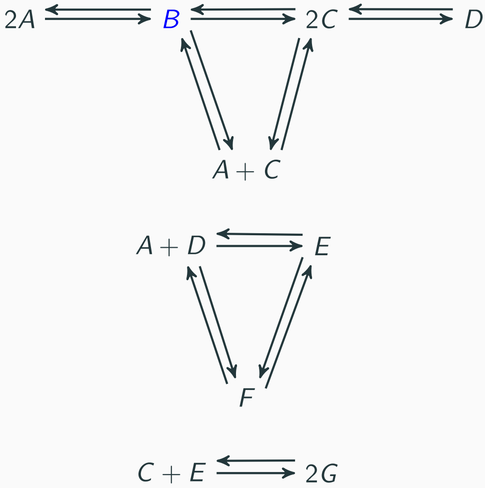
Select Reversible Networks



An example of reversible network taken from lecture of Feinberg.

Minimal Subspace Complexity

Select Reversible Networks



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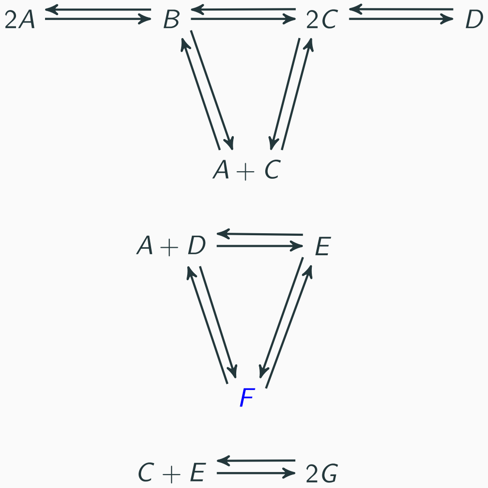
Minimal Subspace Complexity

Select Reversible Networks



An example of reversible network taken from lecture of Feinberg.

Minimal Subspace Complexity

Select Reversible Networks

$$o = \{M\}$$

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Minimal Subspace Complexity

Select Reversible Networks

$o = \{M\}$

For all reversible networks where **any inflow set can trigger all reactions in the system to fire**, the **only organization** in the network is always the **complete set of molecules**

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Minimal Subspace Complexity

Select Reversible Networks

$o = \{M\}$

For all reversible networks where **any inflow set can trigger all reactions in the system to fire**, the **only organization** in the network is always the **complete set of molecules**

Note: not all reversible networks exhibit this behavior.

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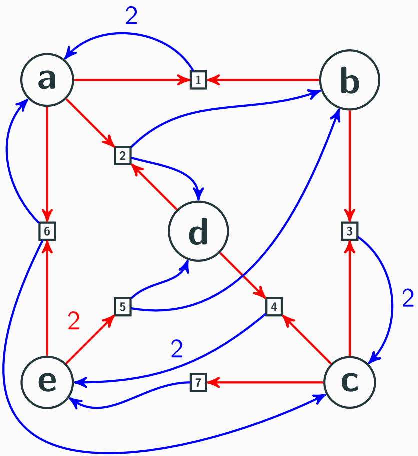
Autocatalytic Network



An abstract example of an autocatalytic network

Autocatalytic Network

Inflow	Closure	Molecules set per organization
$\{a\}$	$\{a\}$	$o_1 : \{a\}$
		$o_2 : \{a, b, c, d, e\}$
$\{e\}$	$\{b, d, e\}$	$o_1 : \{b, d, e\}$
$\{b, e\}$		$o_2 : \{b, c, d, e\}$
$\{d, e\}$		$o_3 : \{a, b, c, d, e\}$
$\{b, d, e\}$		

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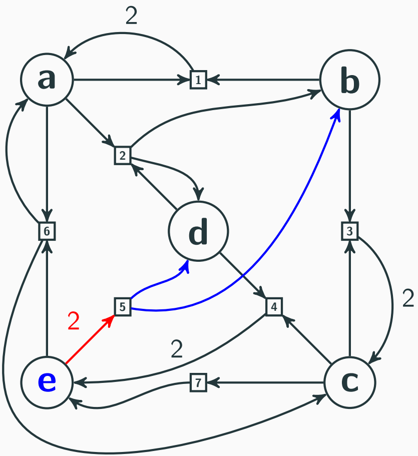
Autocatalytic Network



An abstract example of an autocatalytic network

Autocatalytic Network

Inflow	Closure	Molecules set per organization
$\{b\}$	$\{b\}$	$o_1 : \{b\}$
		$o_2 : \{a, b\}$
		$o_3 : \{b, c, d, e\}$
		$o_4 : \{a, b, c, d, e\}$
$\{b, d\}$	$\{b, d\}$	$o_1 : \{b, d\}$
		$o_2 : \{a, b, d\}$
		$o_3 : \{b, c, d, e\}$
		$o_4 : \{a, b, c, d, e\}$

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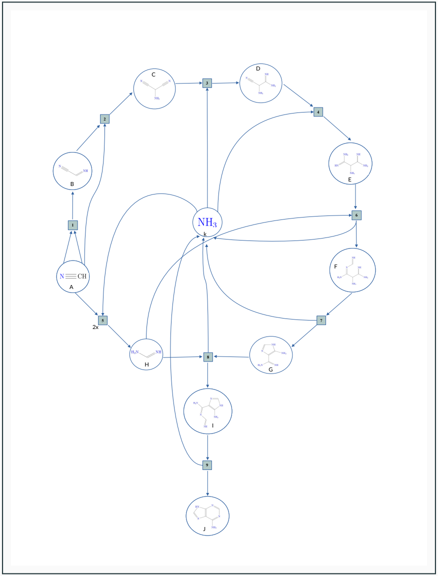
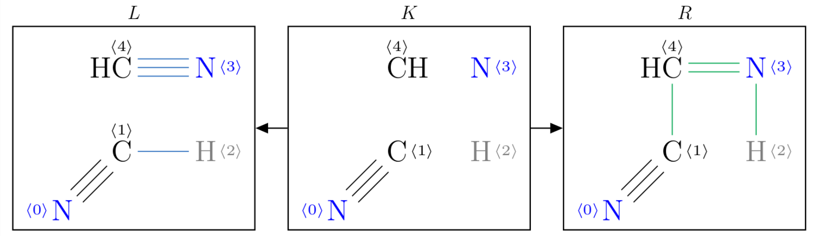
Adenine Chemistry



Formation of adenine from hydrogen cyanide and ammonia

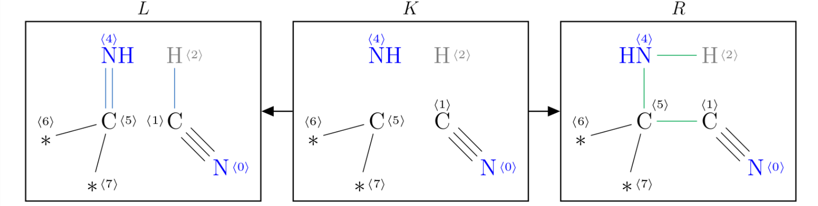
Adenine Chemistry

$$Rule\,1 : [N]1\#[C]2[H]3.[N]4\#[C]5[H]6 >> [N]1\#[C]2[C]5([H]6) = [N]4[H]3$$



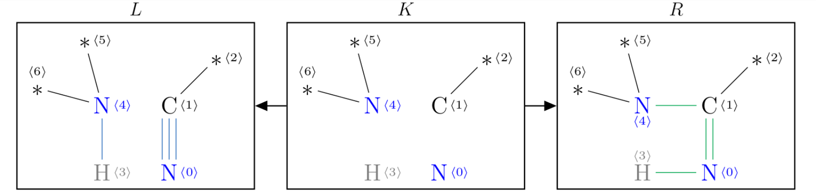
Adenine Chemistry

$$\mathit{Rule}\,2 : [N]1\#[C]2[H]3.[N]4([H]6) = [C]5([*]7)[*]8 \;>>$$
$$[N]1\#[C]2[C]5([*]7)([*]8)[N]4([H]6)[H]3$$



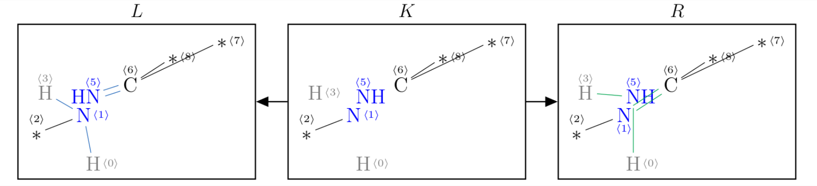
Adenine Chemistry

$$\mathit{Rule}\,3 : [N]1\#[C]2[*]3.[H]4[N]5([*]6)[*]7 >>$$
$$[H]4[N]1 = [C]2([*]3)[N]5([*]6)[*]7$$



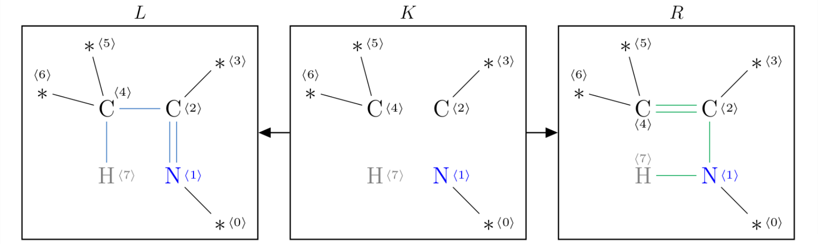
Adenine Chemistry

$$\textit{Rule}\,4 : [H]1[N]2([H]4)[*]3.[H]5[N]6 = [C]7([*]8)[*]9 \;>> $$
$$[*]3[N]2 = [C]7([*]8)[*]9.[H]1[N]6([H]4)[H]5$$



Adenine Chemistry

$$Rule\,5 : [*]1[N]2 = [C]3([*]8)[C]4([H]5)([*]6)[*]7 \;>>$$
$$[*]1[N]2([H]5)[C]3([*]8) = [C]4([*]6)[*]7$$



Adenine Chemistry

The reaction set of the Adenine chemistry:

$$
\begin{aligned}
1\ A + 1\ A &\rightarrow 1\ B \\
1\ A + 1\ B &\rightarrow 1\ C \\
1\ C + 1\ N &\rightarrow 1\ D \\
1\ D + 1\ N &\rightarrow 1\ E \\
1\ A + 1\ N &\rightarrow 1\ H \\
1\ H + 1\ E &\rightarrow 1\ F + 1\ N \\
1\ F &\rightarrow 1\ G + 1\ N \\
1\ H + 1\ G &\rightarrow 1\ I + 1\ N \\
1\ I &\rightarrow 1\ J + 1\ N
\end{aligned}
\tag{1}
$$

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Adenine Chemistry

Subset	Molecules set per organization
(A, D), (A, B, D), (A, C, D), (A, B, C, D)	$o_1 : [A, B, C, D]$, $o_2 : [A, B, C, D, E, F, G, H, I, J, K]$
(A, G), (A, B, G), (A, C, G), (A, B, C, G)	$o_1 : [A, B, C, G]$, $o_2 : [A, B, C, D, E, F, G, H, I, J, K]$

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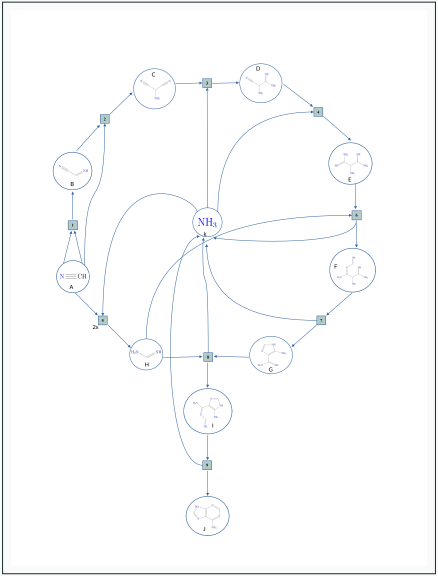
Adenine Chemistry



Formation of adenine from hydrogen cyanide and ammonia

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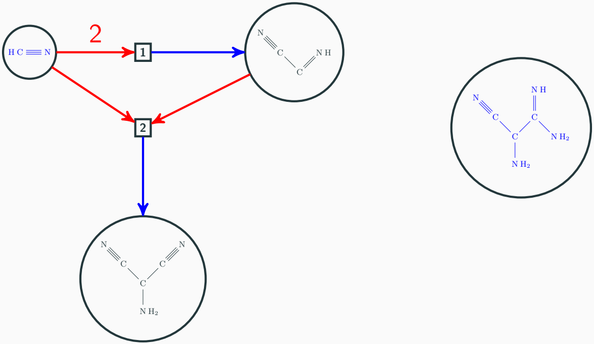
Adenine Chemistry



Adenine Chemistry

Subset	Molecules set per organization
(C, H)	$o_1 : [C, H]$,
	$o_2 : [C, D, E, F, G, H, I, J, K]$
(D, H)	$o_1 : [D, H]$,
	$o_2 : [D, E, F, G, H, I, J, K]$

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Summary and Conclusion

Summary and Conclusion

- the **minimum organization** of a reaction network under flow condition is the **closure of its inflow**

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Summary and Conclusion

- the **minimum organization** of a reaction network under flow condition is the **closure of its inflow**
- in solving the complexity of the reaction network, consider only the **closed set of molecules as inflow**

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Next Step

Focus on rule-based systems

- Given a set of rules, what are the possible inflow sets that can generate the different states of the network?

Thank you!

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