Generalized Algebraic Dynamic Programming: ADP over General Data Structures

Bled 2015

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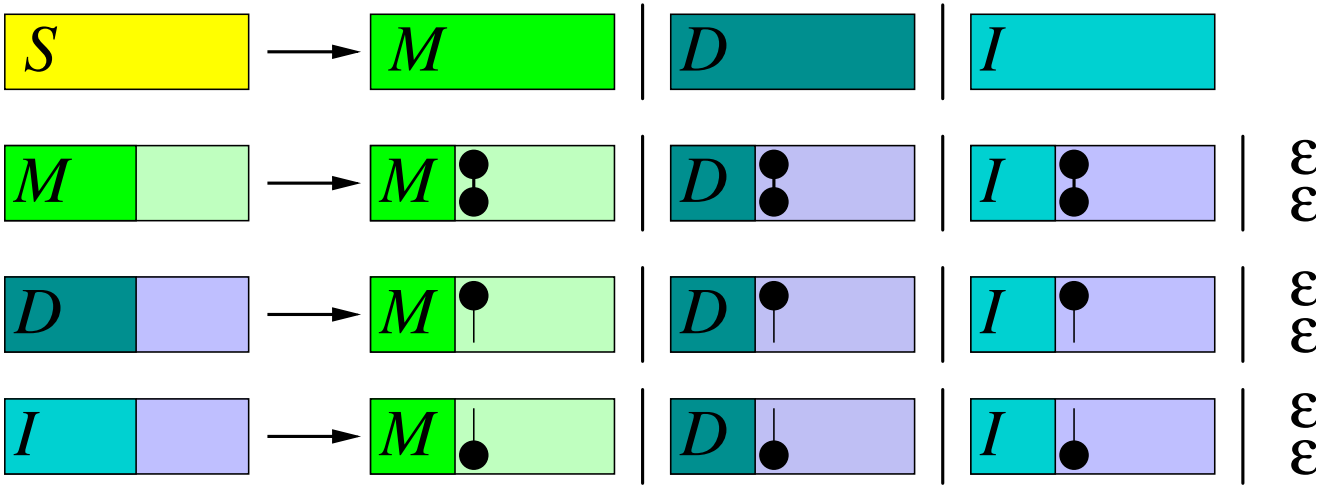
The Church of

Feb 15, 2015 – Feb 22, 2015

Alignment with Affine Gap Costs (Gotoh)



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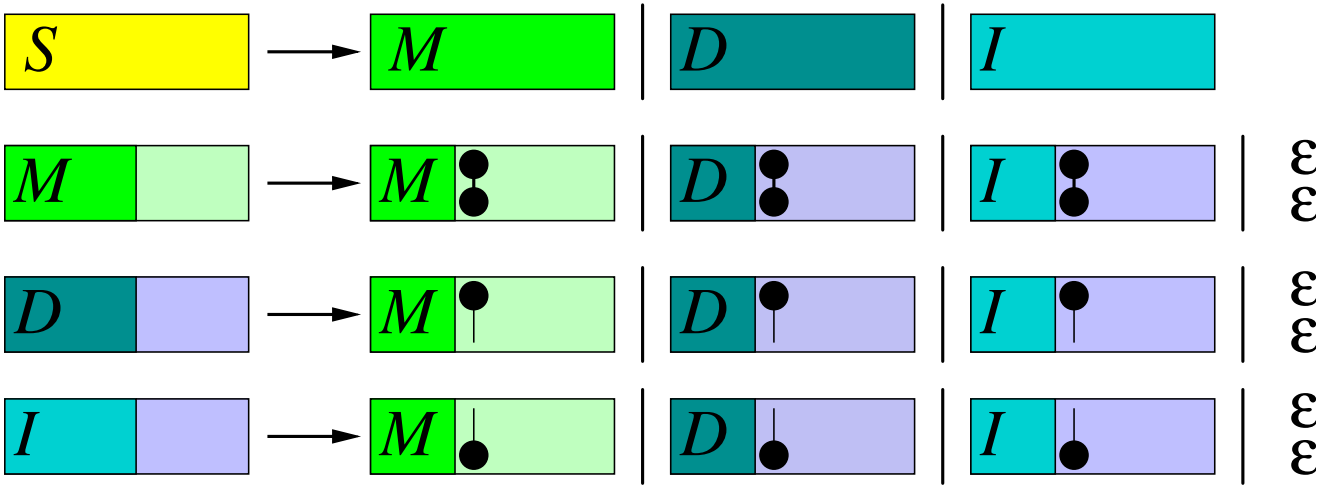

$$
\begin{array}{rcl|c|c|c|c}
S & \to & M & D & I & \\
M & \to & M\binom{u}{v} & D\binom{u}{v} & I\binom{u}{v} & \binom{\varepsilon}{\varepsilon} \\
D & \to & M\binom{u}{-} & D\binom{u}{\cdot} & I\binom{u}{-} & \binom{\varepsilon}{\varepsilon} \\
I & \to & M\binom{-}{v} & D\binom{-}{v} & I\binom{\cdot}{v} & \binom{\varepsilon}{\varepsilon}
\end{array}
$$

Alignment with Affine Gap Costs (Gotoh)

$$
\begin{array}{lcl|l|l|l}
S_{m,n} & \to & M_{m,n} & D_{m,n} & I_{m,n} & \\
M_{i,j} & \to & M_{i-1,j-1}\binom{u_i}{v_j} & D_{i-1,j-1}\binom{u_i}{v_j} & I_{i-1,j-1}\binom{u_i}{v_j} & \binom{\varepsilon_0}{\varepsilon_0} \\
D_{i,j} & \to & M_{i-1,j}\binom{u_i}{-} & D_{i-1,j}\binom{u_i}{\cdot} & I_{i-1,j}\binom{u_i}{-} & \binom{\varepsilon_0}{\varepsilon_0} \\
I_{i,j} & \to & M_{i,j-1}\binom{-}{v_j} & D_{i,j-1}\binom{-}{v_j} & I_{i,j-1}\binom{\cdot}{v_j} & \binom{\varepsilon_0}{\varepsilon_0}
\end{array}
$$

$$
\begin{array}{lcl|l|l|l}
S & \to & M & D & I & \\
M & \to & M\binom{u}{v} & D\binom{u}{v} & I\binom{u}{v} & \binom{\varepsilon}{\varepsilon} \\
D & \to & M\binom{u}{-} & D\binom{u}{\cdot} & I\binom{u}{-} & \binom{\varepsilon}{\varepsilon} \\
I & \to & M\binom{-}{v} & D\binom{-}{v} & I\binom{\cdot}{v} & \binom{\varepsilon}{\varepsilon}
\end{array}
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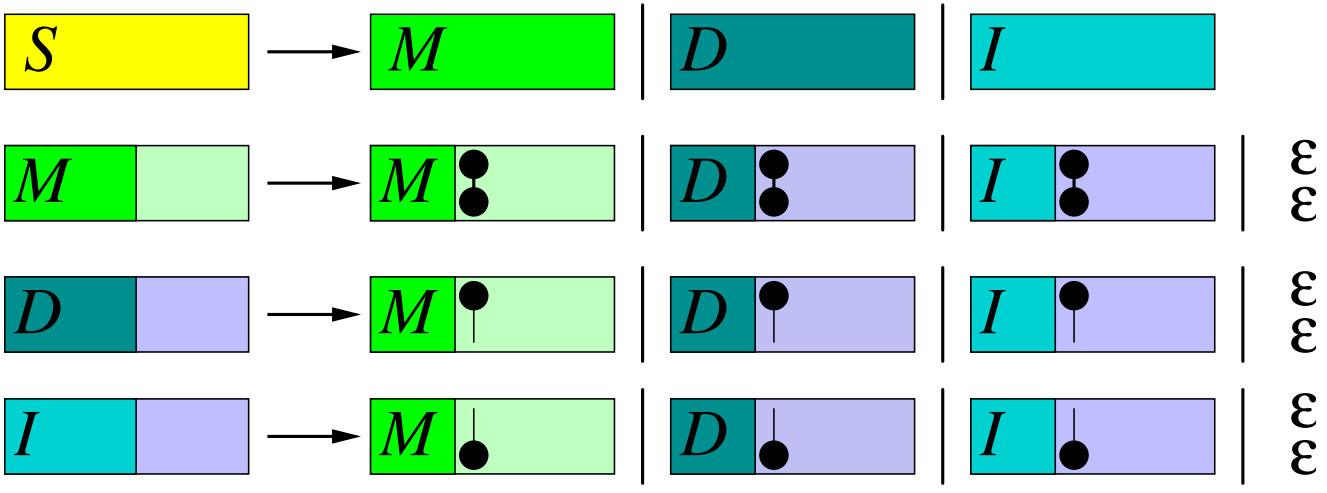
Alignment with Affine Gap Costs (Gotoh)



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\begin{array}{rcl|c|c|c|c}
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M & \to & M\binom{u}{v} & D\binom{u}{v} & I\binom{u}{v} & \binom{\varepsilon}{\varepsilon} \\
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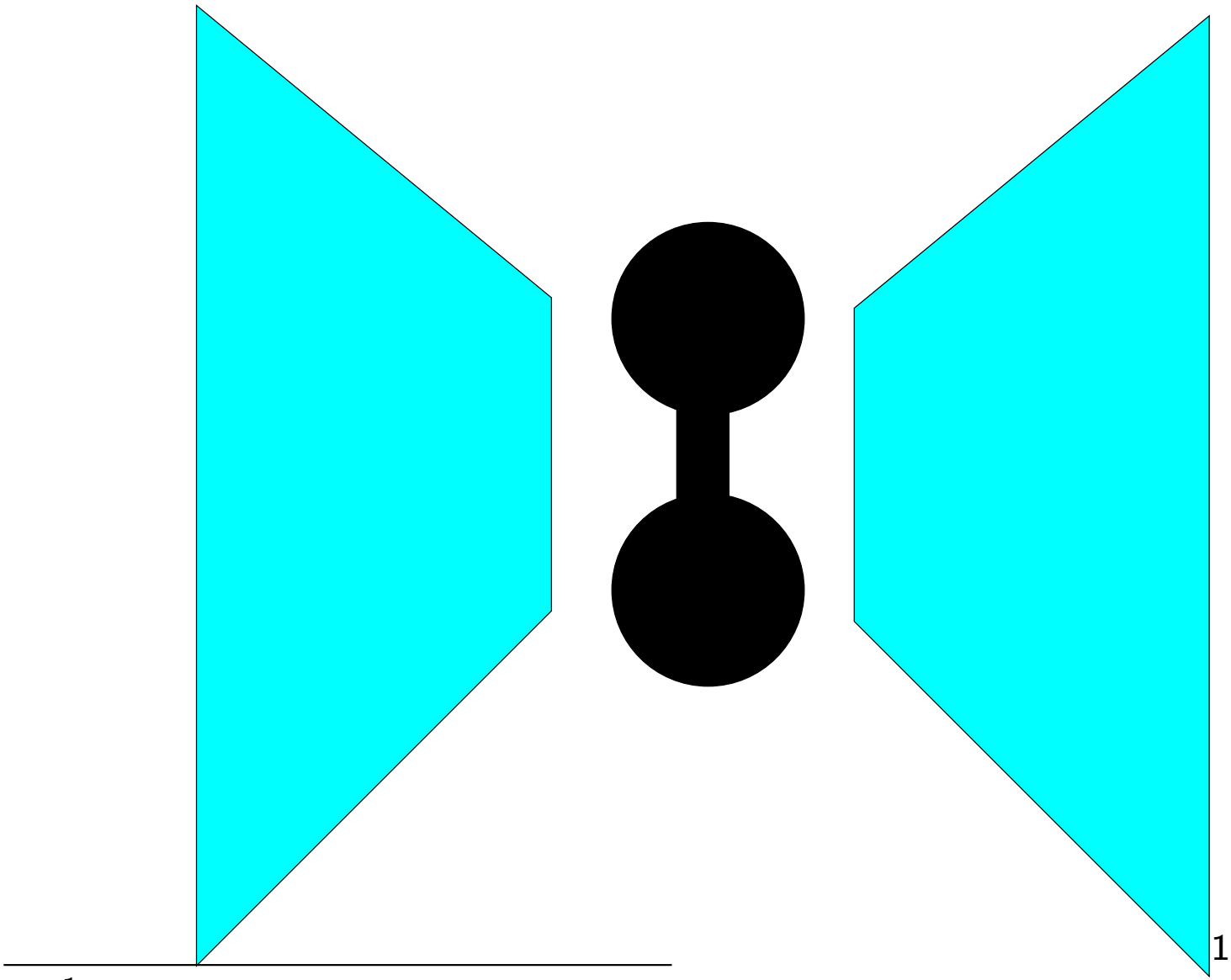
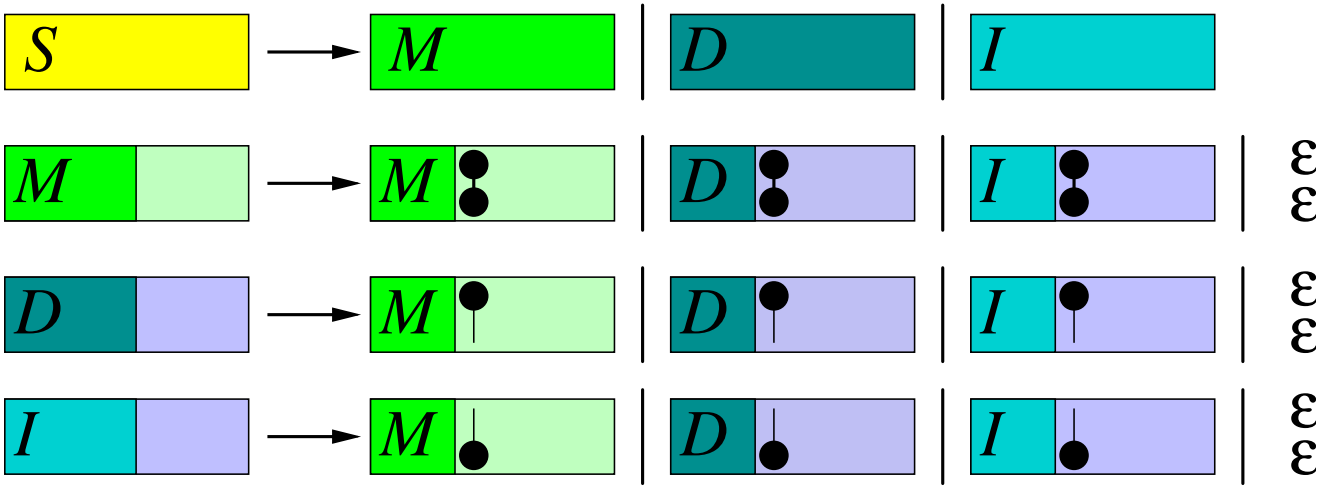
\end{array}
$$

Inside and Outside Combined

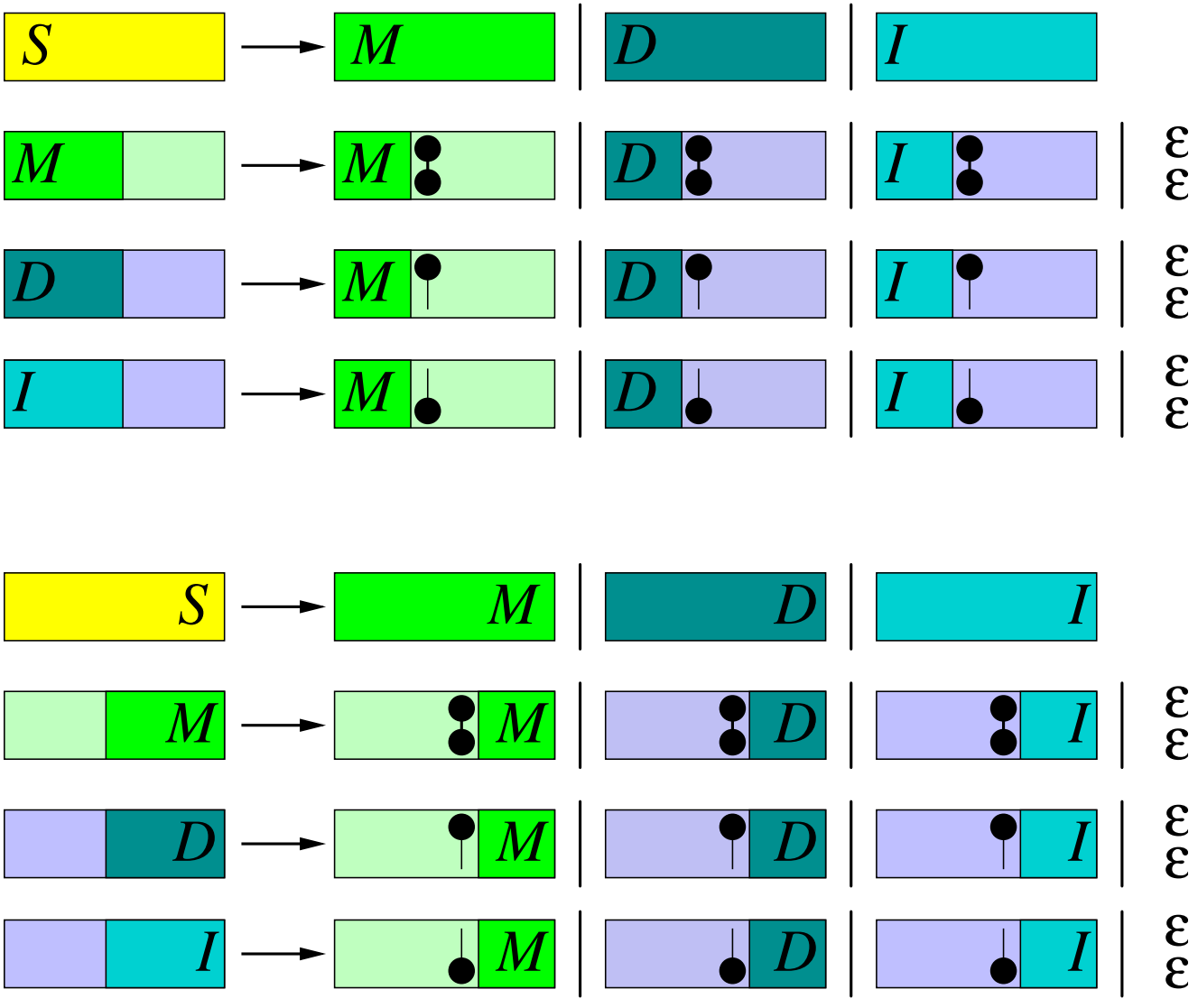


[1] abstract art

Outside Gotoh: Just Mirror the Grammar?

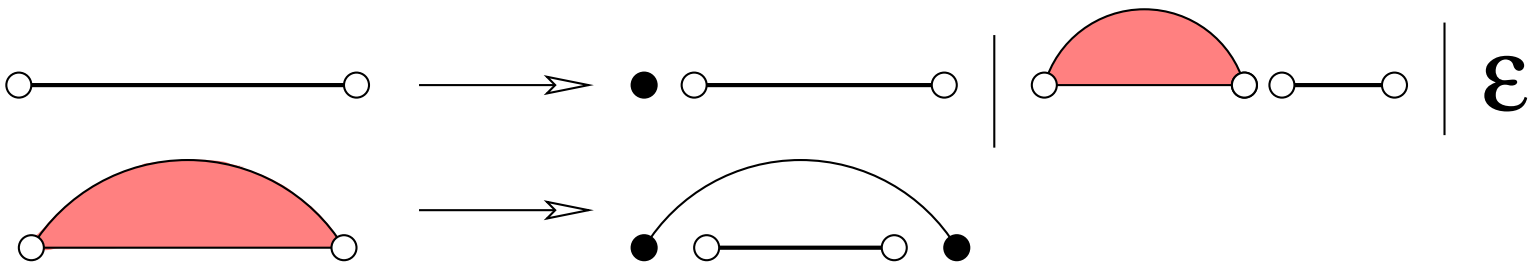


Outside Gotoh: Just Mirror the Grammar?



Unfortunately only a Special Case

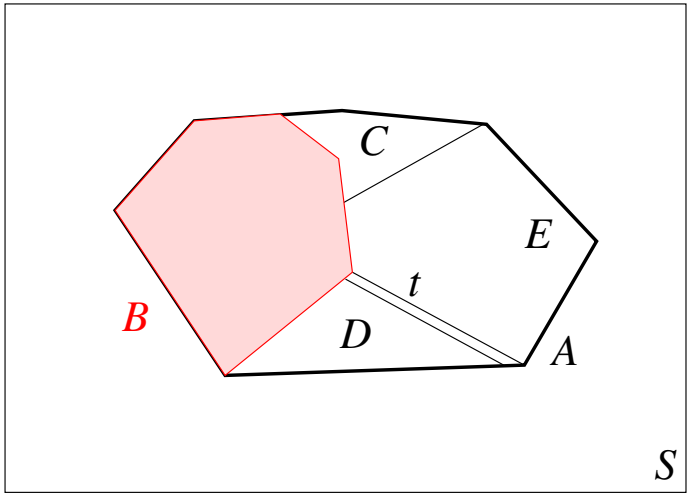
how would you mirror this RNA folding grammar?



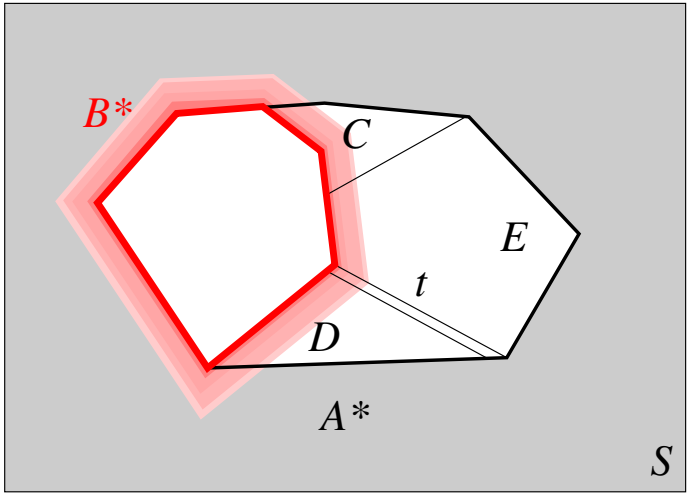
"mirroring" doesn't generalize!

Inside to Outside Productions

Inside



Outside

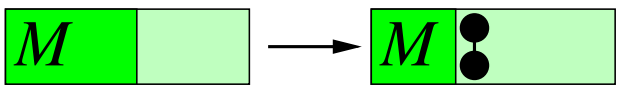


$$A \cup A^* = S$$

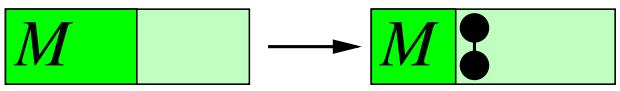
$$\begin{aligned} A &\rightarrow BCDEt \\ B^* &\rightarrow A^*CDEt \end{aligned}$$

$$\begin{aligned} C^* &\rightarrow BA^*DEt \\ &\vdots \end{aligned}$$

Outside Rules Step by Step

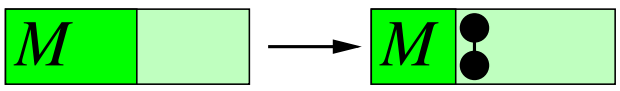


Outside Rules Step by Step



$$M_{i,j} \rightarrow M_{i-1,j-1}\binom{u_i}{v_j}$$

Outside Rules Step by Step



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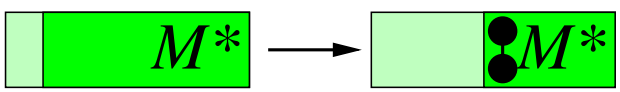
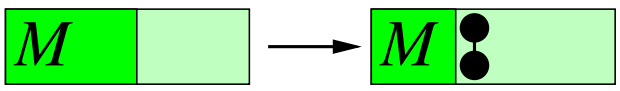
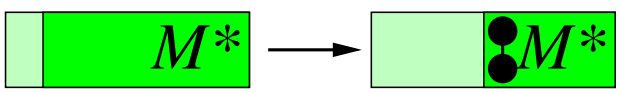


Outside Rules Step by Step

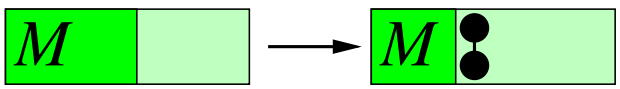


$$M_{i,j} \rightarrow M_{i-1,j-1}\binom{u_i}{v_j}$$



$$M^*_{i-1,j-1} \rightarrow M^*_{i,j}\binom{u_i}{v_j}$$

Outside Rules Step by Step



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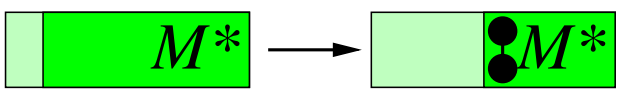
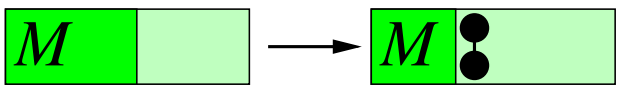


$$M^*_{i-1,j-1} \rightarrow M^*_{i,j}\binom{u_i}{v_j}$$

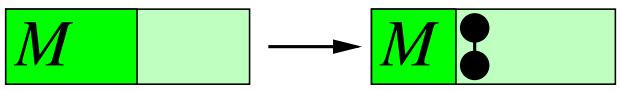
`rewrite indices:  +1/+1`

$$M^*_{i,j} \rightarrow M^*_{i+1,j+1}\binom{u_{i+1}}{v_{j+1}}$$

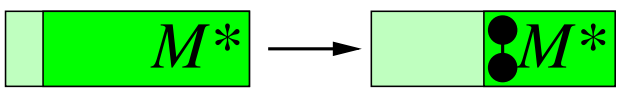
Outside Rules Step by Step



$$M_{i,j} \rightarrow M_{i-1,j-1}\binom{u_i}{v_j}$$



$$M \rightarrow M\binom{u}{v}$$

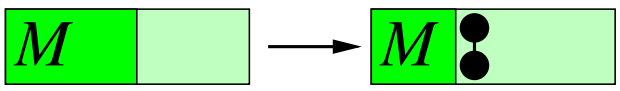


$$M^*_{i-1,j-1} \rightarrow M^*_{i,j}\binom{u_i}{v_j}$$

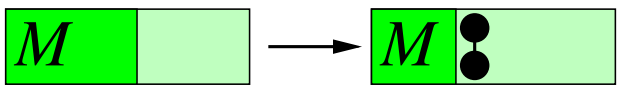
```
rewrite indices:  +1/+1
```

$$M^*_{i,j} \rightarrow M^*_{i+1,j+1}\binom{u_{i+1}}{v_{j+1}}$$

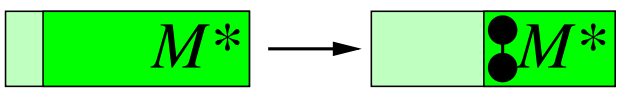
Outside Rules Step by Step



$$M_{i,j} \rightarrow M_{i-1,j-1}\binom{u_i}{v_j}$$



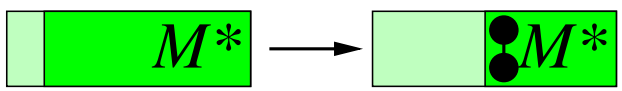
$$M \rightarrow M\binom{u}{v}$$



$$M^*_{i-1,j-1} \rightarrow M^*_{i,j}\binom{u_i}{v_j}$$

rewrite indices: +1/+1

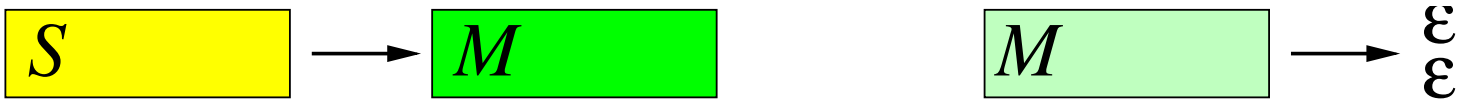
$$M^*_{i,j} \rightarrow M^*_{i+1,j+1}\binom{u_{i+1}}{v_{j+1}}$$



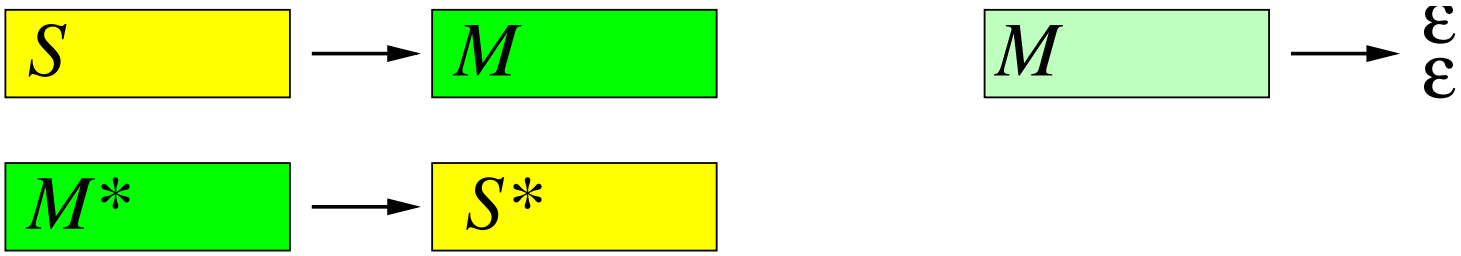
$$M^* \rightarrow M^*\binom{u}{v}$$

no rewrite necessary, let ADPfusion figure it out

Start and End Symbols



Start and End Symbols



Start and End Symbols

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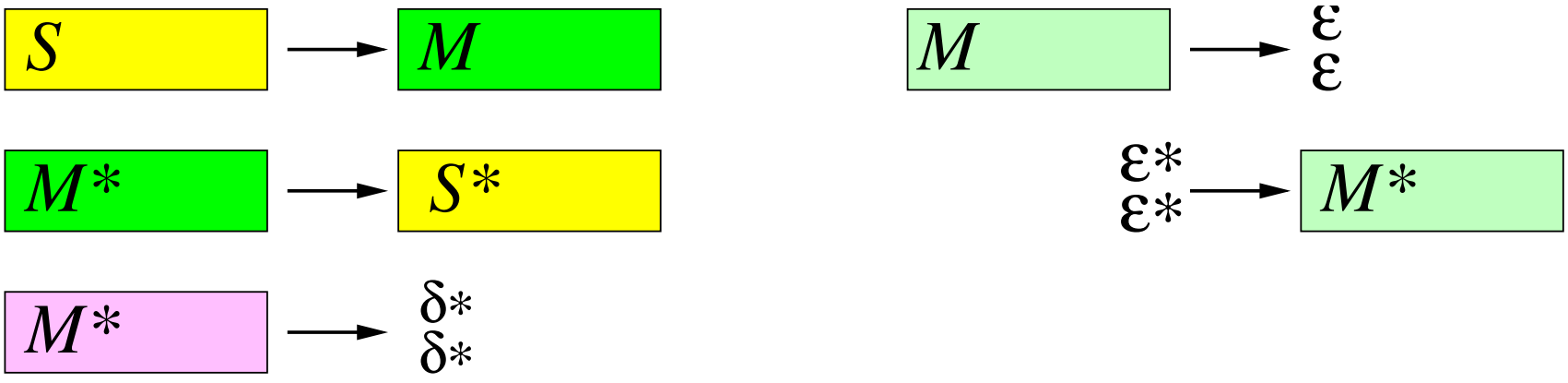
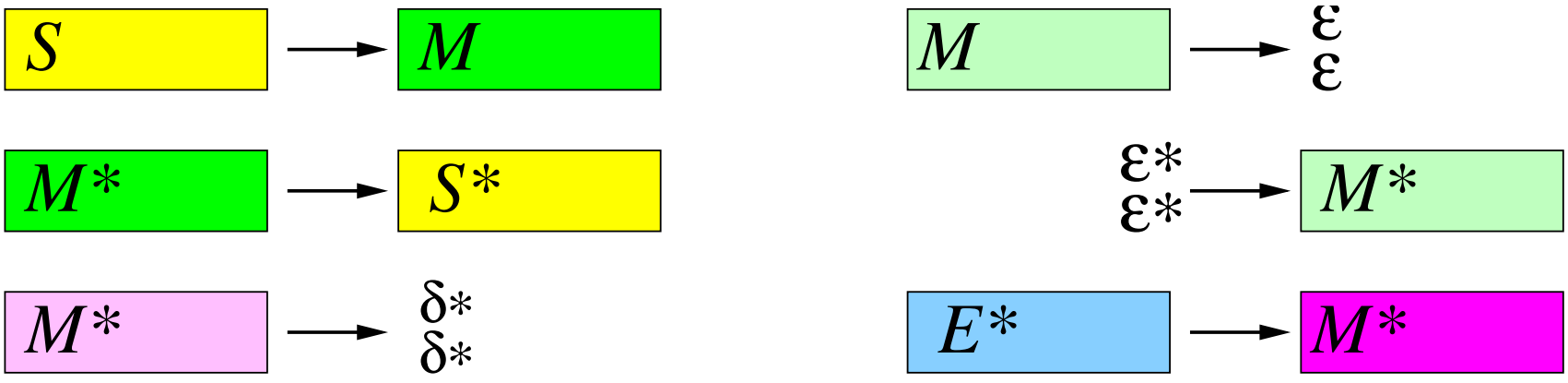
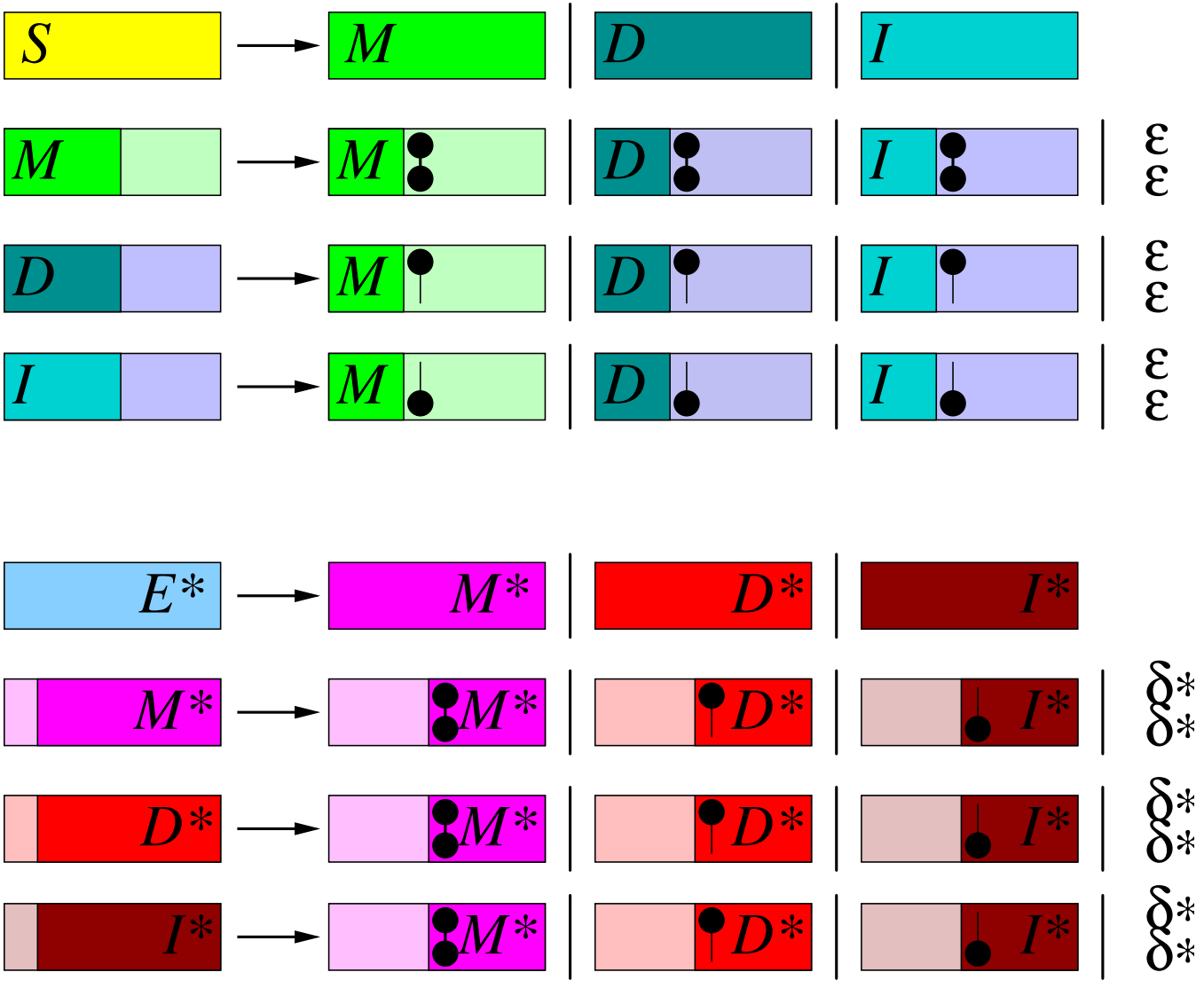


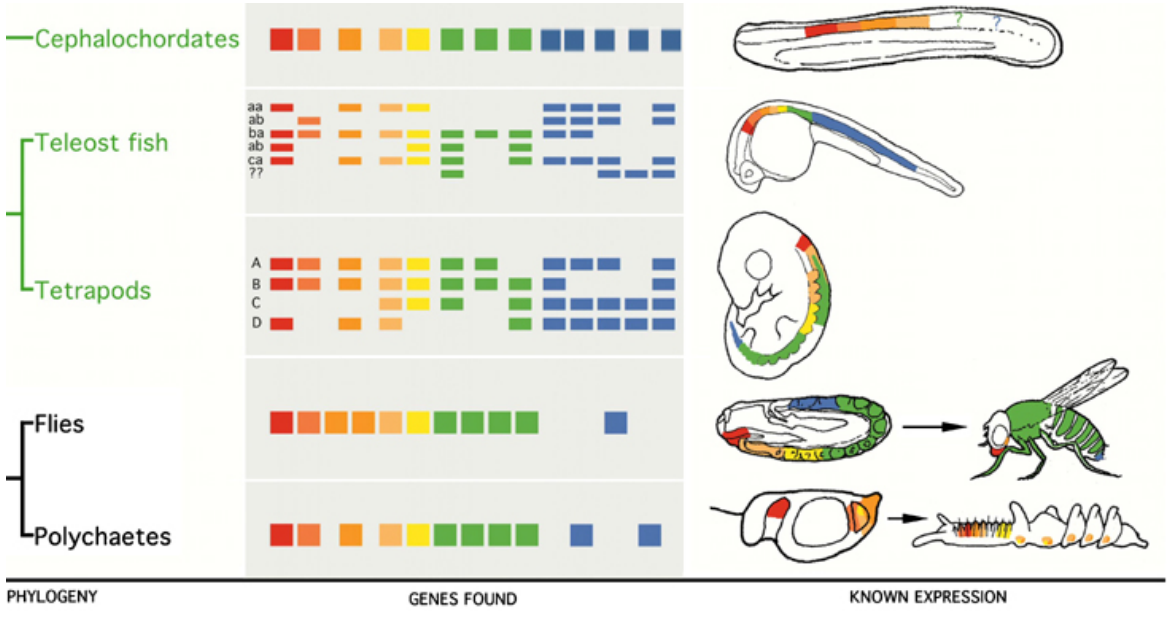
Start and End Symbols

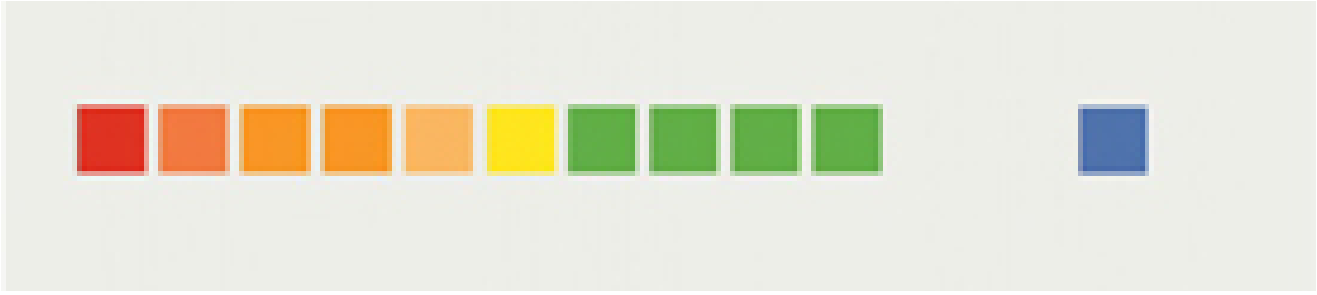


Inside & Outside Gotoh

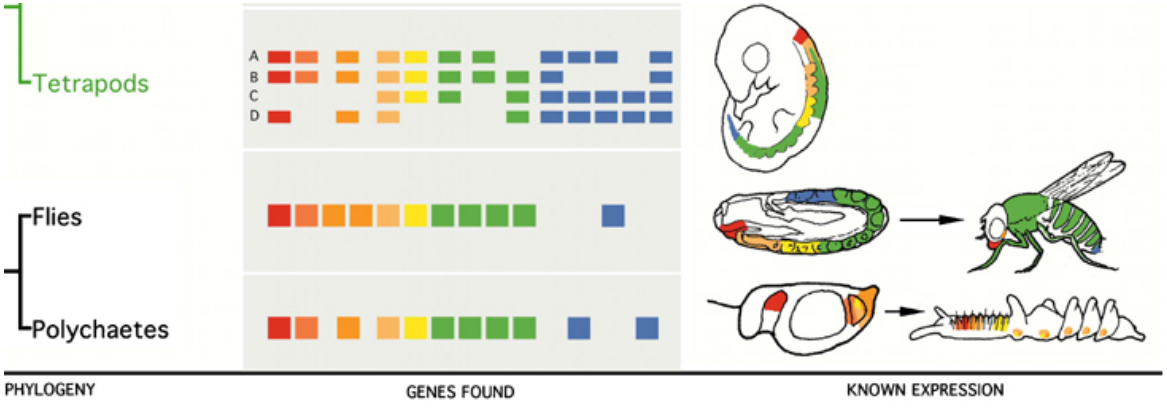


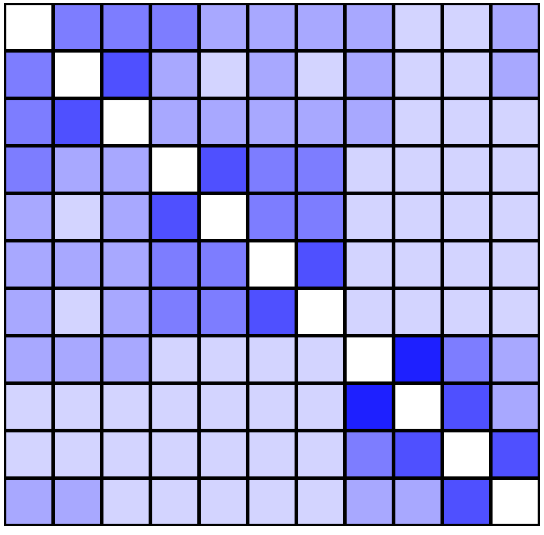
Hox Cluster History

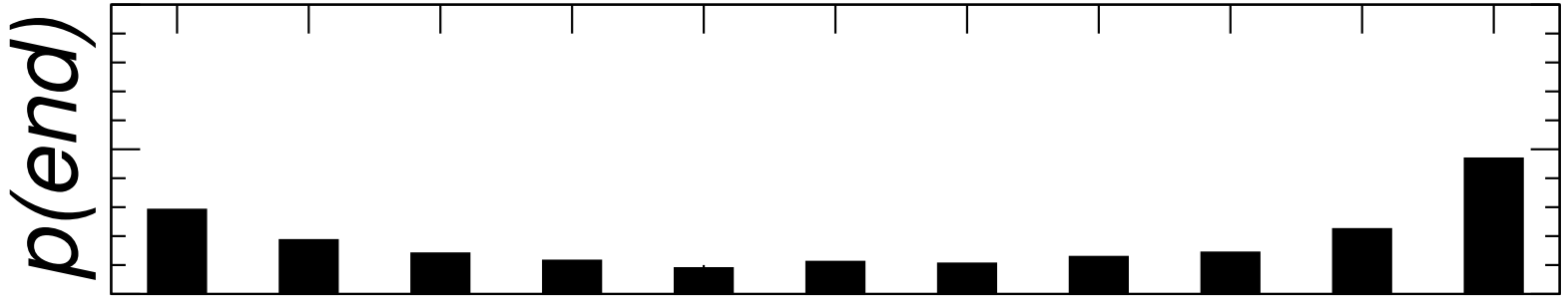




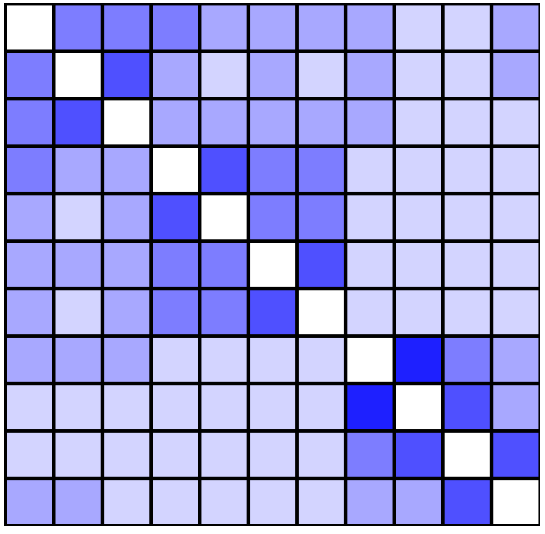
Extracting Summary Information







Calculate Edge Probabilities



$$P(i \sim j) = \frac{1}{Z(S)} \sum_{p,q}$$

$$\sum_{A \subset S} Z([p, A, i]) Z(\langle i, j \rangle) Z([j, S \setminus A, q]) \,.$$

$$S_{p,q} \to A_{p,q}$$

$$A_{p,q} \to \varepsilon$$

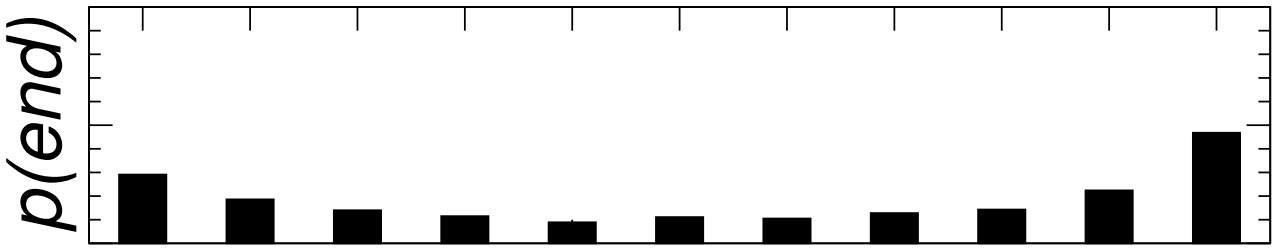
$$A_{p,q} \to A_{p,k}(k, q)$$

$$S \to A_{p,i}(i, j) A^*_{j,q}$$

$A_{p,q}$ `as left`

$A^*_{p,q}$ `mechanically derived`

Calculate End Probabilities



$$P(q \text{ is end}) = \frac{1}{Z(S)} \sum_{p,A \in S} Z([p, A, q])\,.$$

$S_{p,q} \to A_{p,q}$

$A_{p,q} \to \varepsilon$

$A_{p,q} \to A_{p,k}(k, q)$

$S_i \to A_{p,i}$

$A_{p,q}$ `as left`

Generalized ADP

Outside Grammars automatically derived from Inside grammars

Type-Rich DP index-type generic framework (with sequences and sets as examples)

- efficient algorithms as usual
- compatible with grammar products (grammars are objects)

Höner zu Siederdissen, Prohaska, Stadler. 2015
Algebraic Dynamic Programming over General Data Structures.

Hox figure:Building divergent body plans with similar genetic pathways; B J Swalla; Heredity