# 1 + Greedy + DP

Sarah Berkemer

Bioinformatics & MPI MIS Leipzig


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[] {}

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+ () ∘

Species & Data Structures

# Species of Sets: $E$

$E_0 =$ 

# Species of Sets: $E$

$E_0 =$ 

$E_1 =$ 

# Species of Sets: $E$

$E_0 =$ 

$E_1 =$ 

$E_2 =$ 

# Species of Sets: $E$

$E_0$= 

$E_1$= 

$E_2$= 

$E_3$= 

# Species of Sets: $E$

$E_0=$ 

$E_1=$ 

$E_2=$ 

$E_3=$ 

•••

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$E_0 =$ 

$E_1 =$ 

$E_2 =$ 

$E_3 =$ 

...

$E =$  $+$  $+$  $+$  $+ \cdots$

# Species of Lists: $L$

$L_0 =$ 

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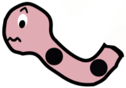
$L_0 =$ 

$L_1 =$ 

# Species of Lists: $L$

$L_0 =$ 

$L_1 =$ 

$L_2 =$ 

# Species of Lists: $L$

$L_0 =$ 

$L_1 =$ 

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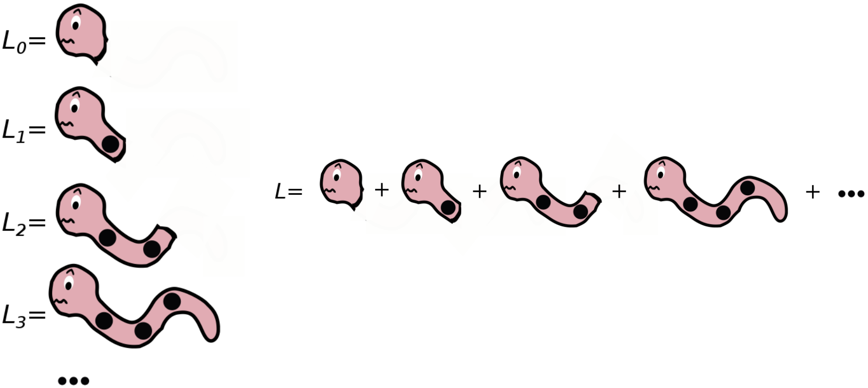

# Combinatorial Species [Joyal, 1981]

A species $s$ consists of a structure $F$, a set $U$ of labels and a mapping $\lambda : U \rightarrow F$.

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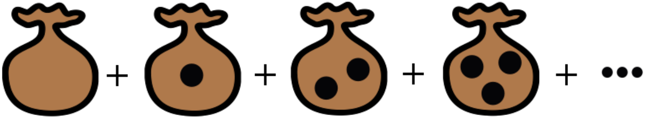
$E =$ 

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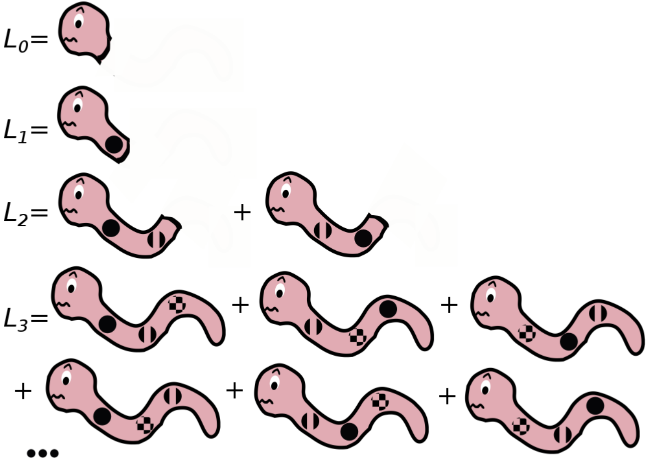


# Basic Species

$0 = \emptyset$

Zero

# Basic Species

$0 = \emptyset$

Zero

$1 =$ 

One

# Basic Species

$0 = \emptyset$ — Zero

$1 =$  — One

$X =$  — Singleton

# Basic Species

0= ∅

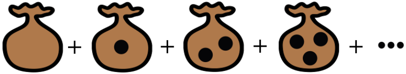
Zero

1= 

One

X= 

Singleton

E = 

Set

# Basic Species

$0 = \emptyset$

Zero

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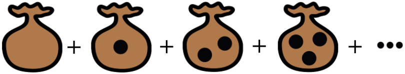
One

$X =$ 

Singleton

$E =$ 

Set

$L =$ 

List

# Basic Species

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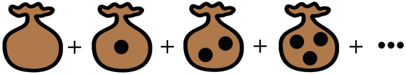
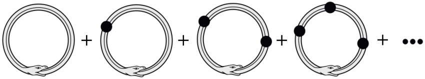
$E =$ 

$L =$ 

$C =$ 

Zero

One

Singleton

Set

List

Cycle

# Sum and Product of Species

$(E + X)_1 =$  *or* 

Sum

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$(E + X)_1 =$  *or* 

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$(X \bullet E)_4 =$  *and* 

Product

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## Examples

# Sum and Product of Species

$(E + X)_1 =$  *or* 

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$(X \bullet E)_4 =$  *and* 

Product

## Examples

List: $L = 1 + X \bullet L = 1 + L_+$

# Sum and Product of Species

$(E + X)_1 =$  *or* 

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$(X \bullet E)_4 =$  *and* 

Product

## Examples

List: $L = 1 + X \bullet L = 1 + L_+$

Elements: $\epsilon = X \bullet E = E_+$

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Product

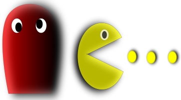
**Examples**

List: $L = 1 + X \bullet L = 1 + L_+$

Elements: $\epsilon = X \bullet E = E_+$

Ordered pairs: $X^2 = X \bullet X = L_2$

# Composition of Species



$F \circ G$

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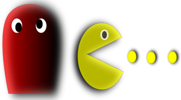

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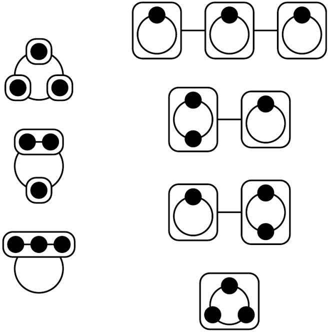
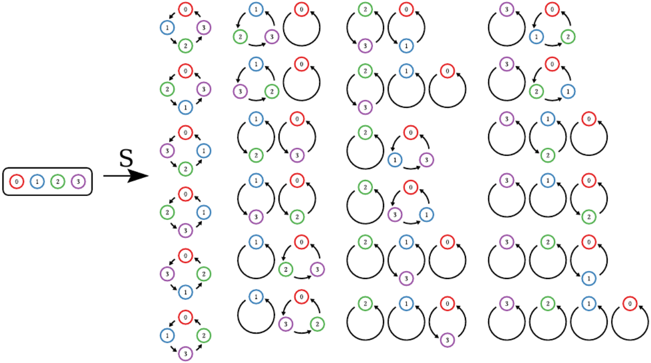


$C \circ L \neq L \circ C$

# Composition of Species




Permutations $S = E \circ C$

# Definitions

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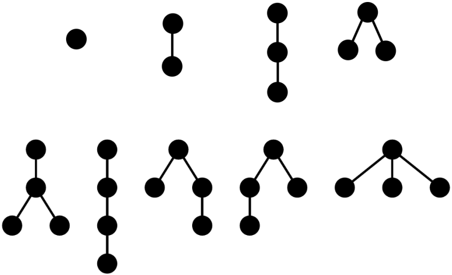

Species of ordered trees $T = 1 + X \bullet (L \circ T)$

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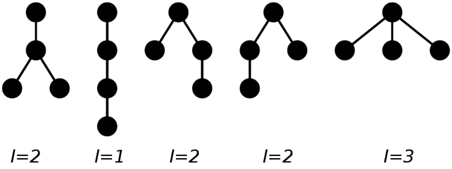


Weight function: $l(t) = \textit{number of leaves of } t$

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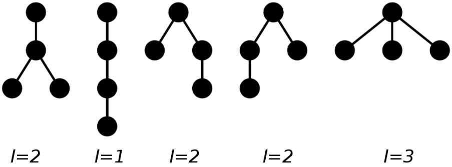


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By taking the maximum as ring operation, we can filter structures!

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*no*

*talk..*

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*no talk..* OR *Greedoids are the underlying structure for most Greedy algorithms!*

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*no talk..* OR *Greedoids are the underlying structure for most Greedy algorithms!* OR *Dynamic Programming and its morphisms*

# Overview

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# Thank you for your attention!


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$(L')_3 =$ 

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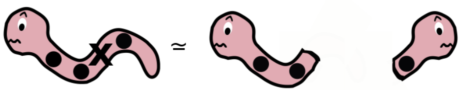
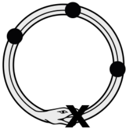
$(L')_3 =$  $= L^2$

# Differentiation of Species

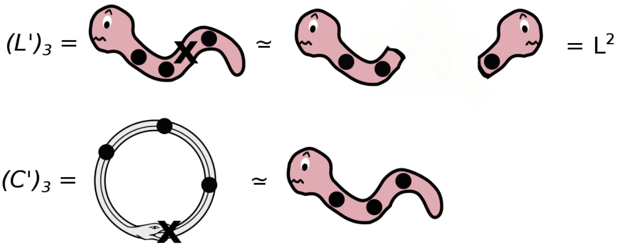
$(L')_3 =$ 

$\simeq$  $= L^2$

$(C')_3 =$ 

$\simeq$ 

# Differentiation of Species




## Examples

| | |
|---|---|
| Zero: | $0' = 0$ |
| One: | $1' = 0$ |
| Singleton: | $X' = 1$ |
| Set: | $E' = E$ |
| List: | $L' \simeq L^2$ |
| Cycle: | $C' \simeq L$ |

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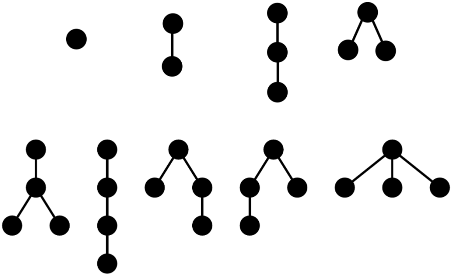

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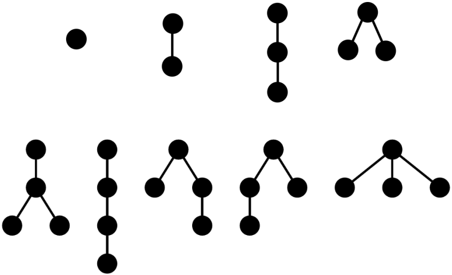

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## Enumeration

$$T = \underbrace{X}_{1\ n.,1\ s.} + \underbrace{X^2}_{2\ n.,1\ s.} + \underbrace{(X^3 + XL_2)}_{3\ nodes,2\ structures} + \underbrace{(3X^4 + X^2L_2 + XL_3)}_{4\ nodes,5\ structures} \dots$$

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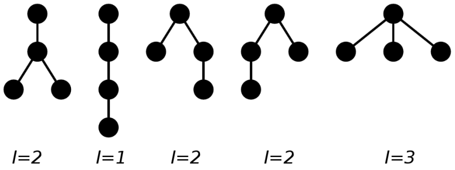


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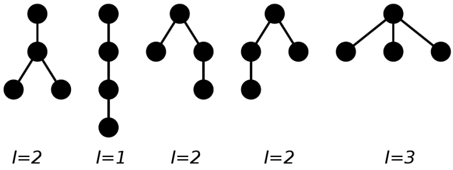


Weight function: $l(t) =$ *number of leaves of t*

By taking the maximum as ring operation, we can filter structures!