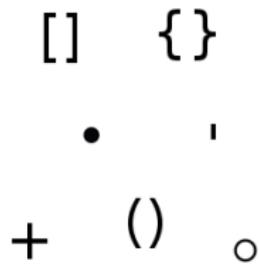
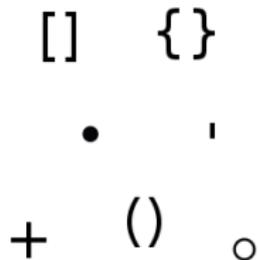


1 + Greedy + DP

Sarah Berkemer

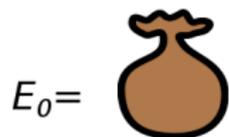
Bioinformatics & MPI MIS Leipzig



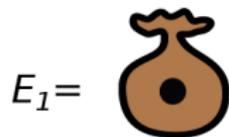
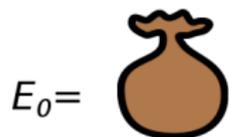


Species & Data Structures

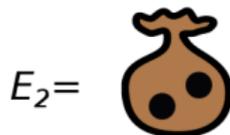
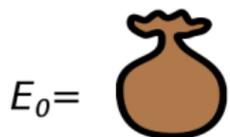
Species of Sets: E



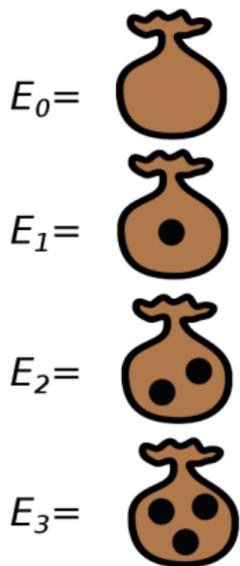
Species of Sets: E



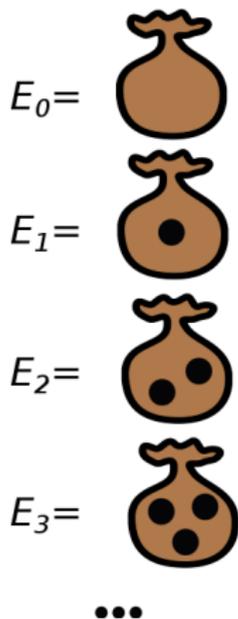
Species of Sets: E



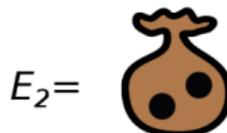
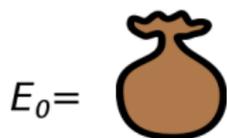
Species of Sets: E



Species of Sets: E



Species of Sets: E



...

$$E = \text{} + \text{} + \text{} + \text{} + \dots$$

Species of Lists: L

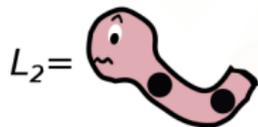
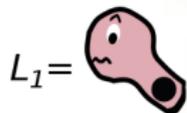
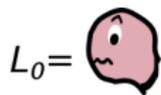
$$L_0 = \text{[Cartoon drawing of a pink blob with a face and a wavy tail]}$$

Species of Lists: L

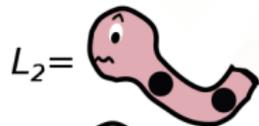
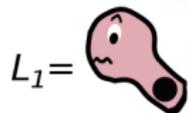
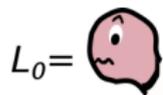
$$L_0 = \text{[blob]}$$

$$L_1 = \text{[blob with tail]}$$

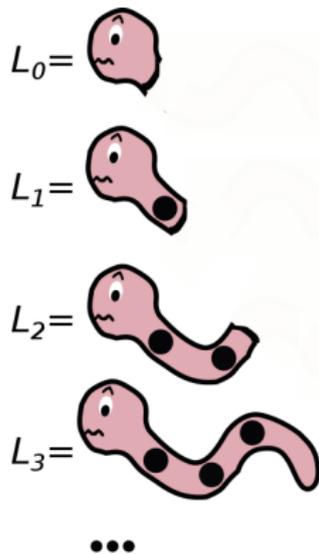
Species of Lists: L



Species of Lists: L



Species of Lists: L



Species of Lists: L

$$L_0 = \text{[head]}$$

$$L_1 = \text{[head, tail]}$$

$$L_2 = \text{[head, tail, tail]}$$

$$L_3 = \text{[head, tail, tail, tail]}$$

...

$$L = \text{[head]} + \text{[head, tail]} + \text{[head, tail, tail]} + \text{[head, tail, tail, tail]} + \dots$$

Combinatorial Species [Joyal, 1981]

A species s consists of a structure F , a set U of labels and a mapping $\lambda : U \rightarrow F$.

Combinatorial Species [Joyal, 1981]

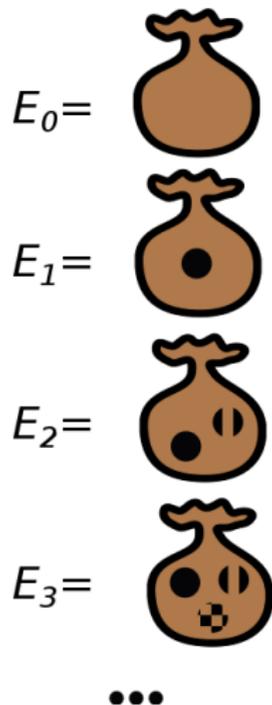
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$$E = \text{[empty bag]} + \text{[bag with 1 dot]} + \text{[bag with 2 dots]} + \text{[bag with 3 dots]} + \dots$$

$$L = \text{[empty worm]} + \text{[worm with 1 dot]} + \text{[worm with 2 dots]} + \text{[worm with 3 dots]} + \dots$$

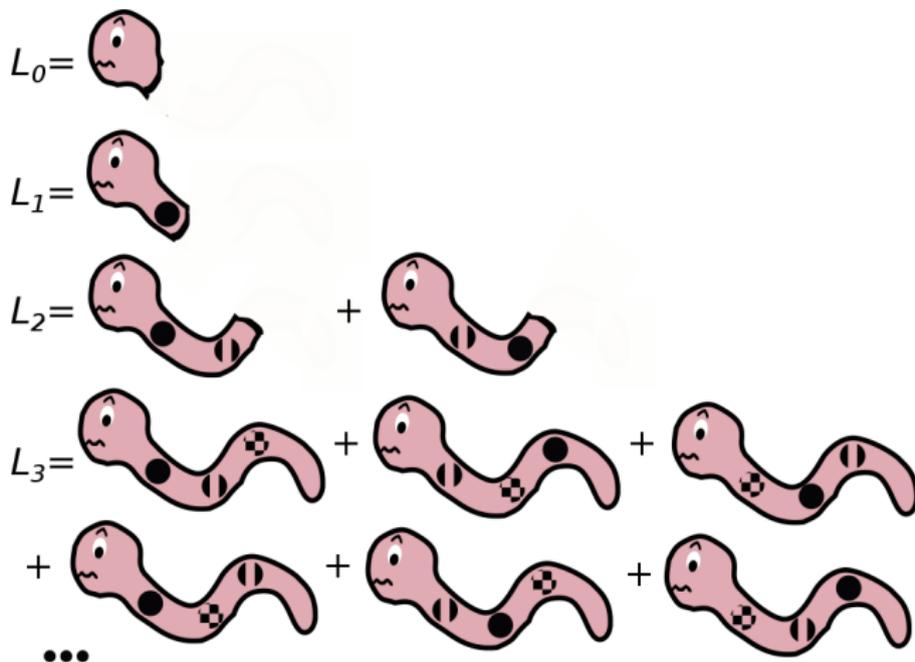
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Basic Species

$$0 = \emptyset$$

Zero

Basic Species

$$0 = \emptyset$$

Zero

$$1 = \cup$$

One

Basic Species

$$0 = \emptyset$$

Zero

$$1 = \text{V}$$

One

$$X = \text{unicorn}$$

Singleton

Basic Species

$$0 = \emptyset$$

Zero

$$1 = \cup$$

One

$$X = \text{unicorn}$$

Singleton

$$E = \text{bag} + \text{bag} + \text{bag} + \text{bag} + \dots$$

Set

Basic Species

$$0 = \emptyset$$

Zero

$$1 = \text{V}$$

One

$$X = \text{unicorn}$$

Singleton

$$E = \text{bag}_0 + \text{bag}_1 + \text{bag}_2 + \text{bag}_3 + \dots$$

Set

$$L = \text{worm}_1 + \text{worm}_2 + \text{worm}_3 + \text{worm}_4 + \dots$$

List

Basic Species

$$o = \emptyset$$

Zero

$$1 = \text{V}$$

One

$$X = \text{unicorn}$$

Singleton

$$E = \text{bag} + \text{bag} + \text{bag} + \text{bag} + \dots$$

Set

$$L = \text{snake} + \text{snake} + \text{snake} + \text{snake} + \dots$$

List

$$C = \text{circle} + \text{circle} + \text{circle} + \text{circle} + \dots$$

Cycle

Sum and Product of Species

$$(E + X)_1 = \text{img} \text{ or } \text{img}$$

The equation shows the first power of the sum of two species, $(E + X)_1$. It is equal to the sum of the two species themselves. The first species is represented by a brown sack with a black dot, and the second species is represented by a colorful unicorn with a black dot on its side. The word "or" is placed between the two images.

Sum

Sum and Product of Species

$$(E + X)_1 = \text{[bag]} \text{ or } \text{[unicorn]}$$

Sum

$$(X \bullet E)_4 = \text{[unicorn]} \text{ and } \text{[bag]}$$

Product

Sum and Product of Species

$$(E + X)_1 = \text{[bag]} \text{ or } \text{[unicorn]}$$

Sum

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Product

Examples

Sum and Product of Species

$$(E + X)_1 = \text{[bag]} \text{ or } \text{[unicorn]}$$

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Product

Examples

List: $L = 1 + X \bullet L = 1 + L_+$

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Examples

List: $L = 1 + X \bullet L = 1 + L_+$

Elements: $\epsilon = X \bullet E = E_+$

Ordered pairs: $X^2 = X \bullet X = L_2$

Composition of Species

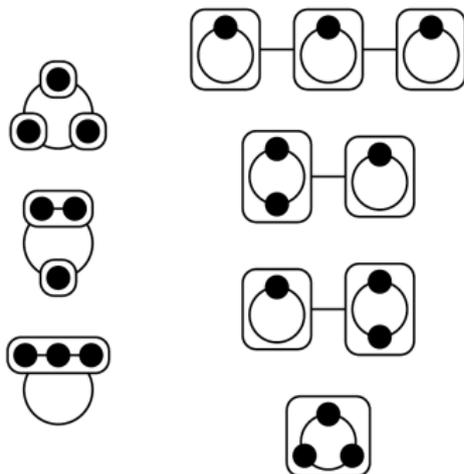


$$F \circ G$$

Composition of Species

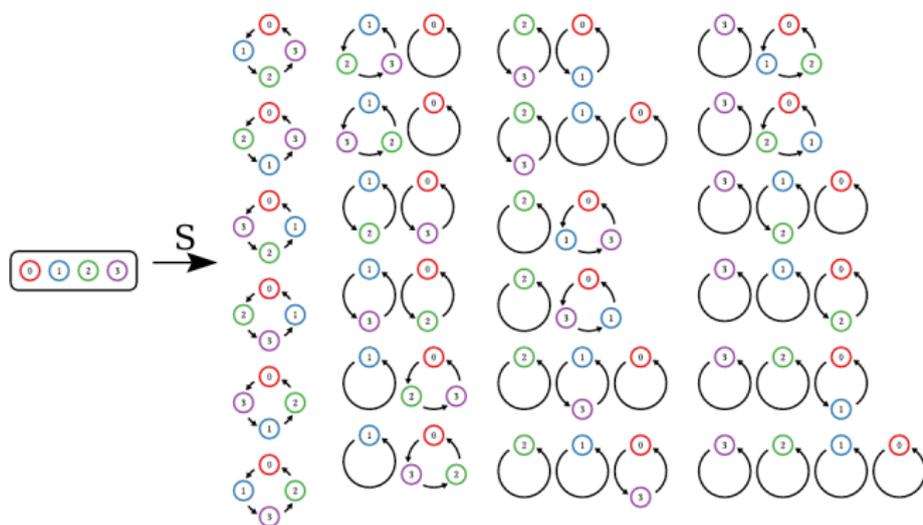


$$F \circ G$$



$$C \circ L \neq L \circ C$$

Composition of Species



Permutations $S = E \circ C$

Definitions

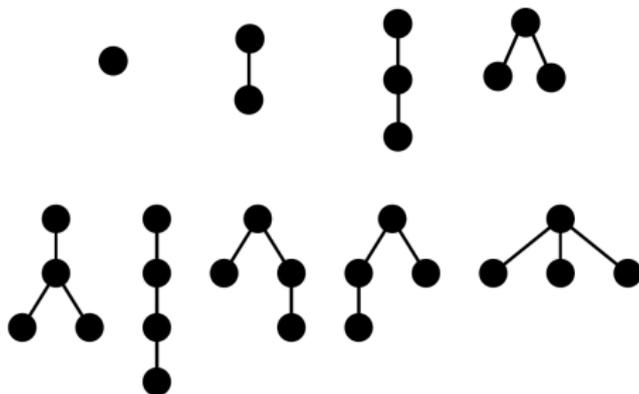
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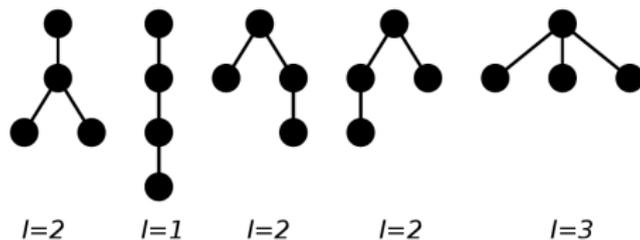
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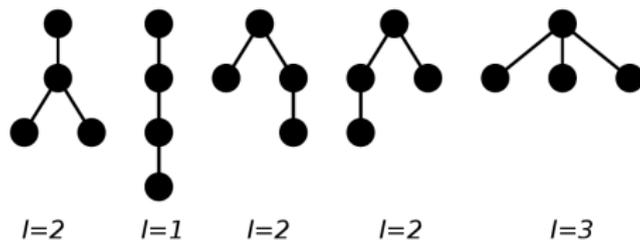
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1 + Greedy + DP

1 + Greedy + DP

|
no
talk.

1 + Greedy + DP

no
talk.

OR

*Greedoids are
the underlying
structure for most
Greedy algorithms!*

1 + Greedy + DP

no
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OR

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OR

*Dynamic
Programming
and its
morphisms*

Overview

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Thank you for your attention!



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Differentiation of Species

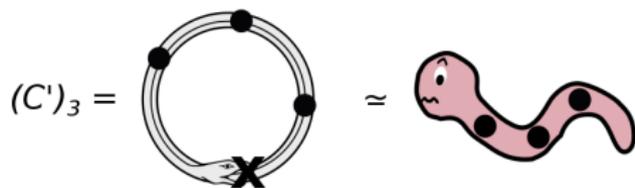
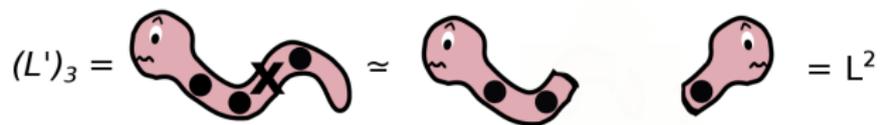


Differentiation of Species

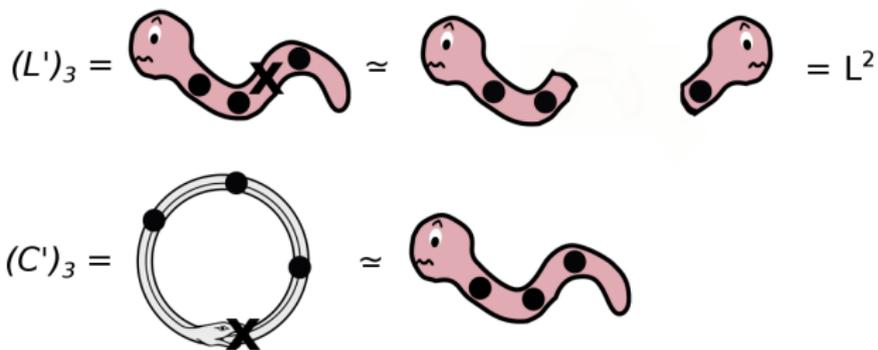
$$(L^1)_3 = \text{[Diagram of a worm with 3 black spots and a cross]} \approx \text{[Diagram of a worm with 2 black spots]} \quad \text{[Diagram of a worm with 1 black spot]} = L^2$$

The diagram illustrates the differentiation of the species L^1 at order 3. On the left, $(L^1)_3$ is represented by a pink worm with three black spots and a black cross on its body. This is shown to be isomorphic (\approx) to a pink worm with two black spots. To the right, a pink worm with one black spot is shown to be equal to the species L^2 .

Differentiation of Species



Differentiation of Species



Examples

Zero: $0' = 0$

One: $1' = 0$

Singleton: $X' = 1$

Set: $E' = E$

List: $L' \simeq L^2$

Cycle: $C' \simeq L$

Definitions

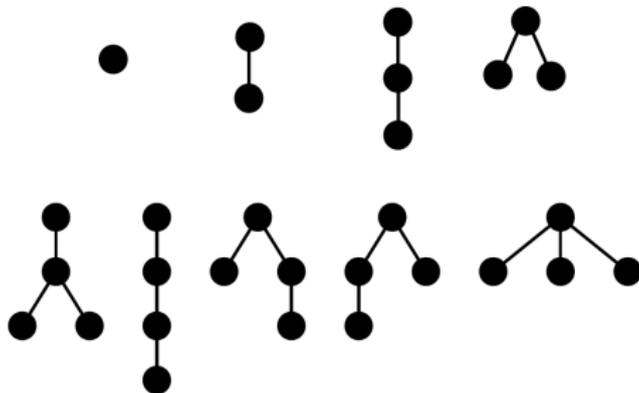
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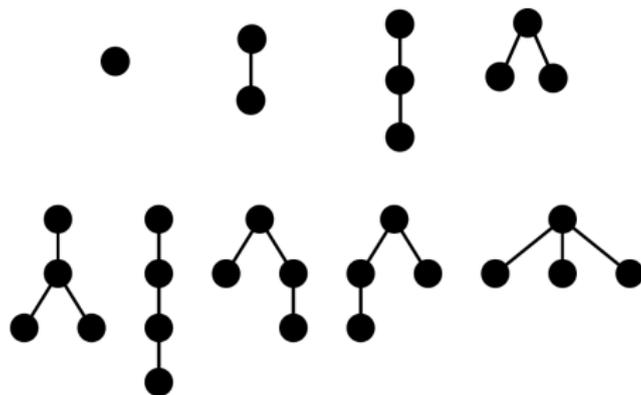


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Species of ordered trees $T = 1 + X \bullet (L \circ T)$

Enumeration

$$T = \underbrace{X}_{1 \text{ node}, 1 \text{ structure}} + \underbrace{X^2}_{2 \text{ nodes}, 1 \text{ structure}} + \underbrace{(X^3 + XL_2)}_{3 \text{ nodes}, 2 \text{ structures}} + \underbrace{(3X^4 + X^2L_2 + XL_3)}_{4 \text{ nodes}, 5 \text{ structures}} \dots$$

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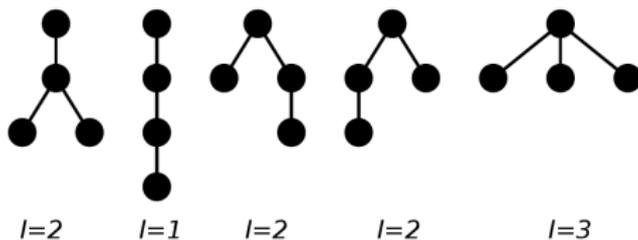
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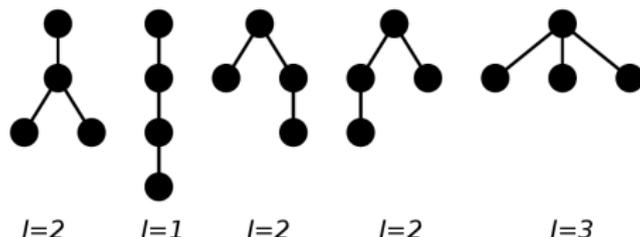
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