Sorting by TDRL and iTDRL

Bruno Schmidt, Tom Hartmann, Peter Stadler

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- The **Tandem Duplication Random Loss/inverse Tandem Duplication Random Loss model** can mimic most "popular" rearrangements, and we developed a polynomial time algorithm for the sorting problem with TDRL/iTDRL

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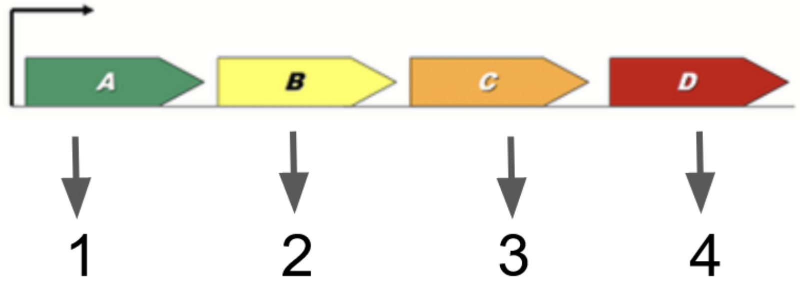

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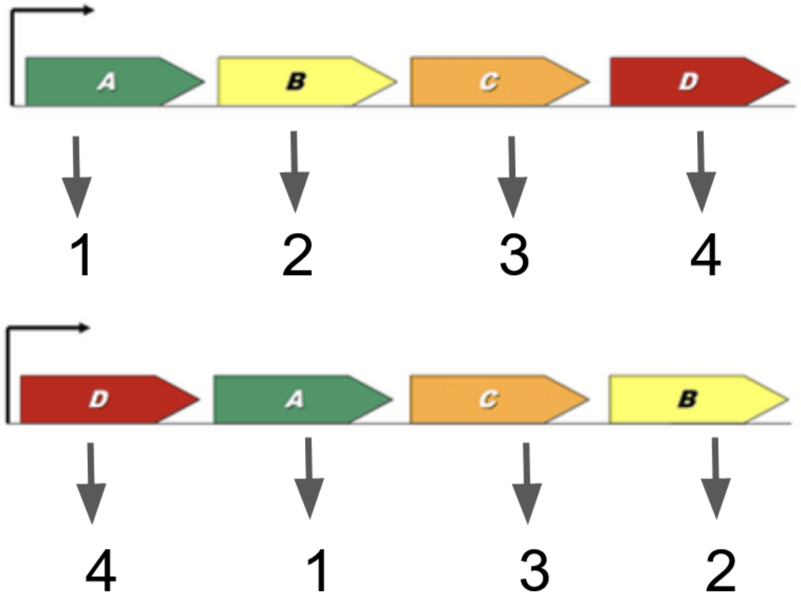

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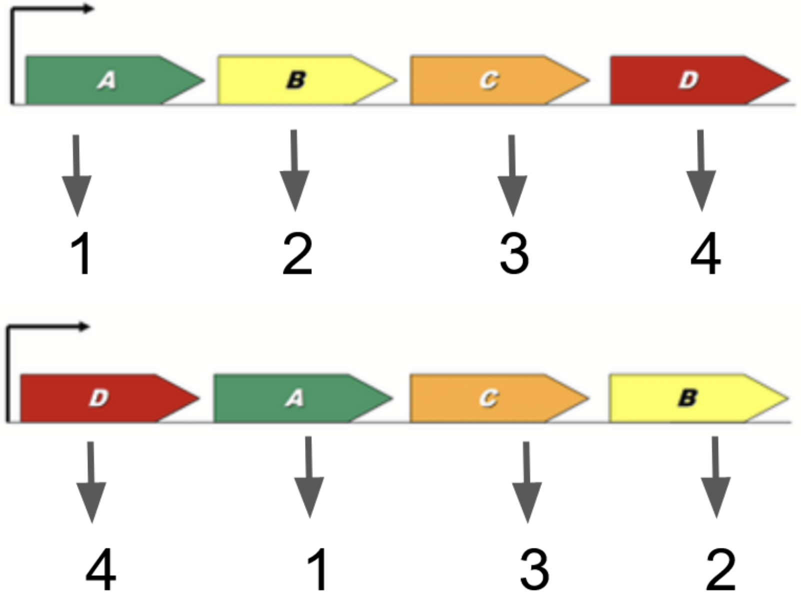

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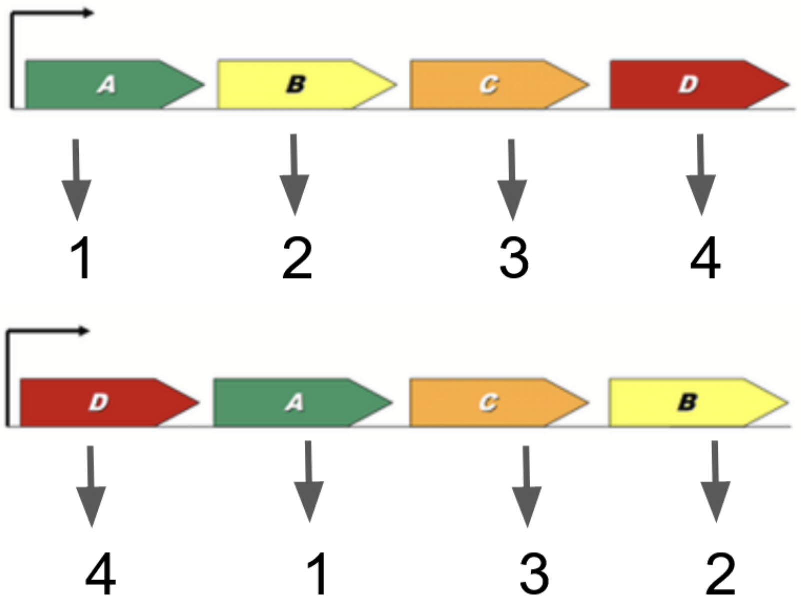

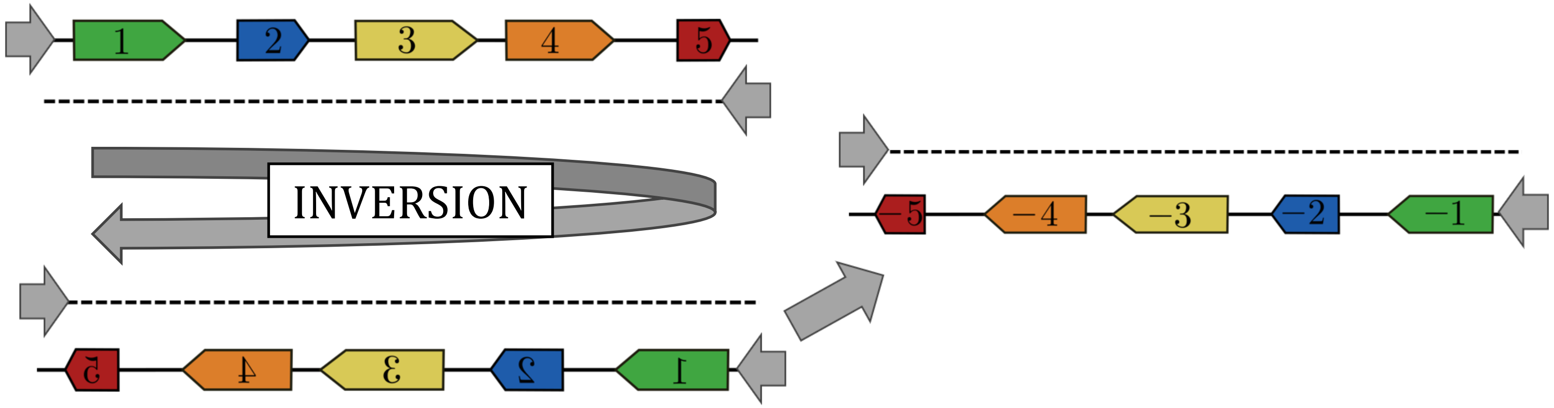

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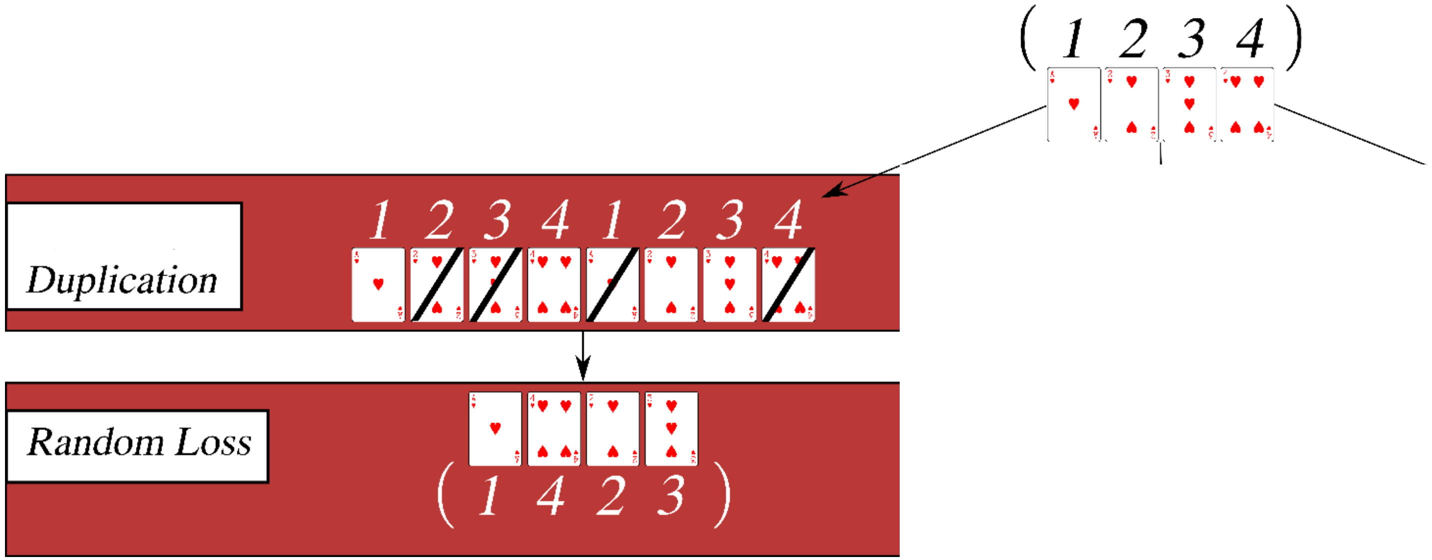

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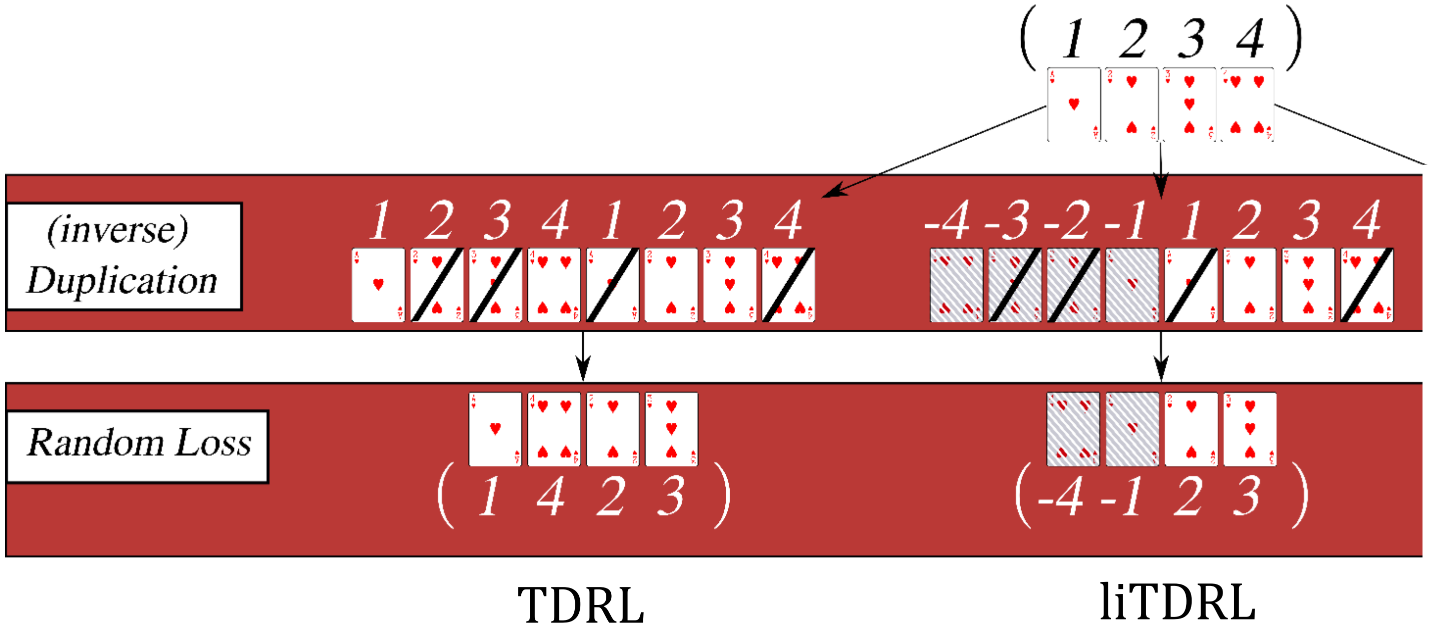

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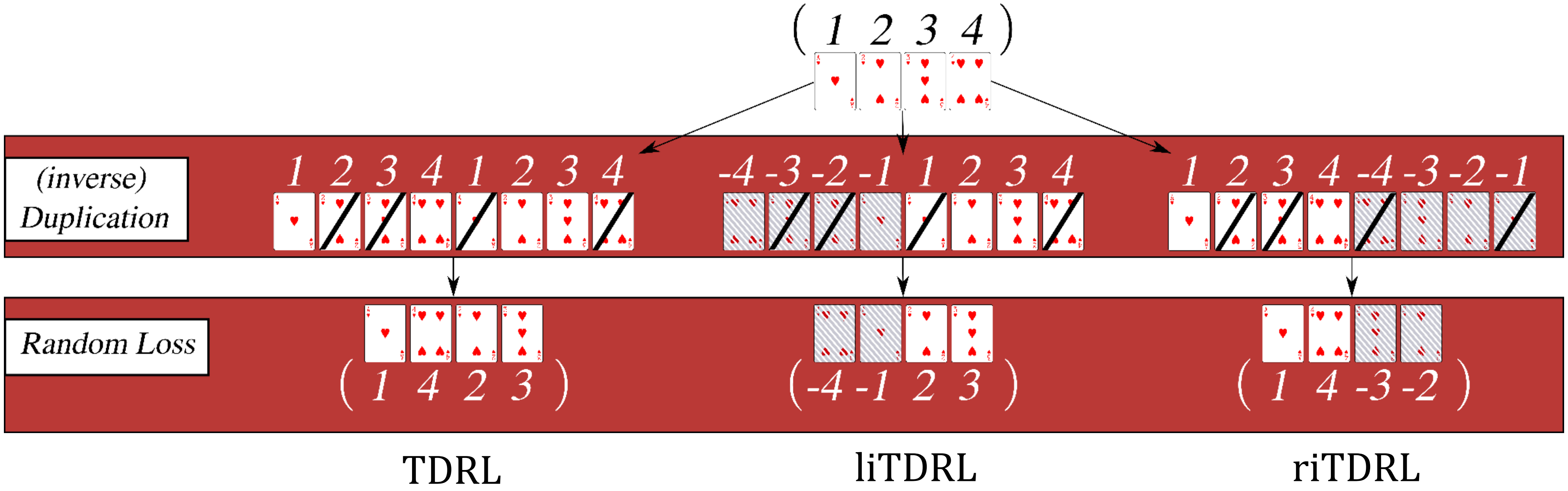

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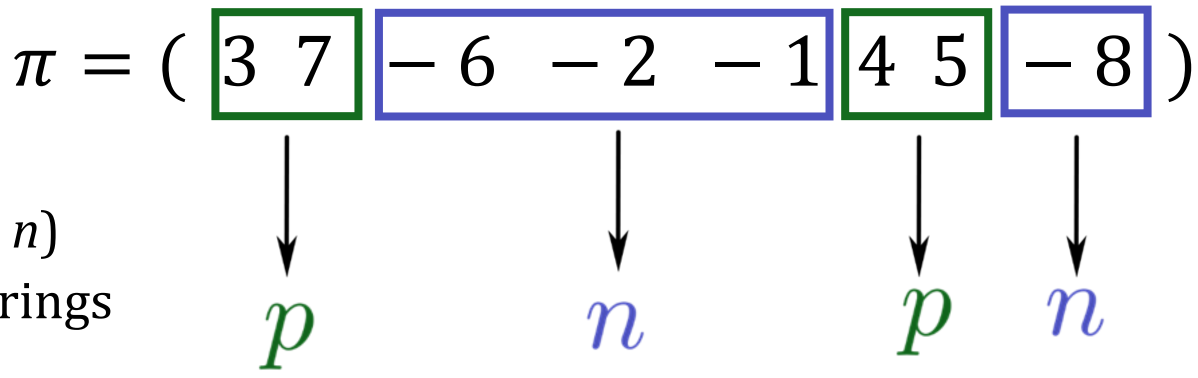

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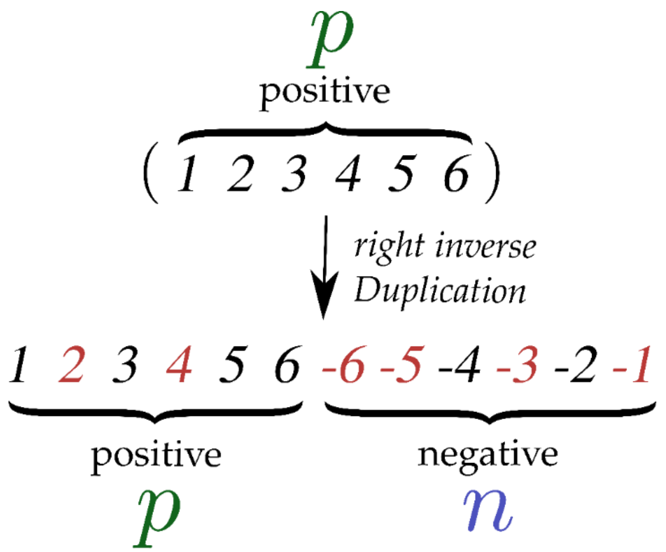

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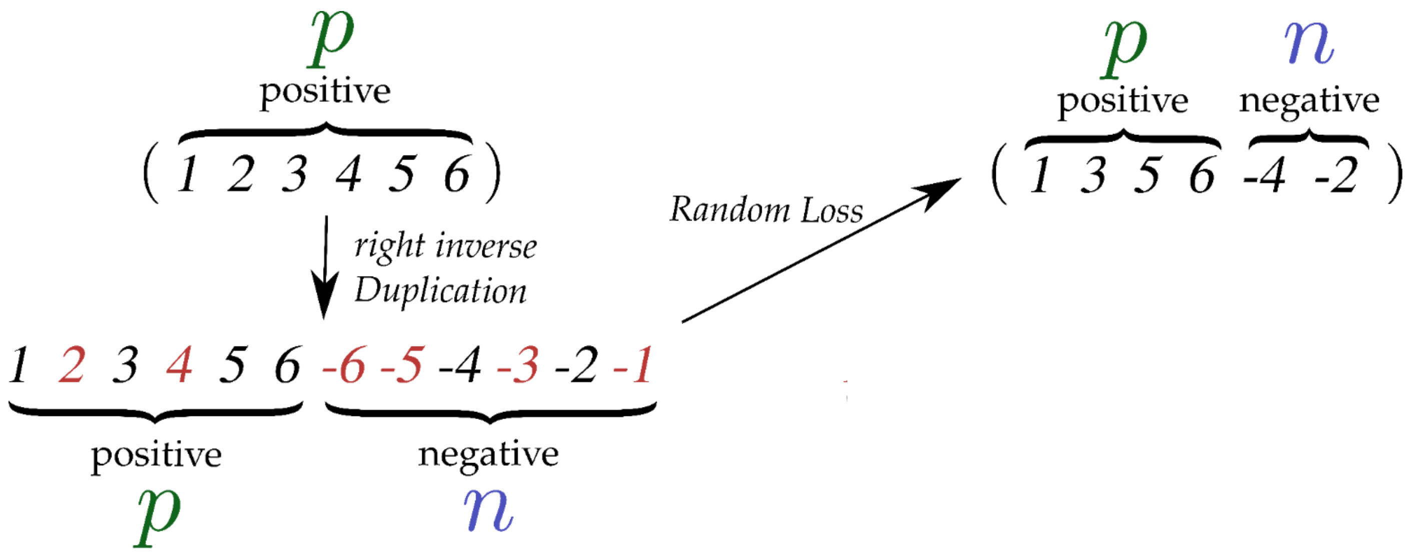

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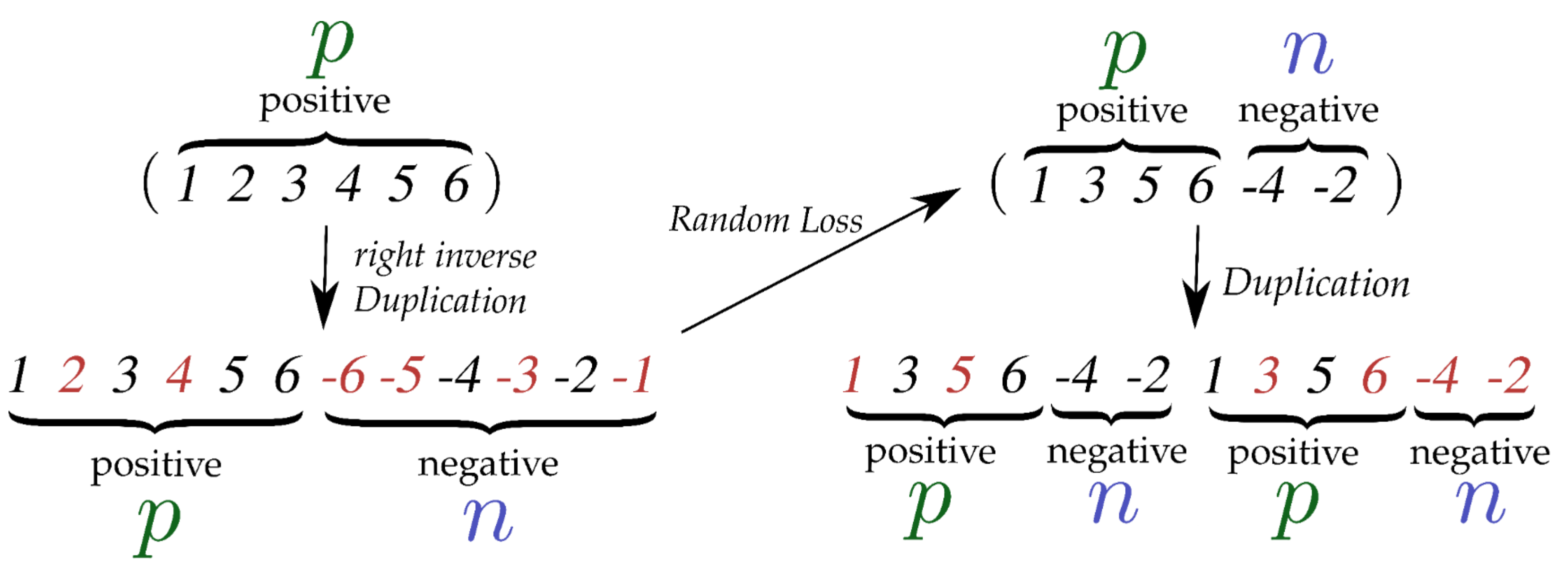

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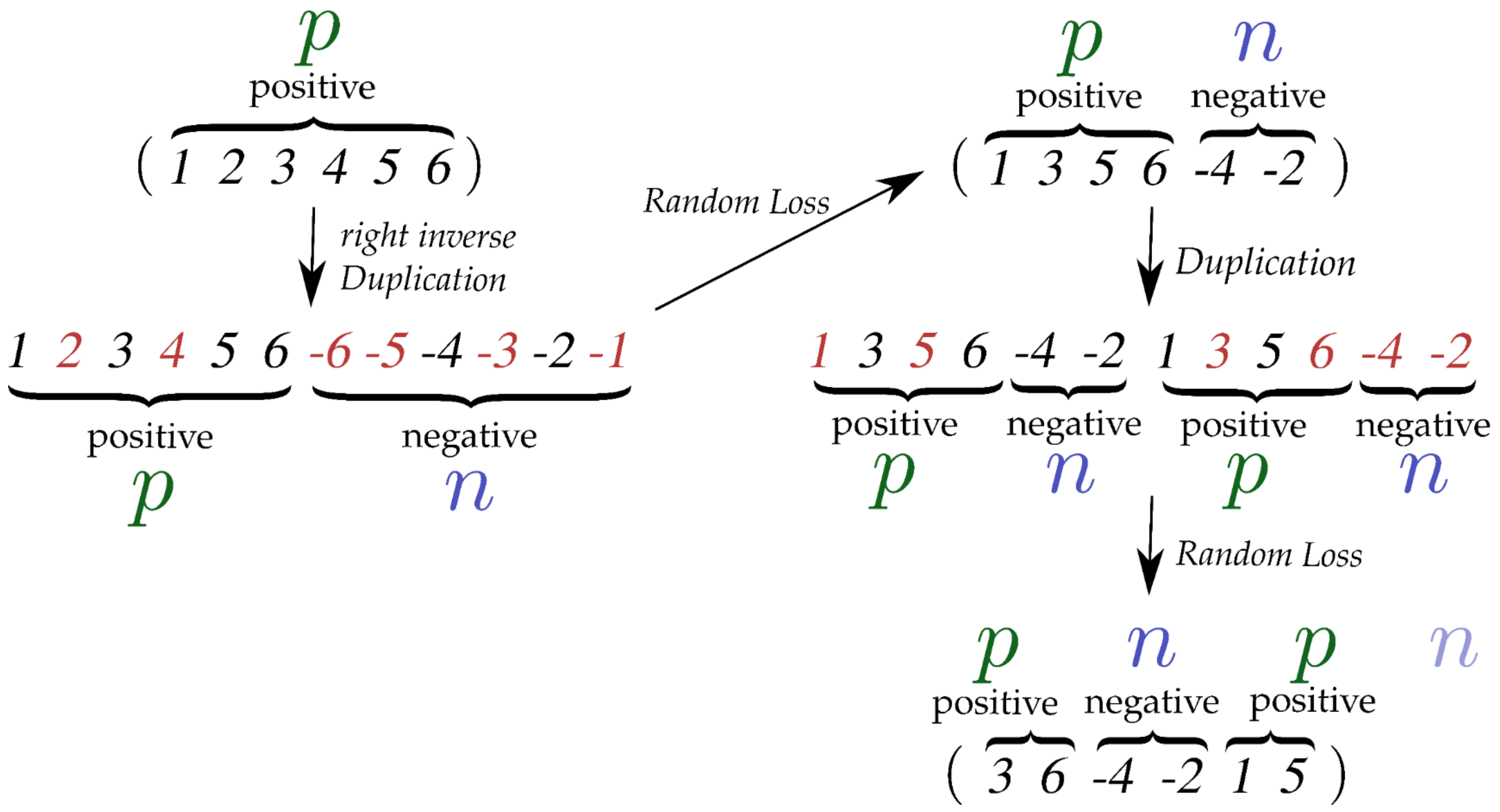

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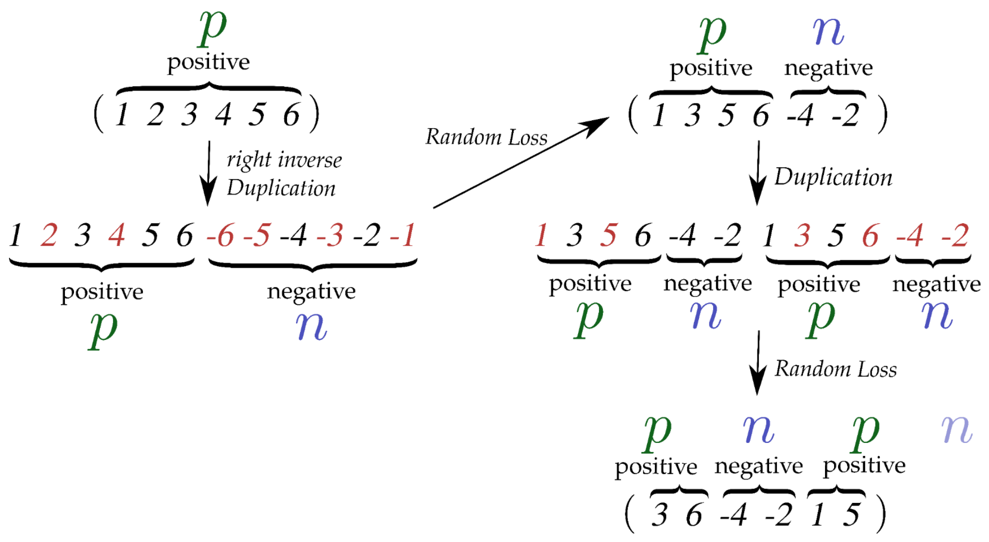

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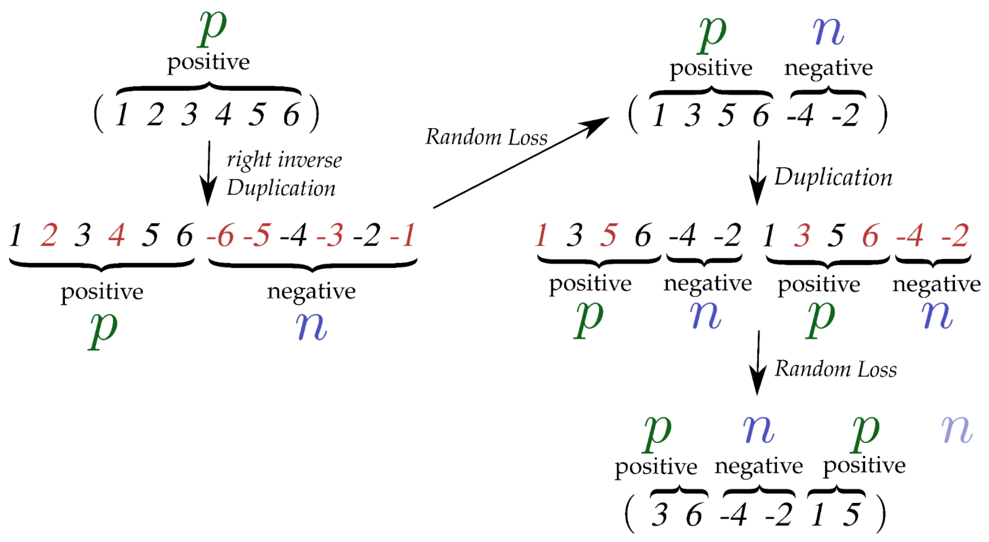

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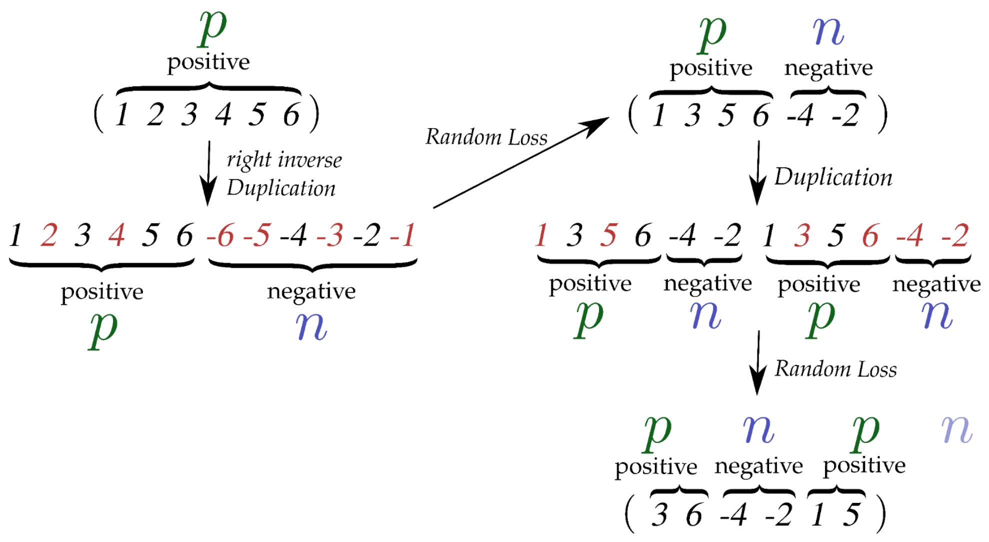

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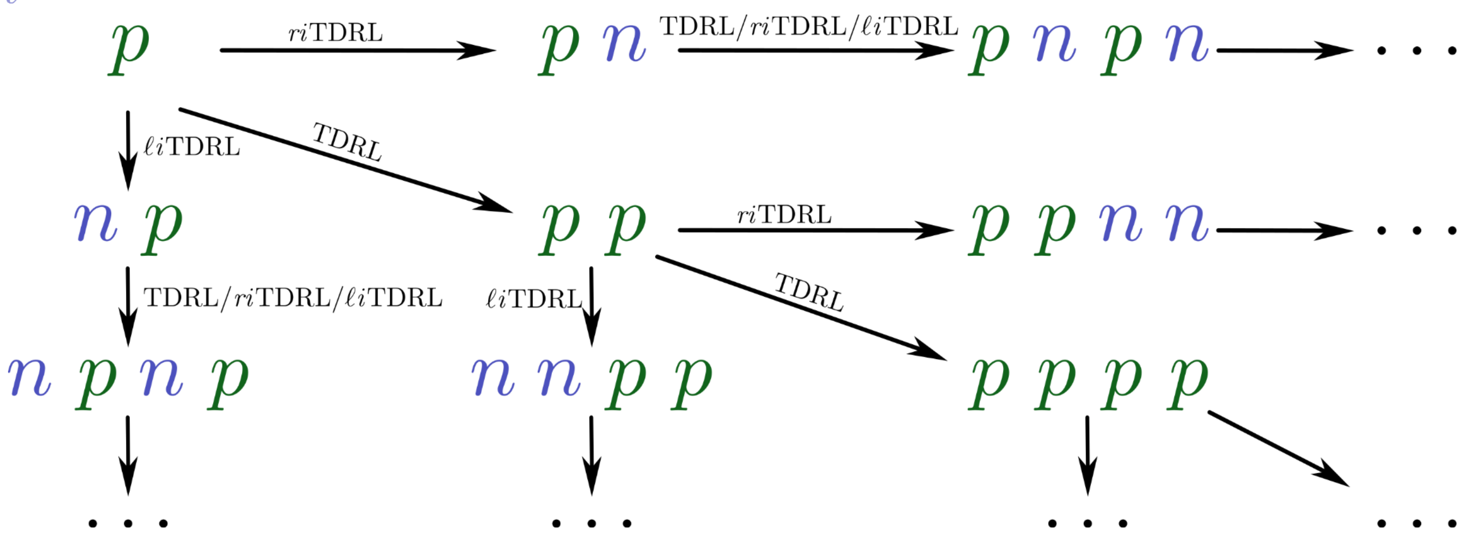

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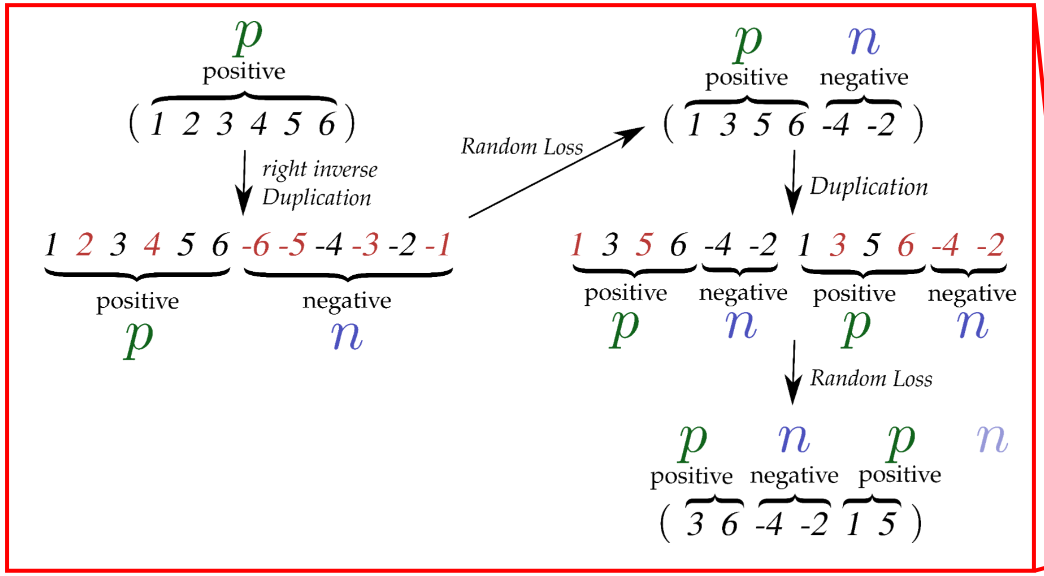

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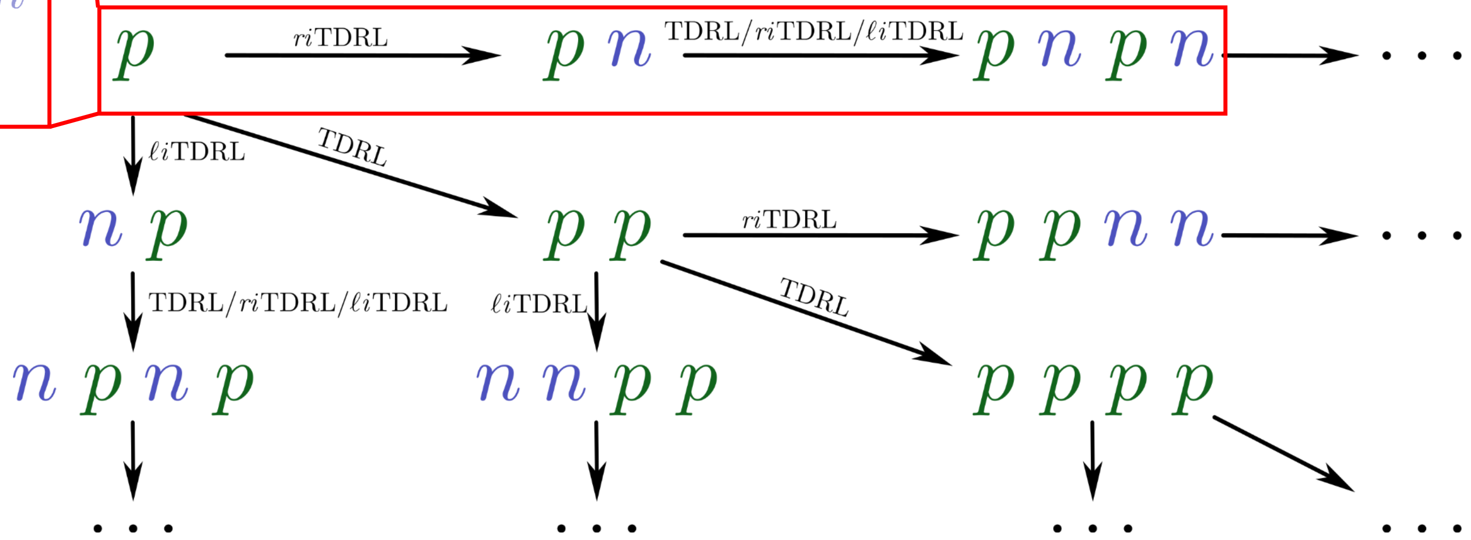

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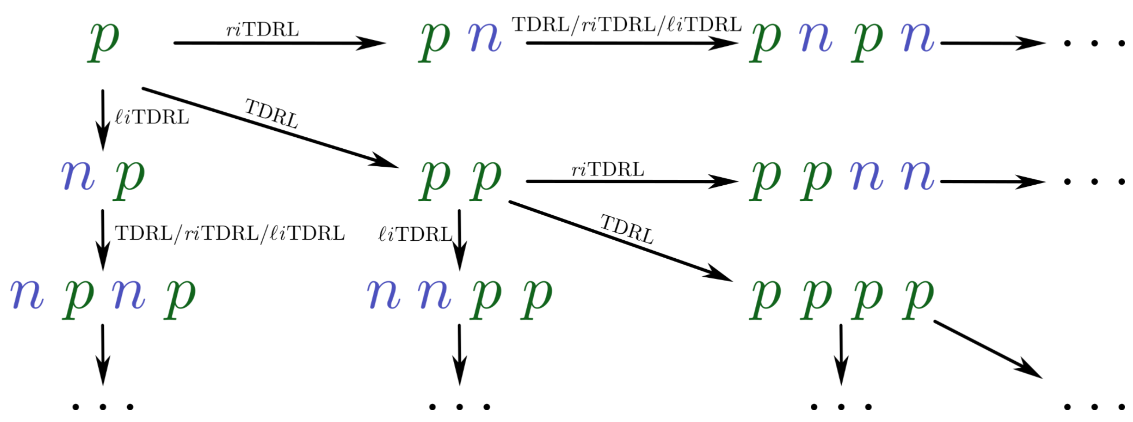

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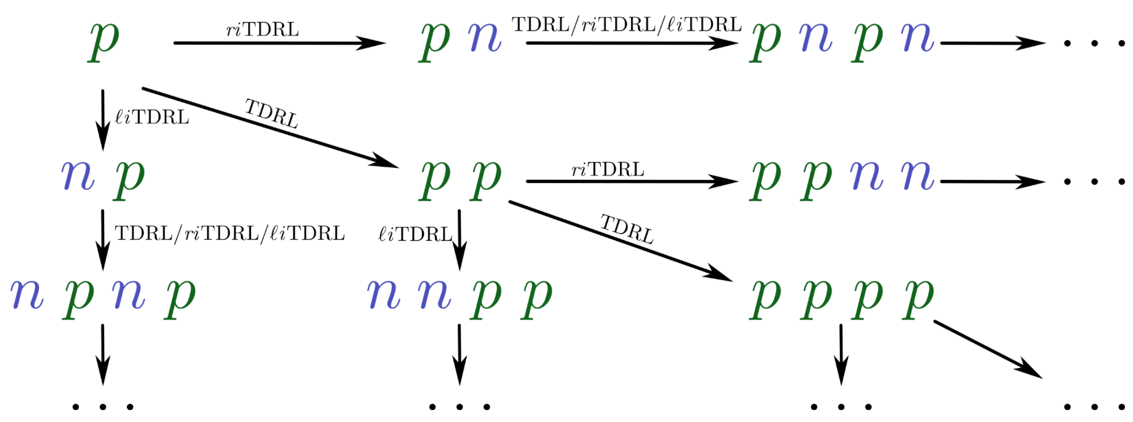

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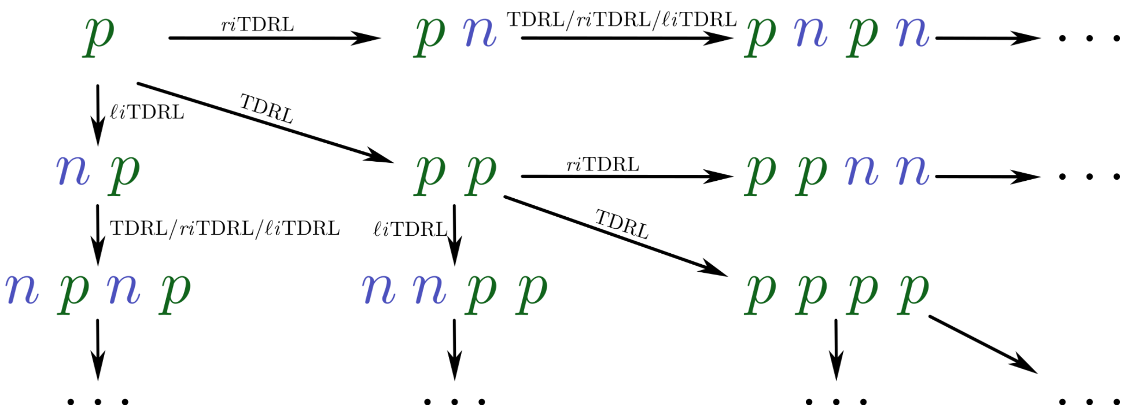

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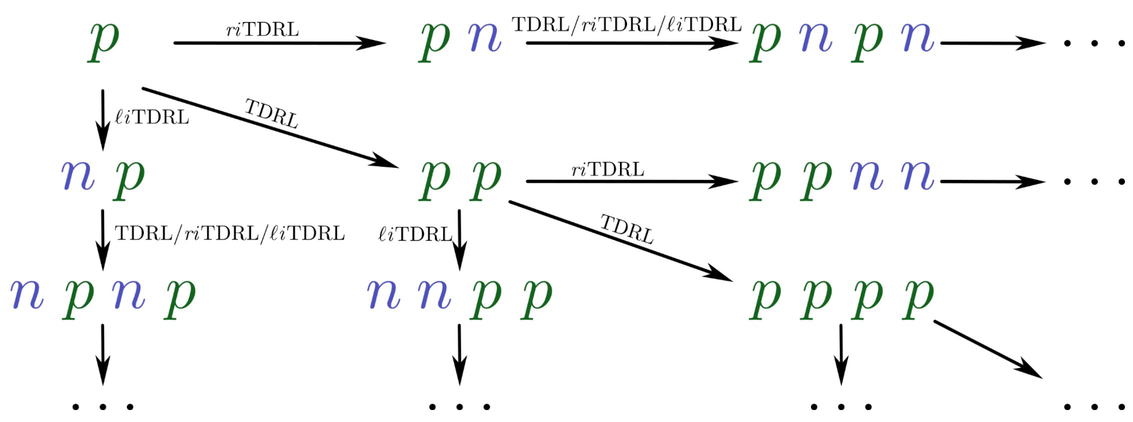

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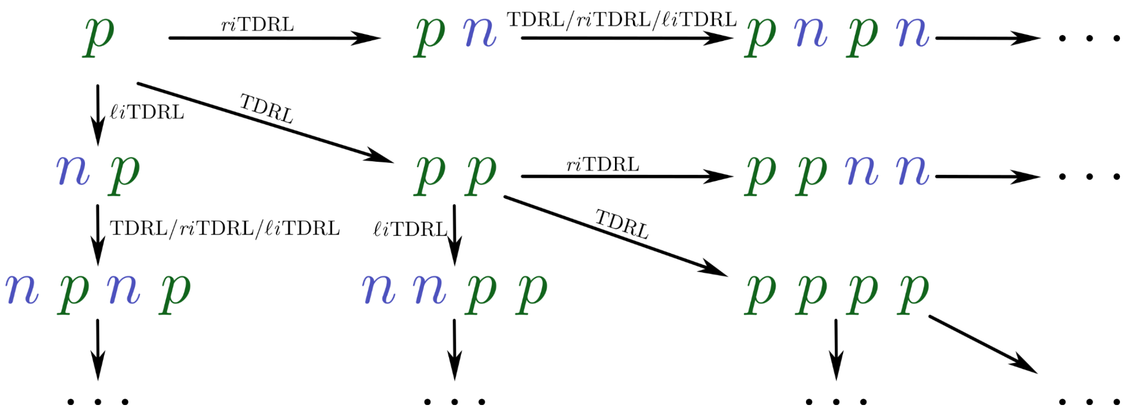

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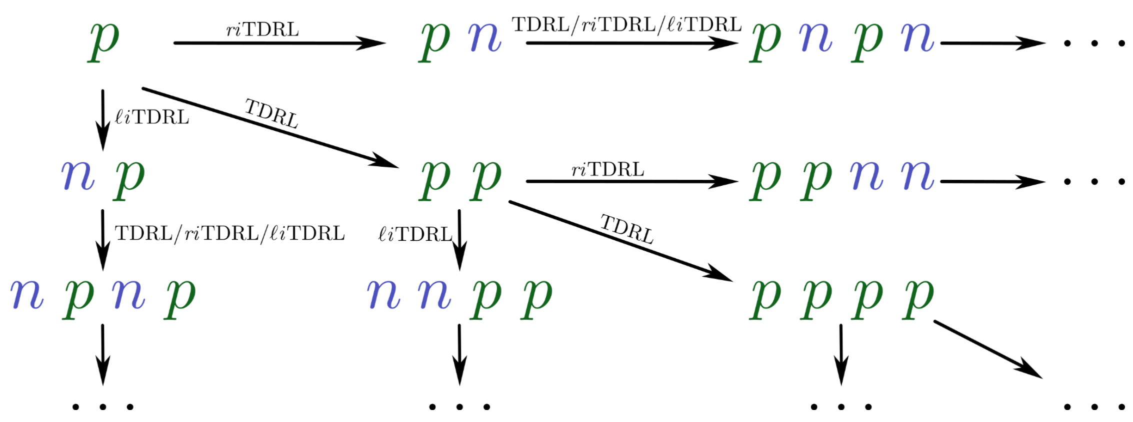

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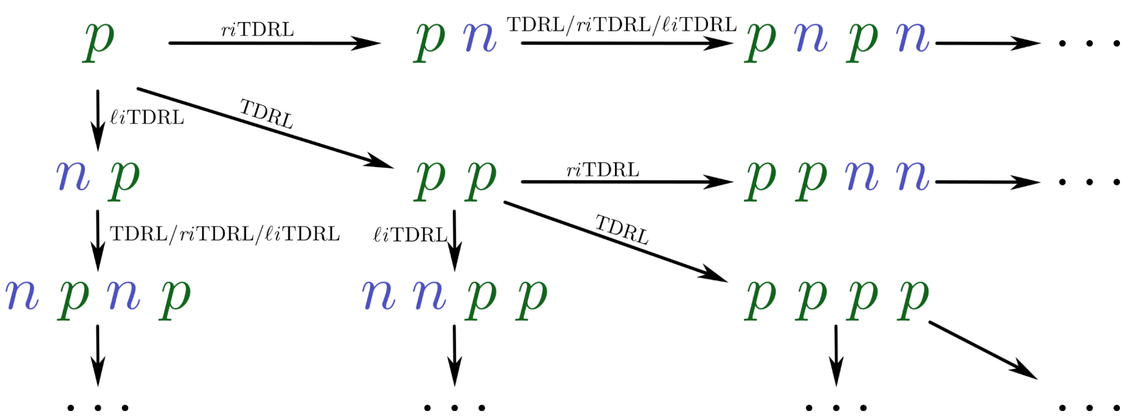

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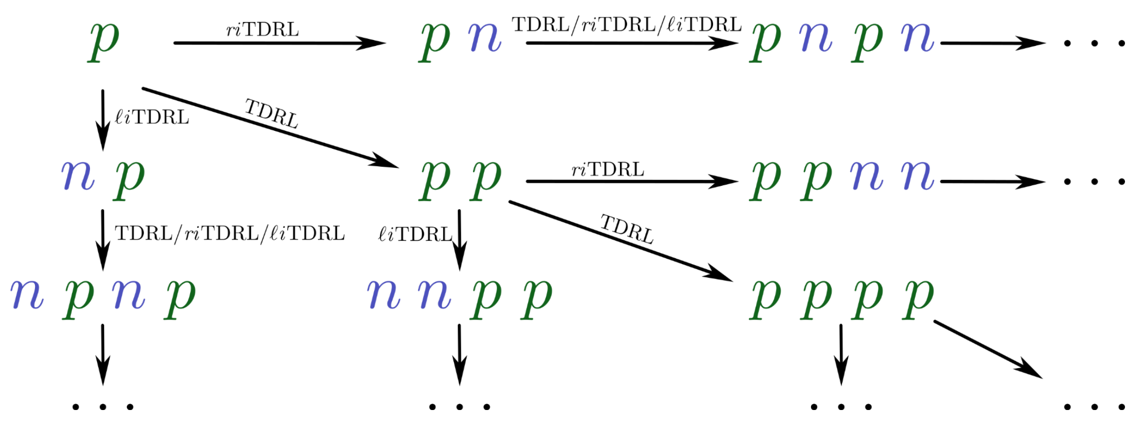

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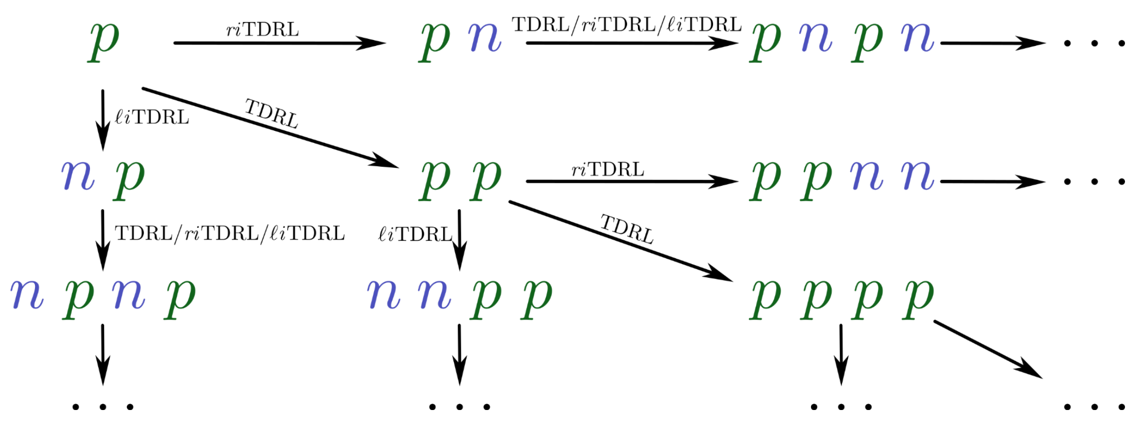

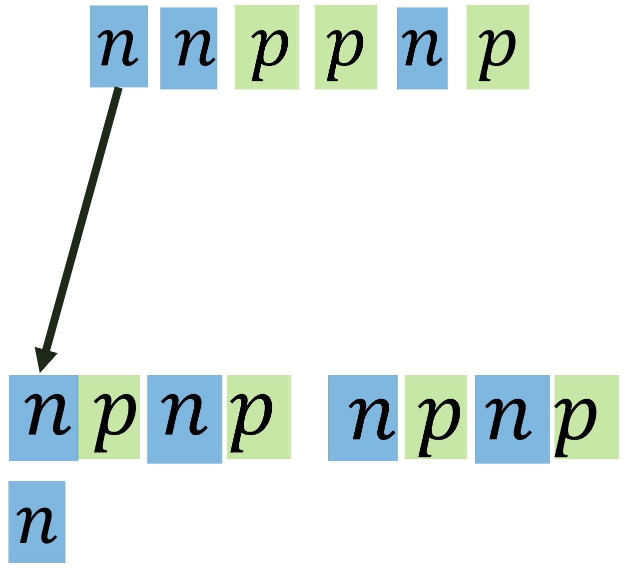

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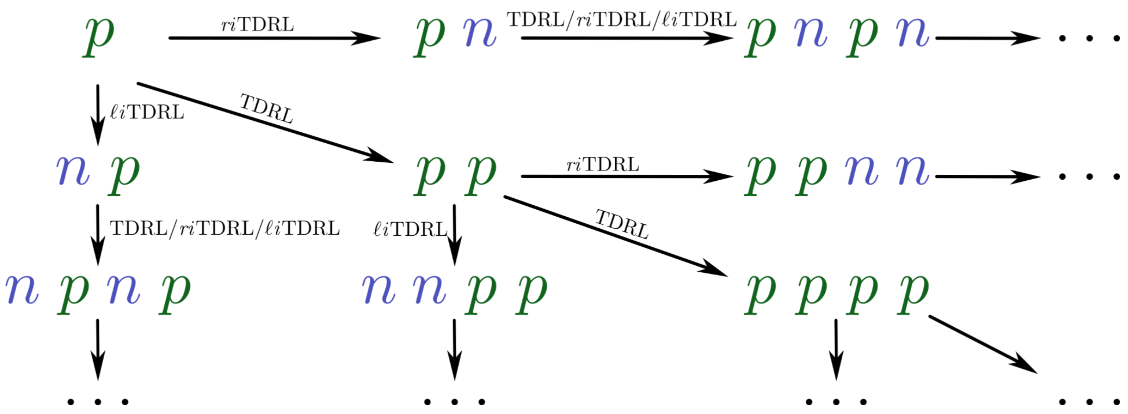

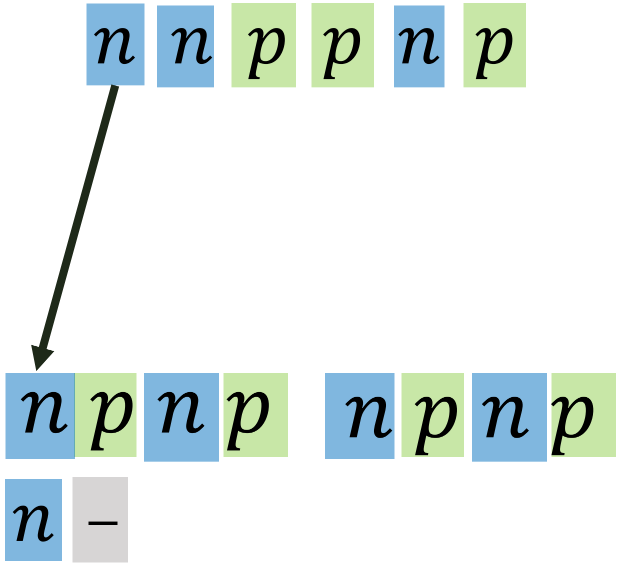

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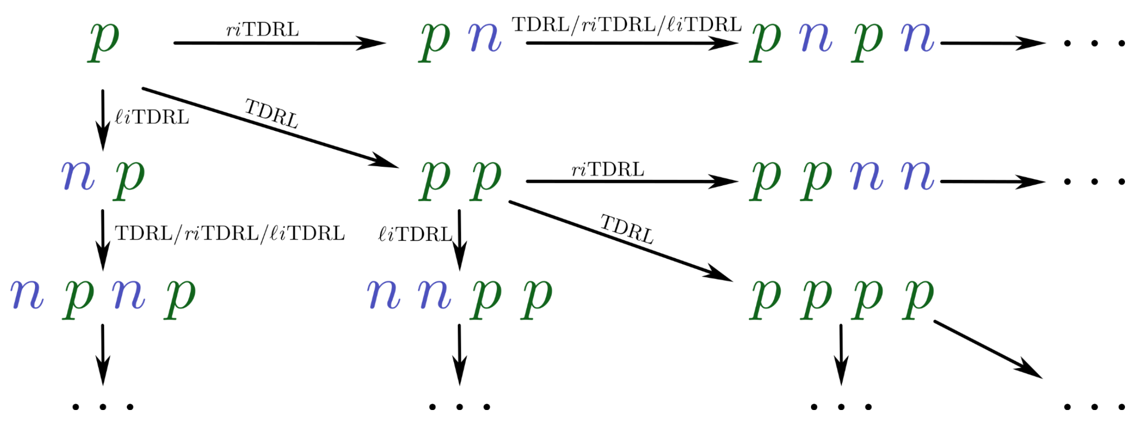

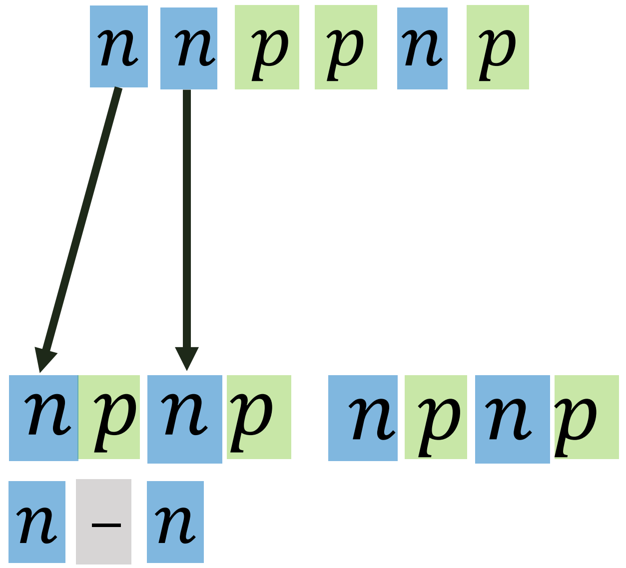

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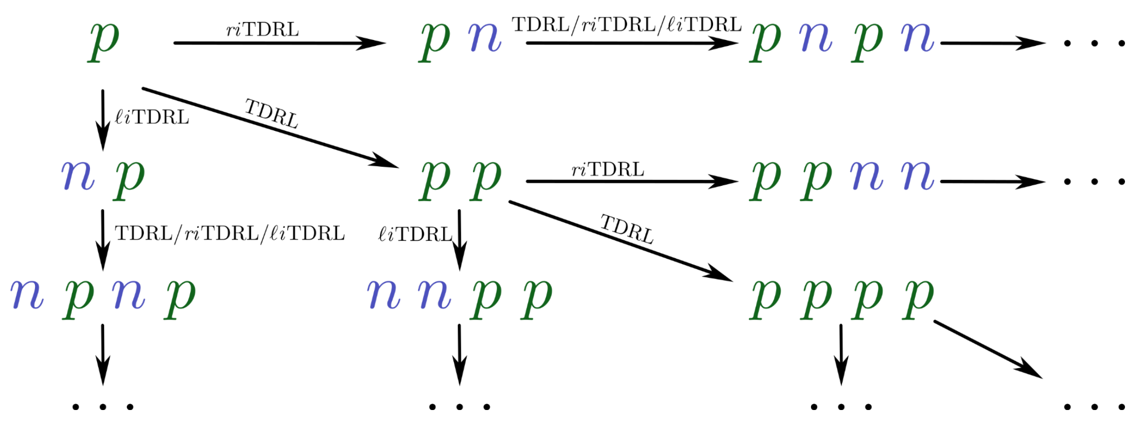

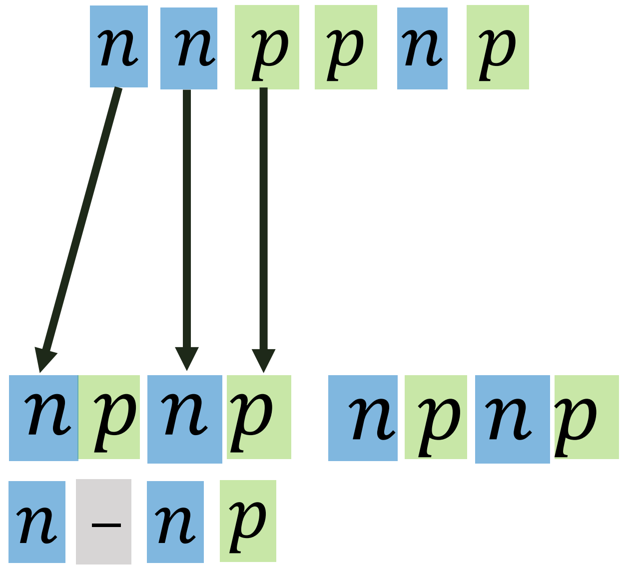

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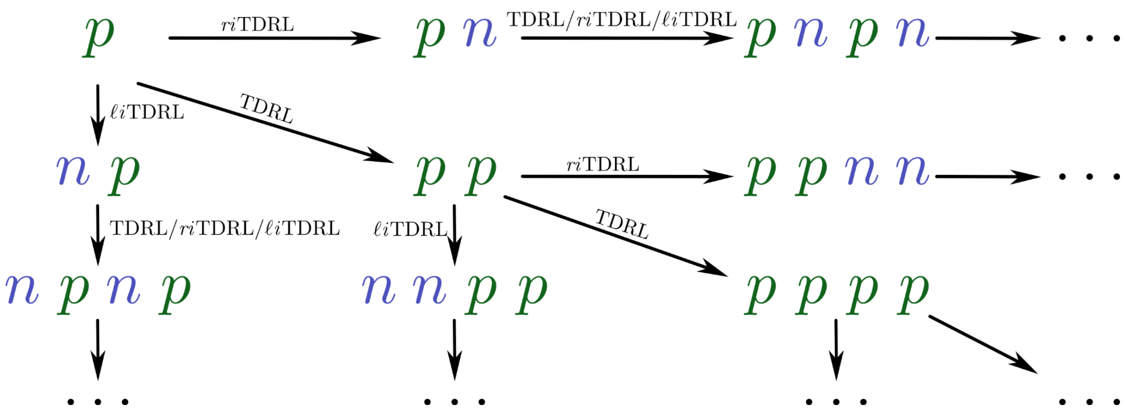

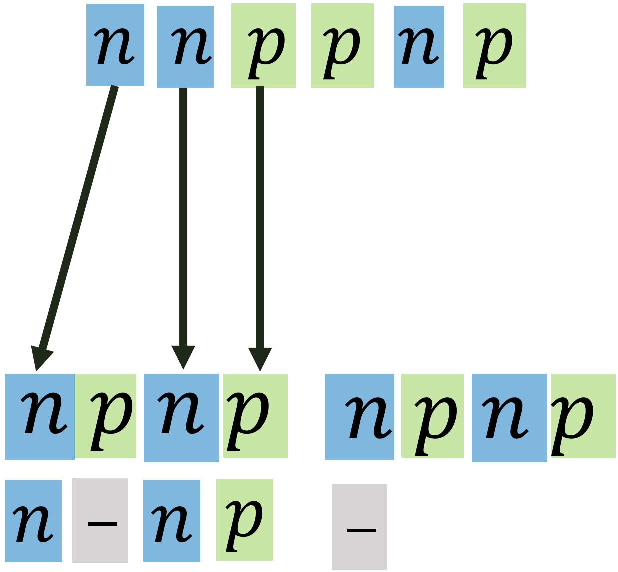

An Algorithm

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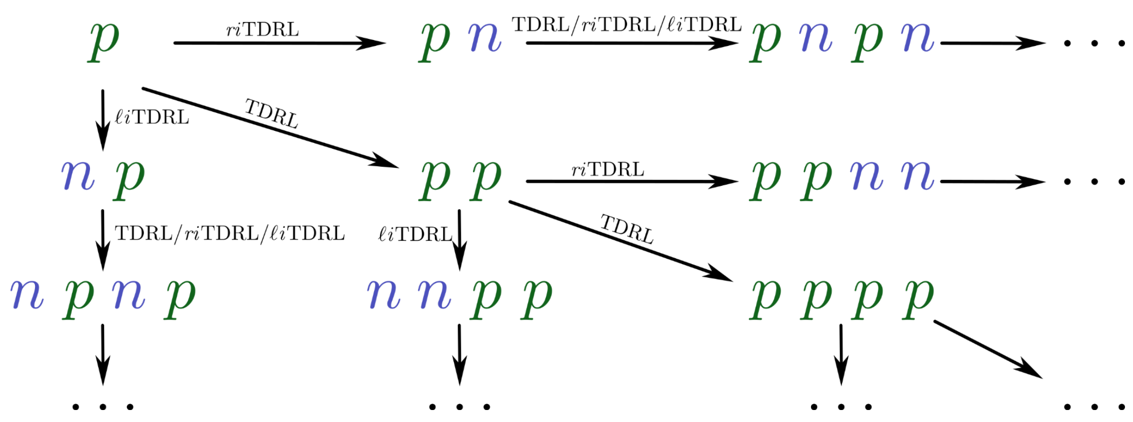

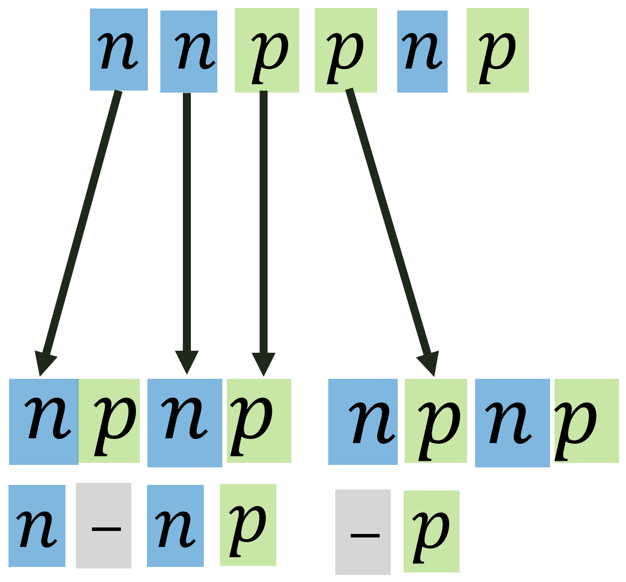

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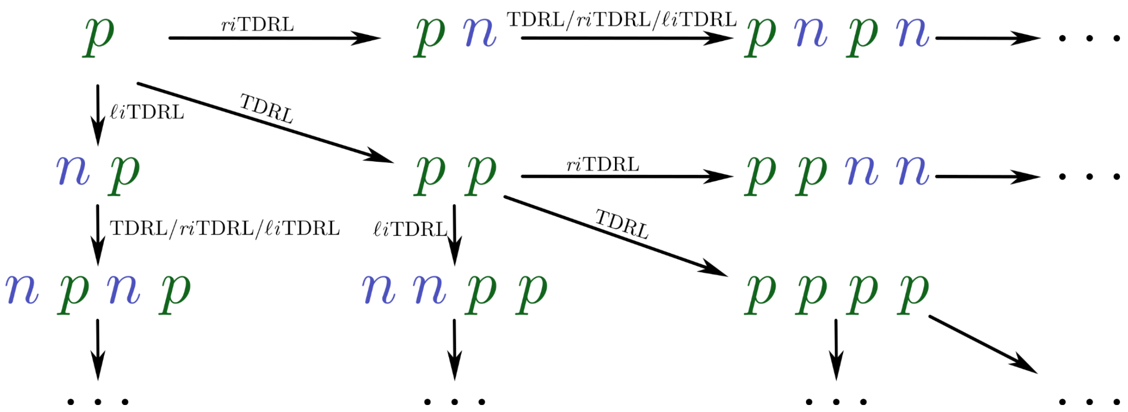

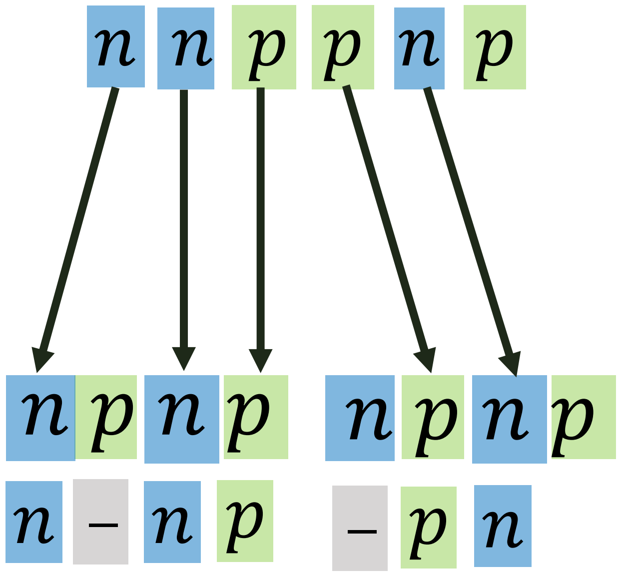

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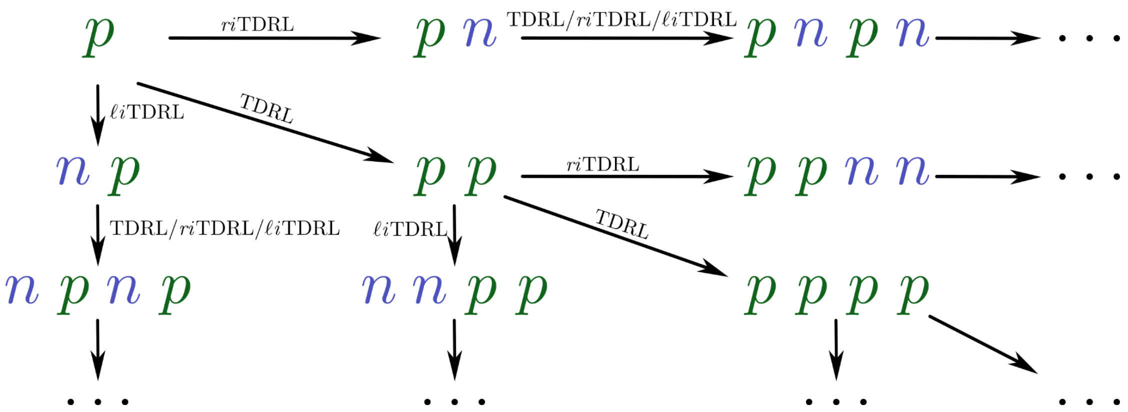

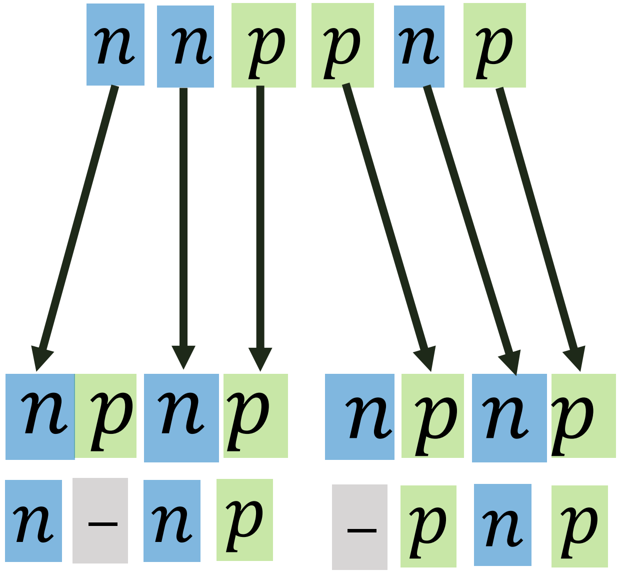

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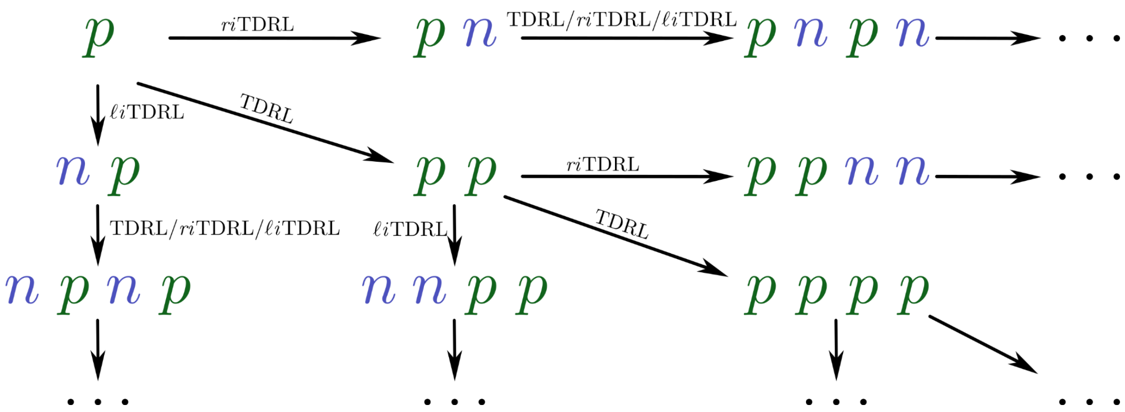

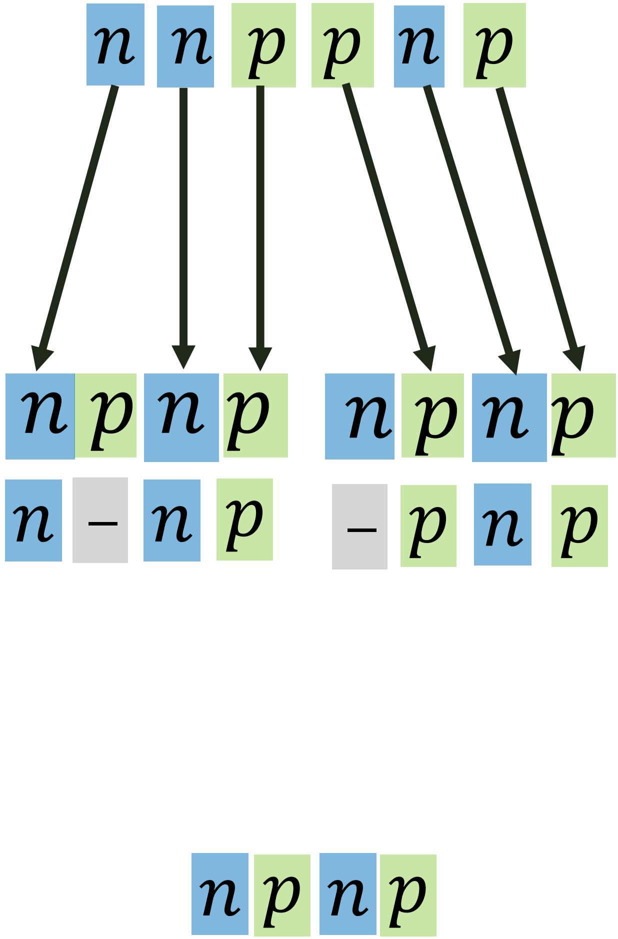

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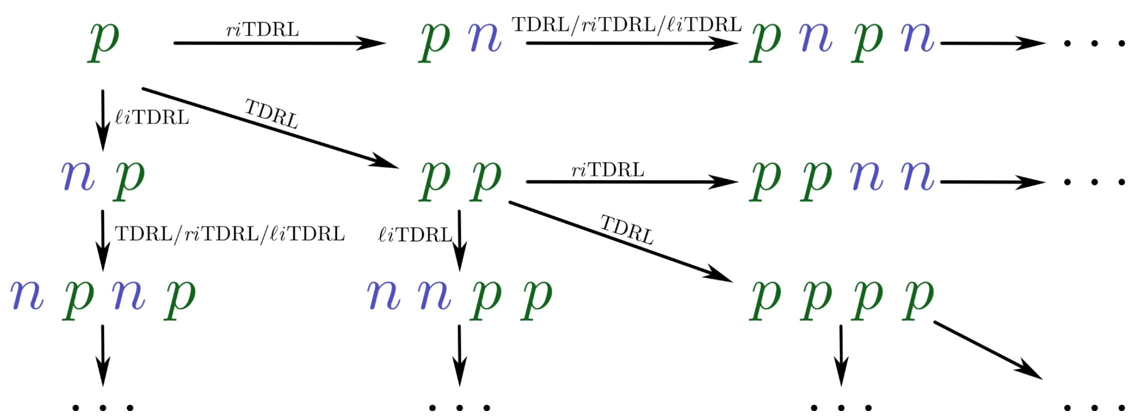

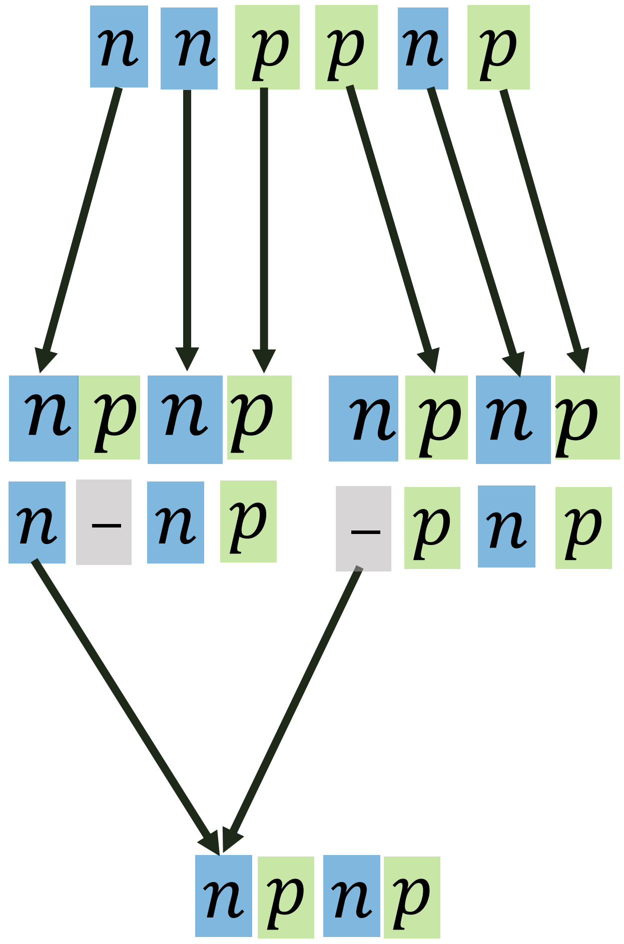

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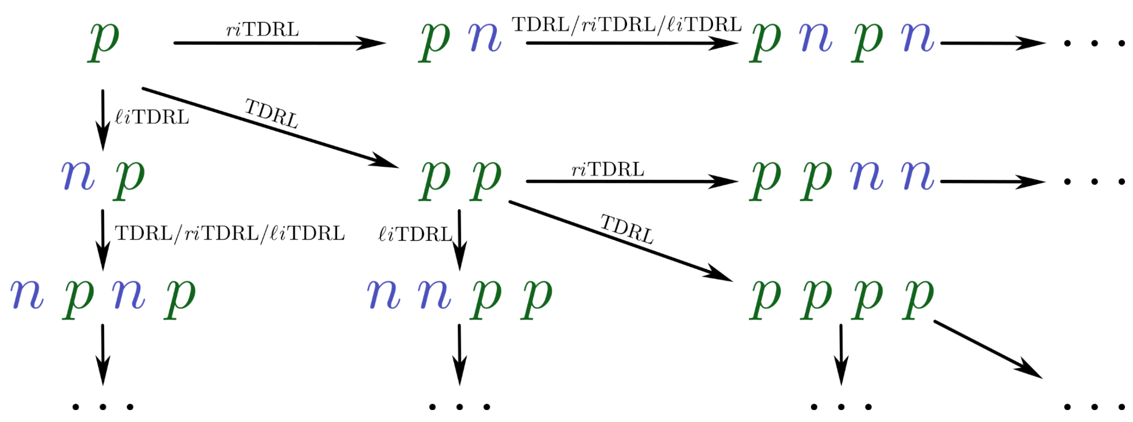

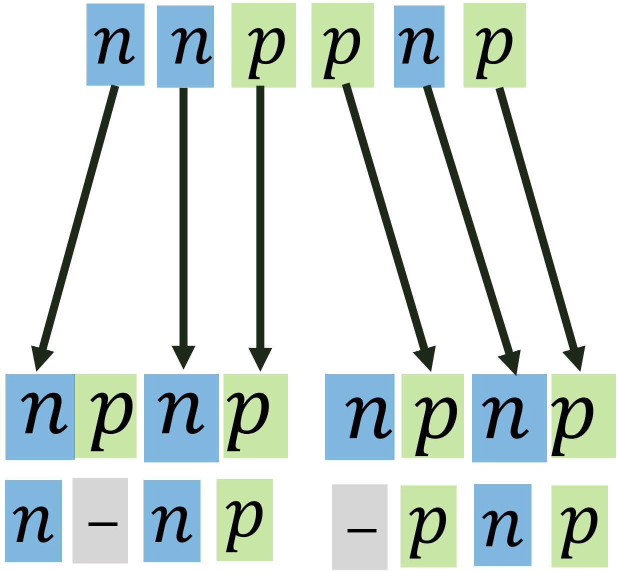

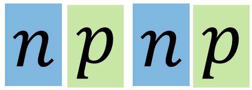

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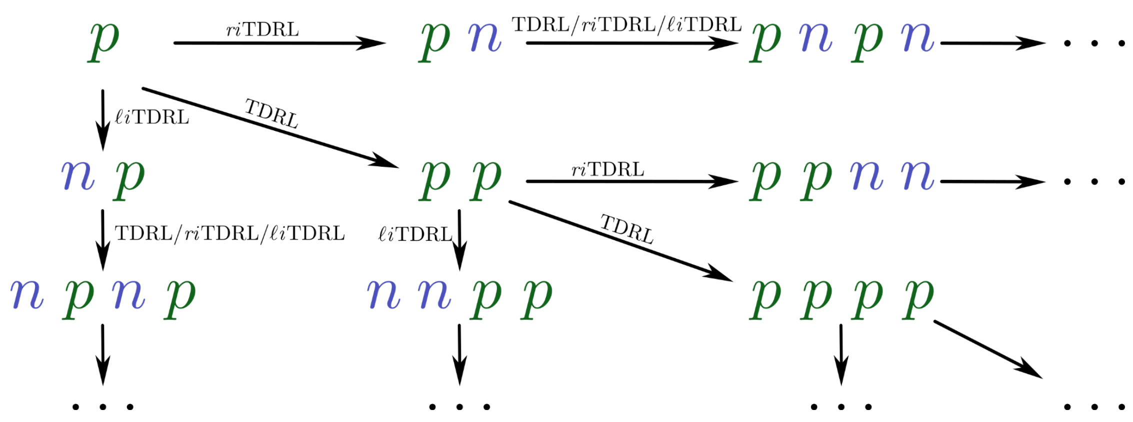

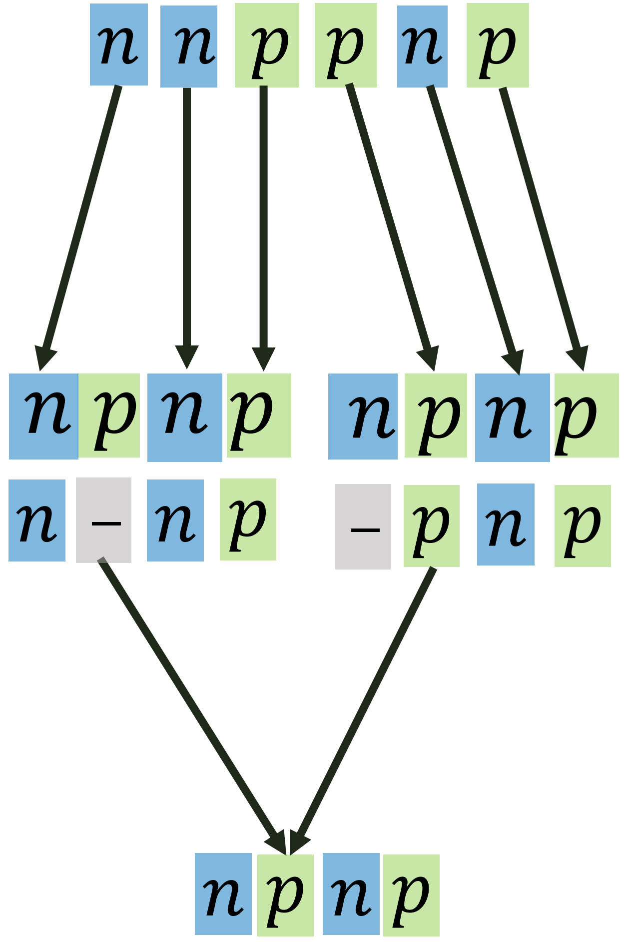

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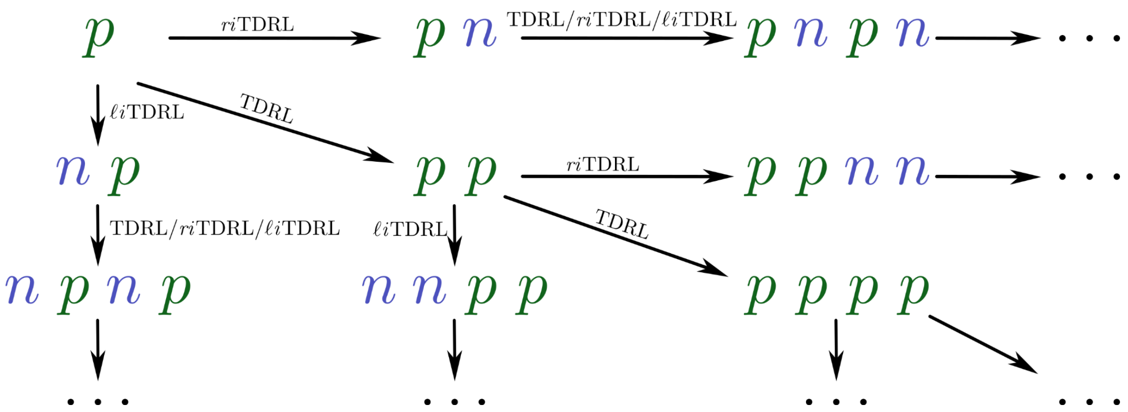

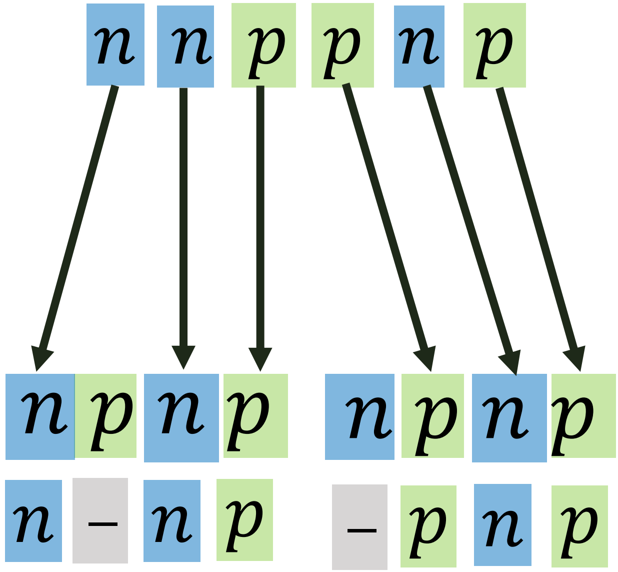

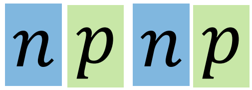

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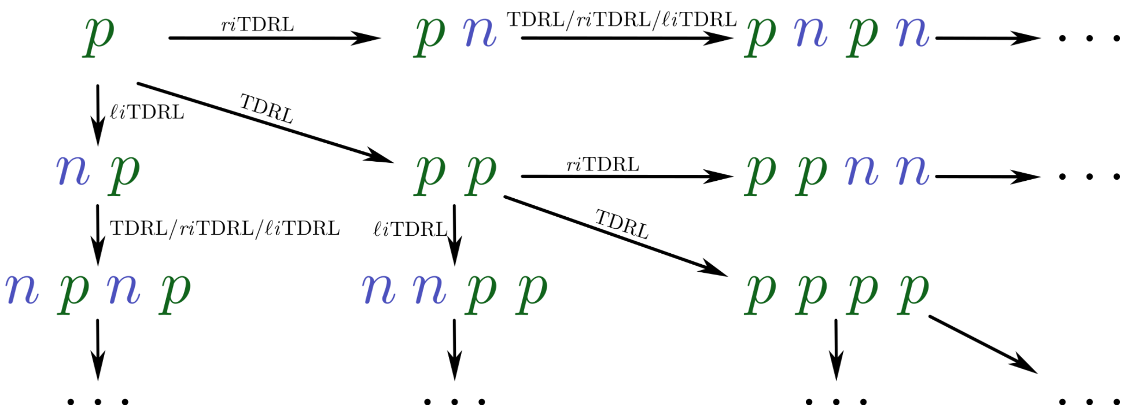

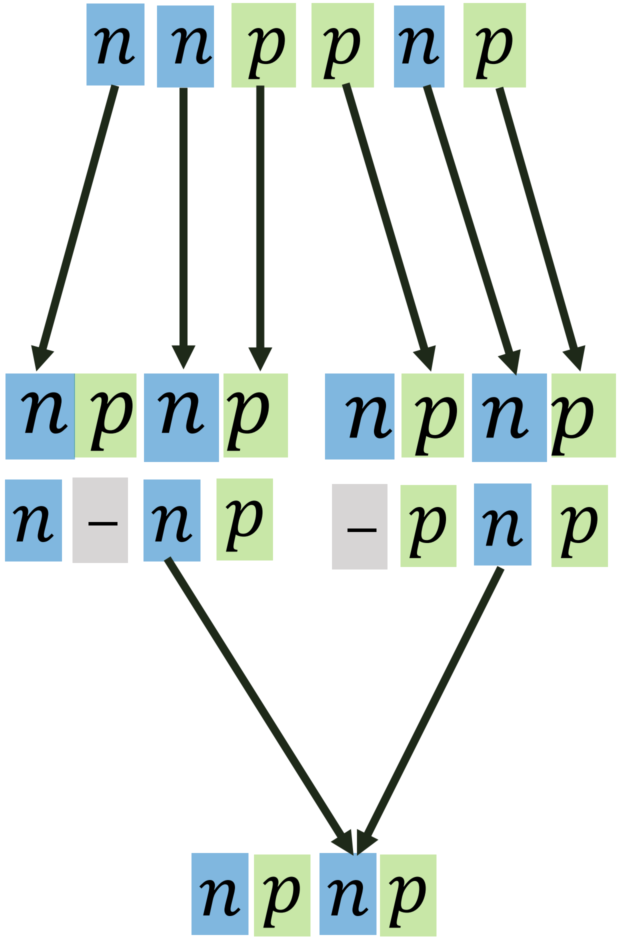

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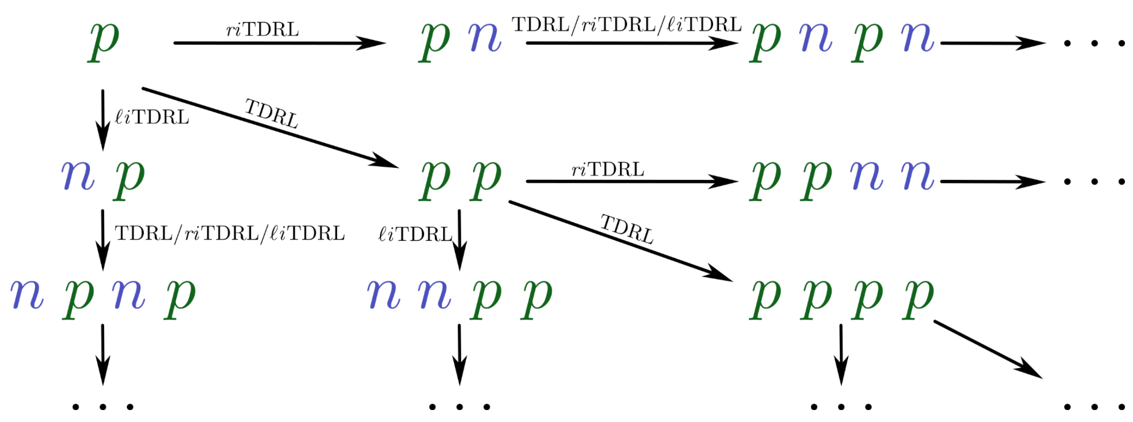

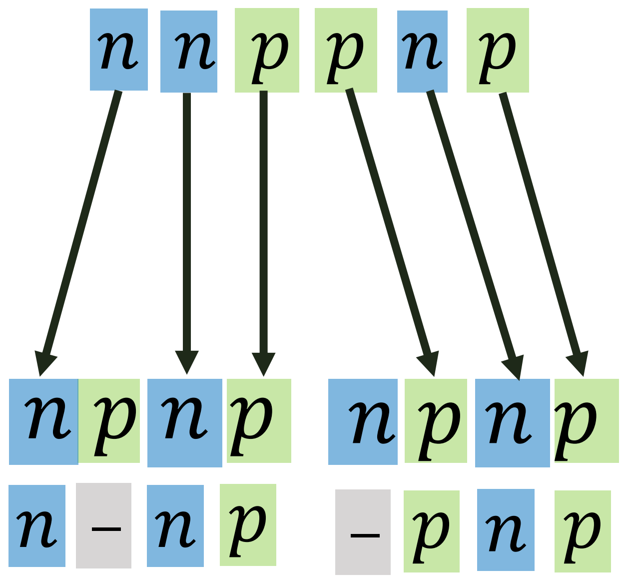

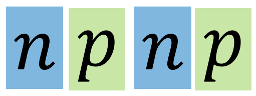

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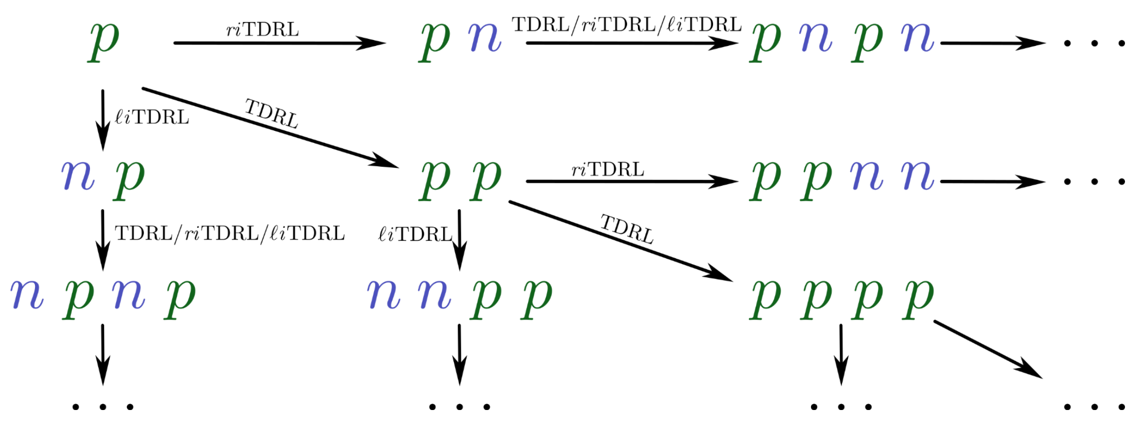

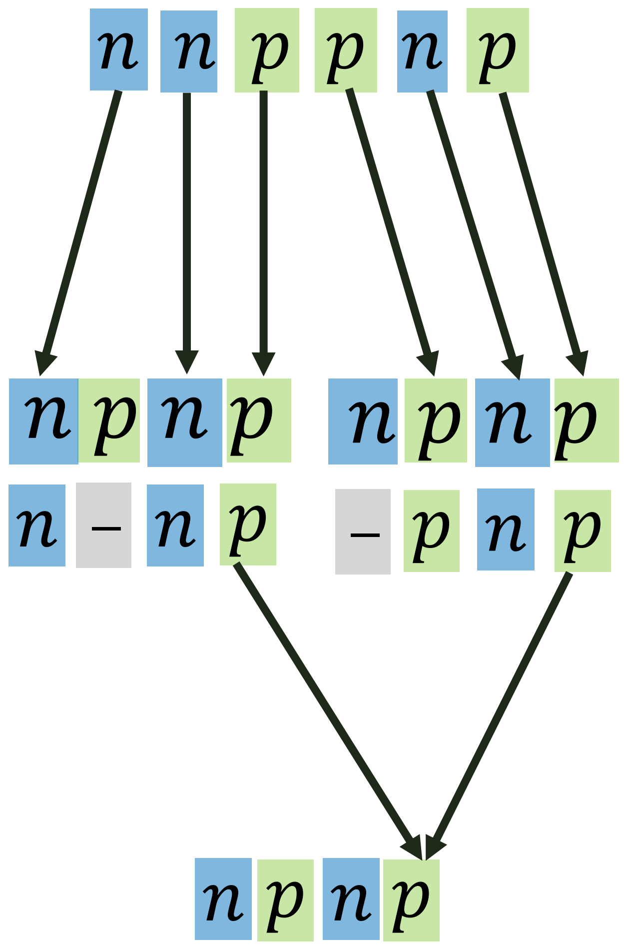

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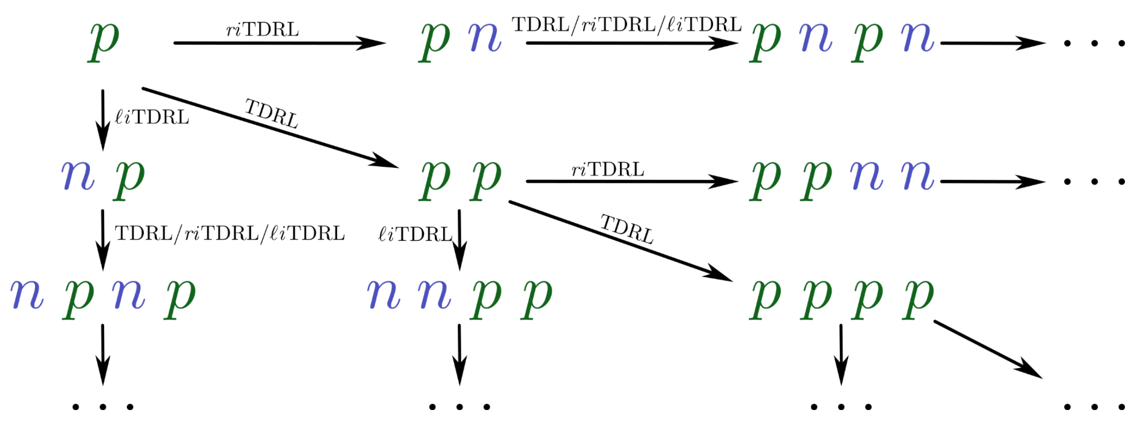

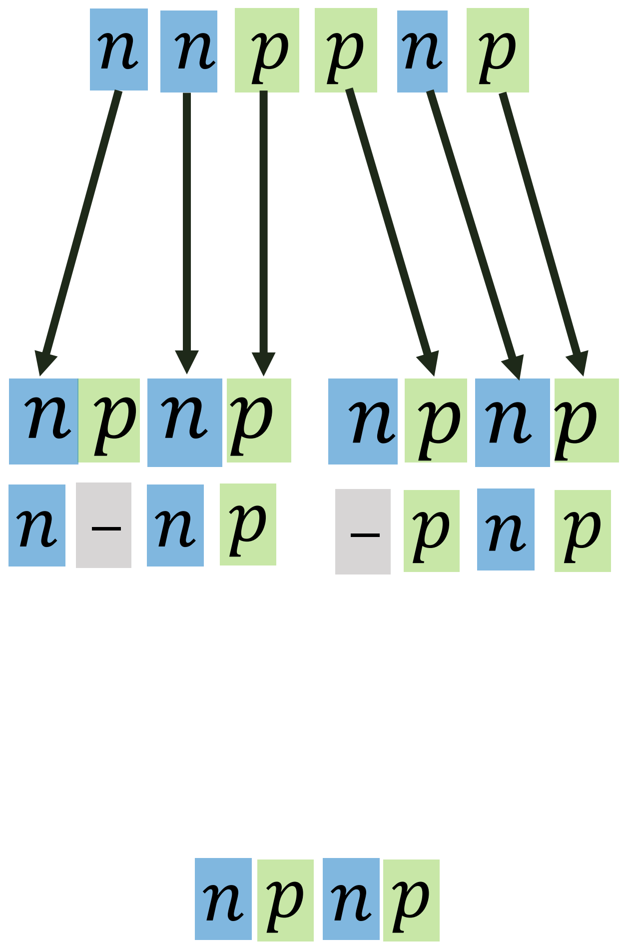

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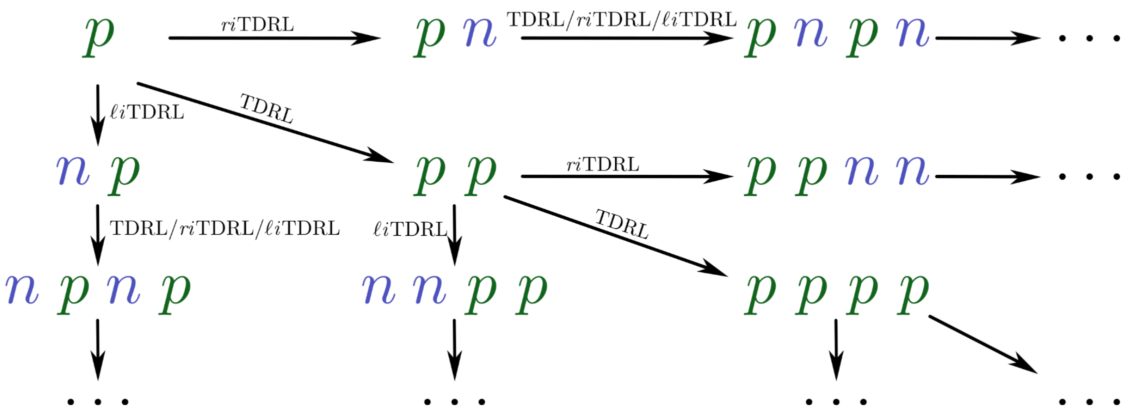
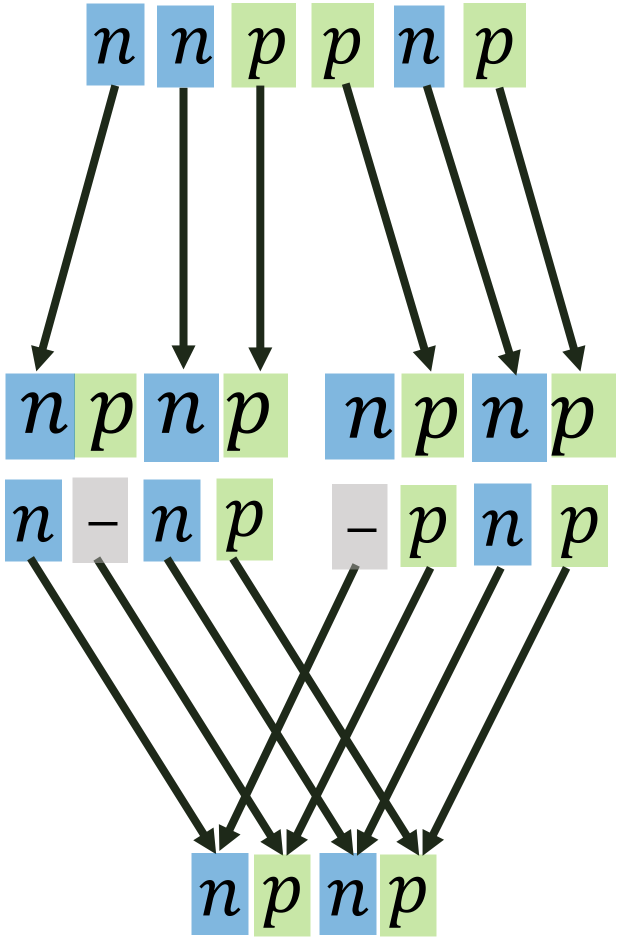
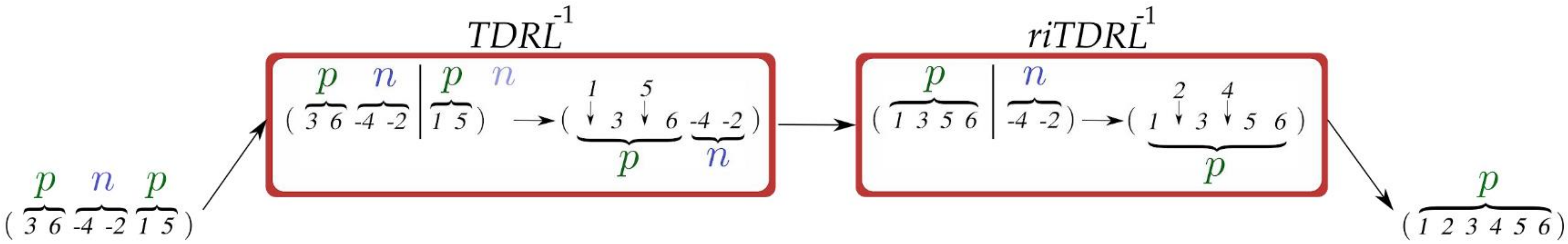




Sorting by TDRL and iTDRL



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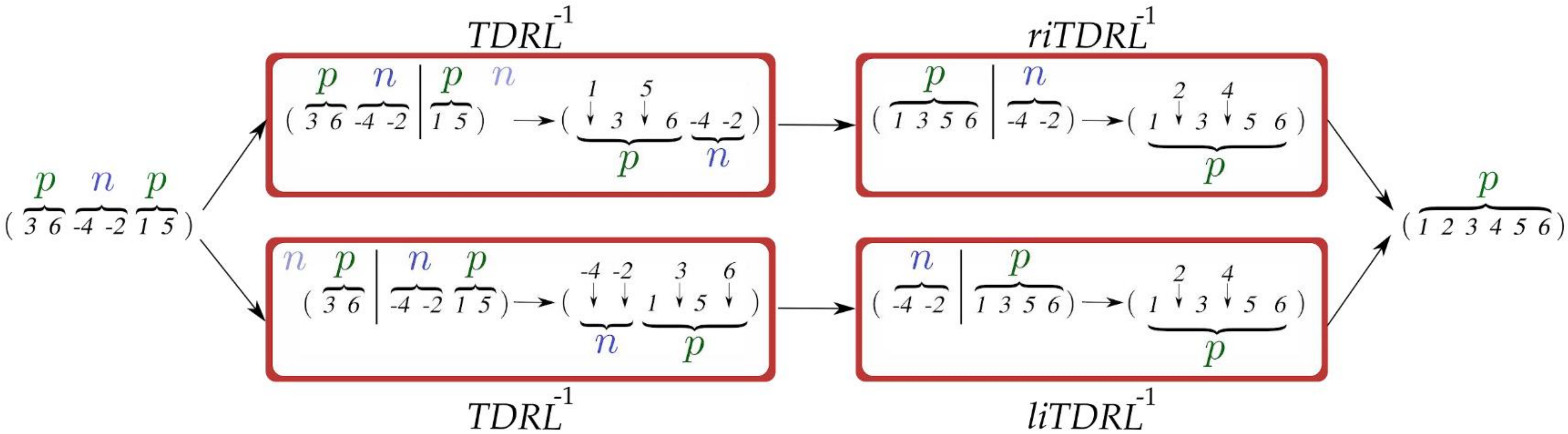
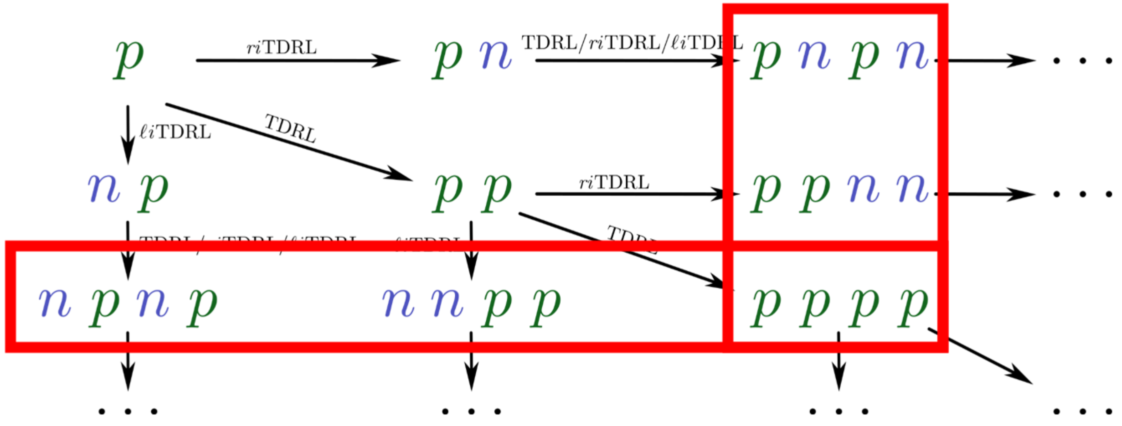
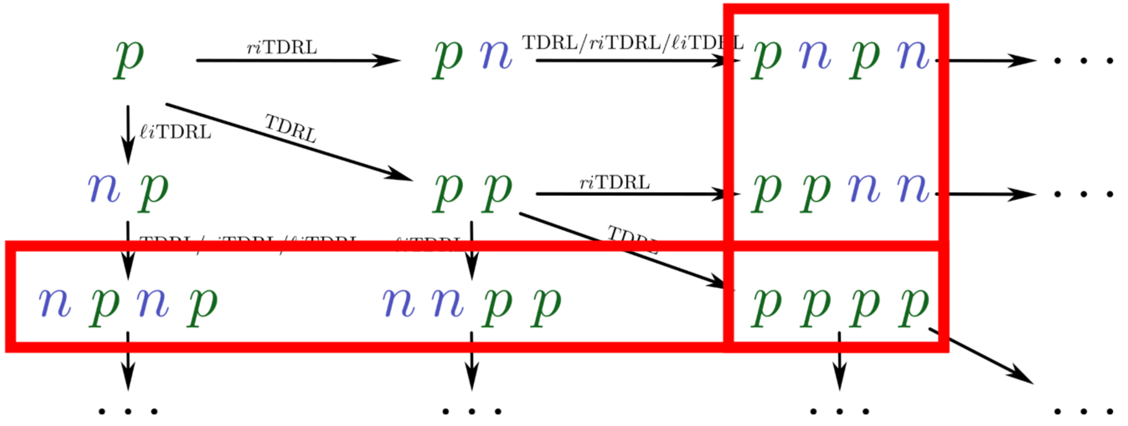


Complexity


p
riTDRL
p n
TDRL/riTDRL/ℓiTDRL
p n p n
ℓiTDRL
TDRL
n p
p p
riTDRL
p p n n
n p n p
n n p p
p p p p
. . .

Complexity



- Number of characters in MISC-encoding: $\mathcal{O}(n)$

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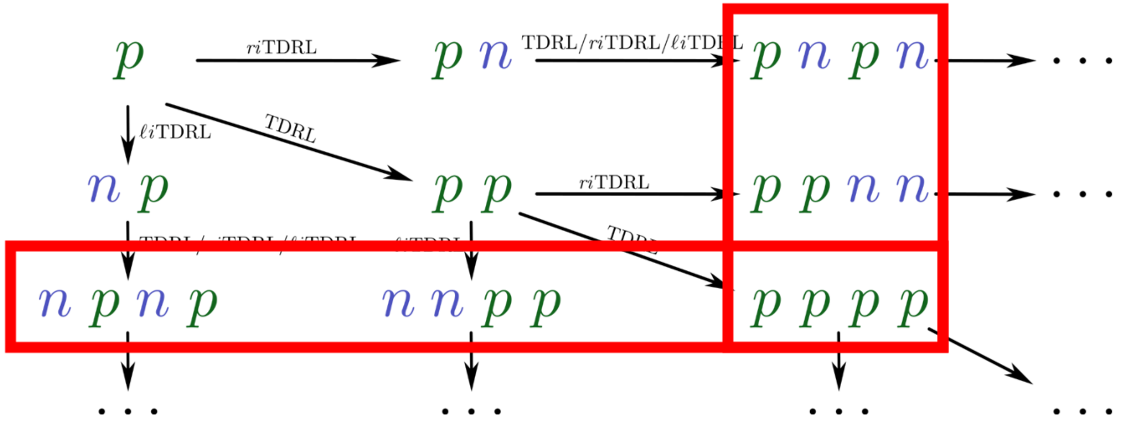

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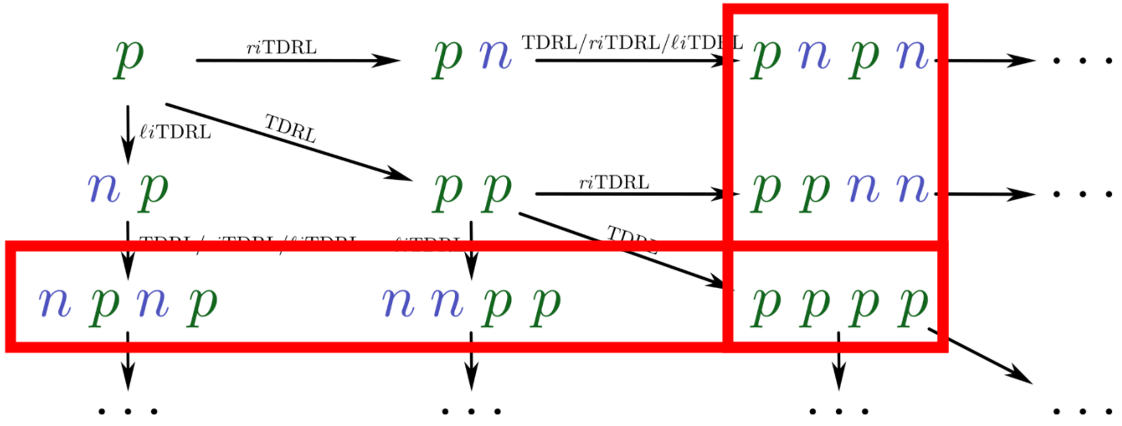

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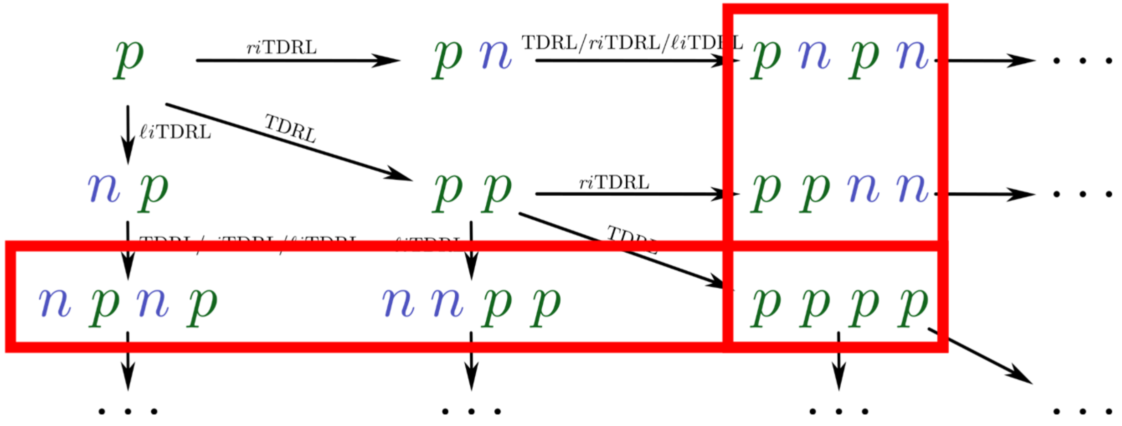

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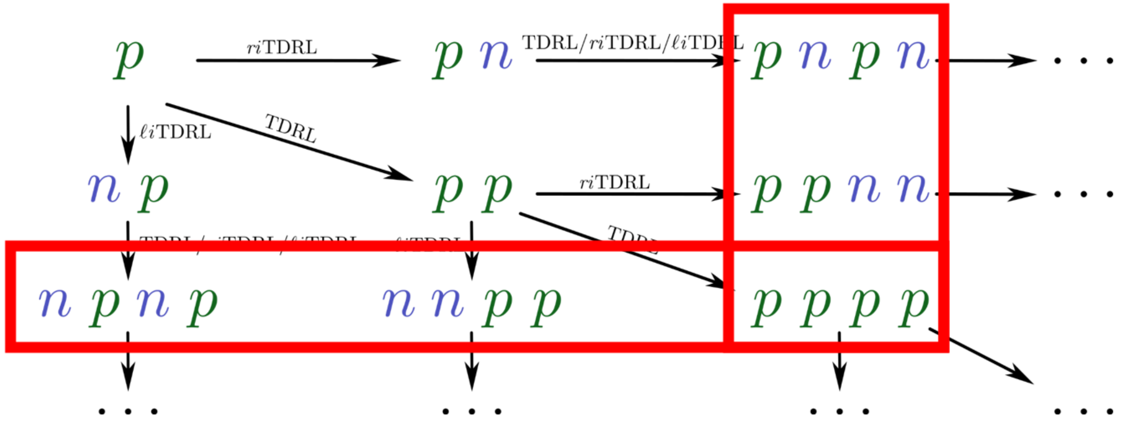

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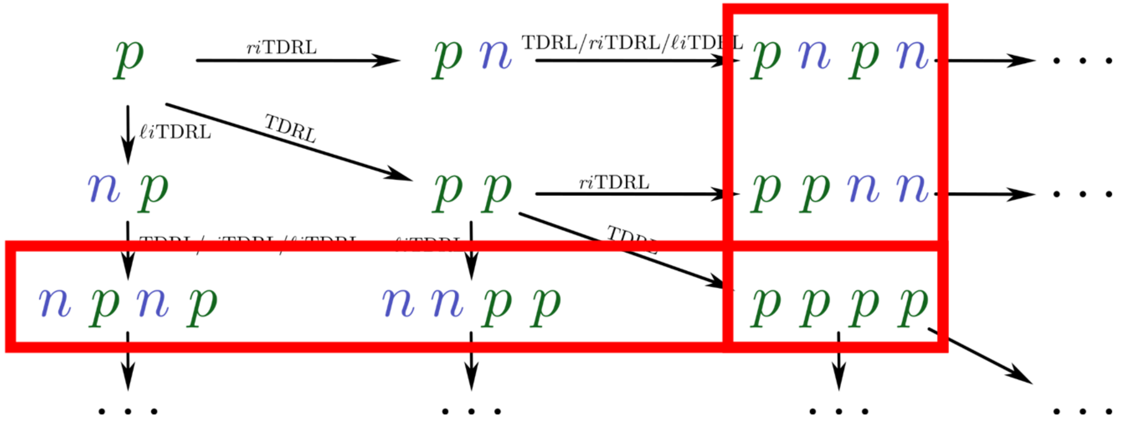

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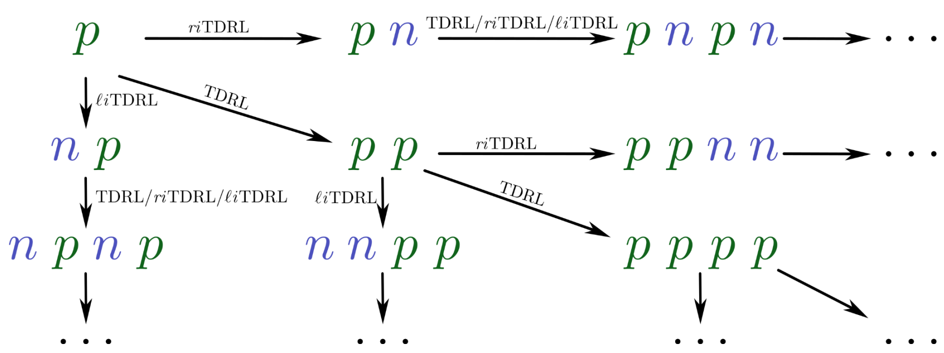
Conclusion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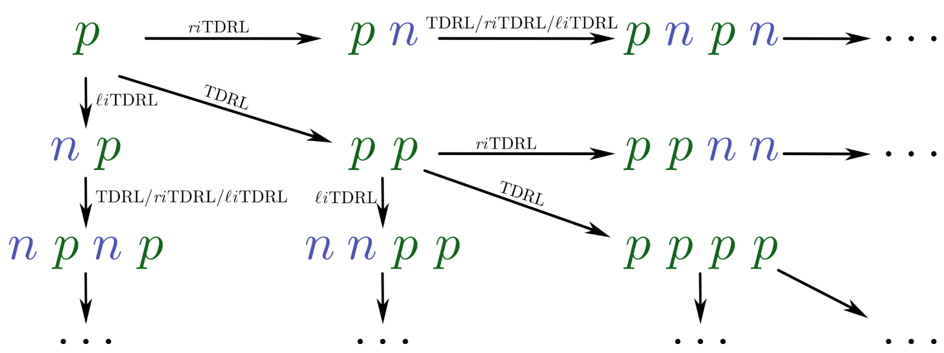

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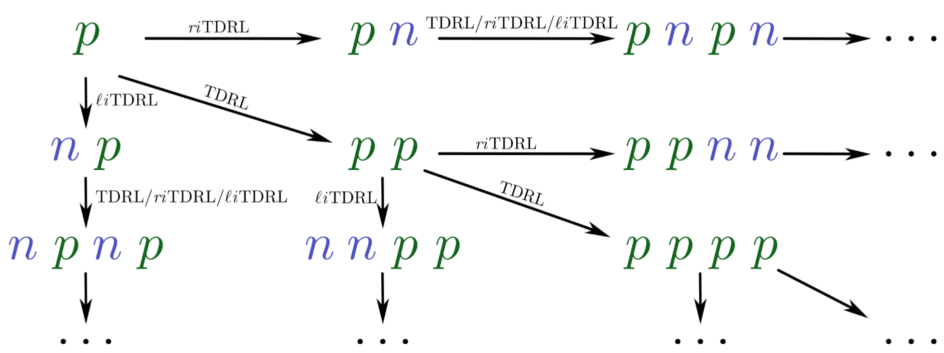

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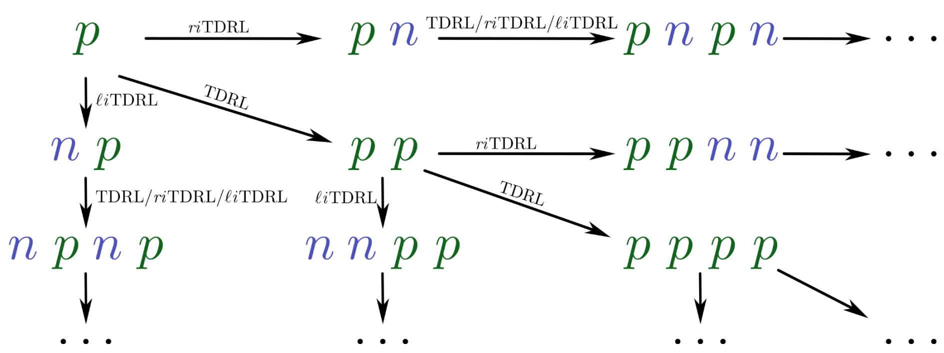

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- Biological constraints need to be considered
  → Keep common gene clusters at each sorting step
  → restrict number of genes on which a TDRL/iTDRL can act on