Partition function for circular RNA and Application

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1. Partition function for linear RNAs
   - Structure decomposition
2. Extension for circular RNAs
   - Structure decomposition
3. Application
   - Occurence of circular RNAs
   - Metastable Structures

- let S be the set of all possible secondary structures for a given RNA sequence
- each $s \in S$ is associated with a free energy G_s

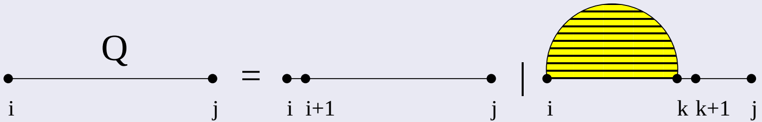
The partition function Q for a given sequence

$$Q = \sum_{s \in S} e^{\frac{-G_S}{RT}} \tag{1}$$

- i and j do not pair
- i and j form a hairpin loop, with closing pair [i,j],
- there exist two positions k,l with $i < k < l < j$ so that the subsequence i,j forms an interior loop with closing pairs i,j and k,l
- i and j are the closing pair of a multiloop

dynamic programming approach, that calculates the partition function for all subsequences $x[i,j]$ and ends with the partition function for the whole sequence...

Decomposition of paired/unpaired i,j



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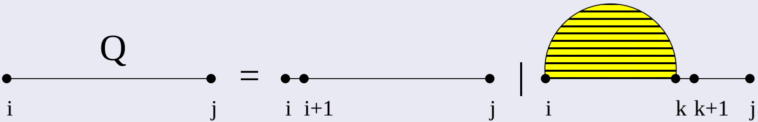

$$Q_{i,j} = 1 + \sum_{1<k<j} Q^b_{i,k} \cdot Q_{k+1,j} \tag{2}$$

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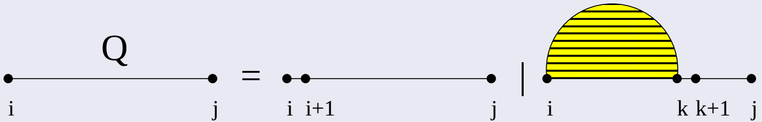

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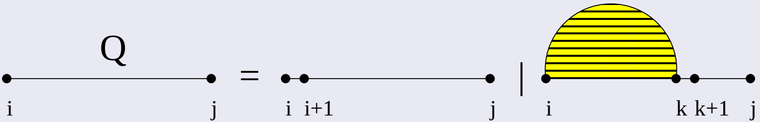

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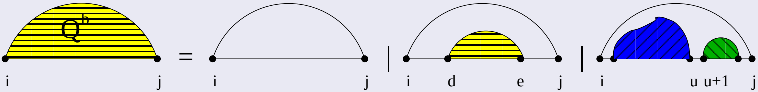
Decomposition with constraint that i pair with j



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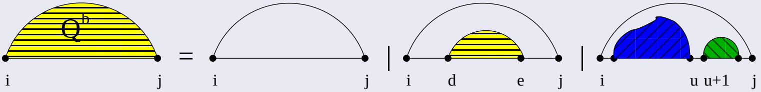

$$Q^b_{i,j} = e^{\frac{-G^{H_{i,j}}}{RT}} + \sum_{i<d<e<j} e^{\frac{-G^{I_{i,d,e,j}}}{RT}} \cdot Q^b_{d,e} + \sum_{i<u<j} Q^m_{i+1,u} \cdot Q^{m1}_{u+1,j-1} \cdot e^{\frac{-a}{RT}} \quad (3)$$

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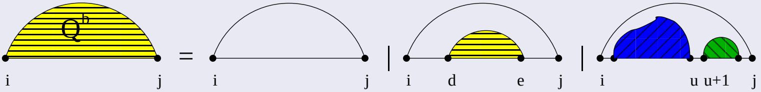

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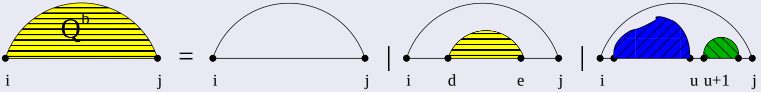

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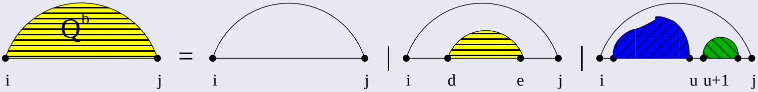

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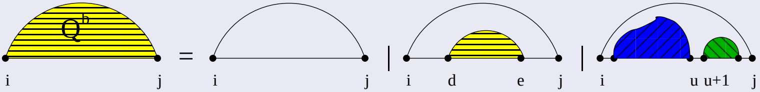

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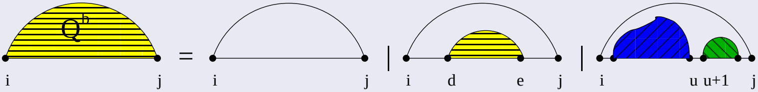

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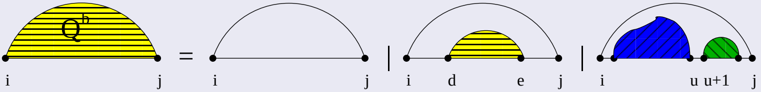

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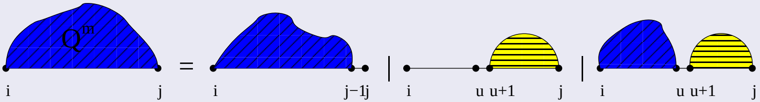
Decomposition of multiloop



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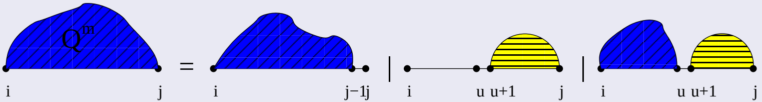

$$Q^m_{i,j} = Q^m_{i,j-1} \cdot e^{\frac{-c}{RT}} + \sum_{i<u<j} e^{\frac{-(u-i-1)\cdot c - b}{RT}} \cdot Q^b_{u+1,j} + \sum_{i<u<j} Q^m_{i,u} \cdot Q^b_{u+1,j} \cdot e^{\frac{-b}{RT}} \quad (4)$$

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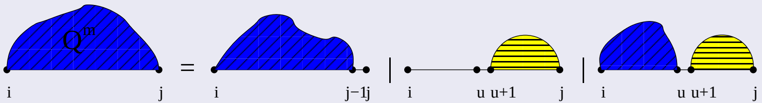

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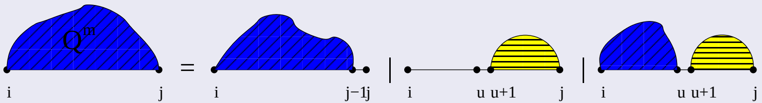

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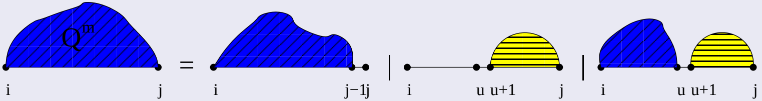

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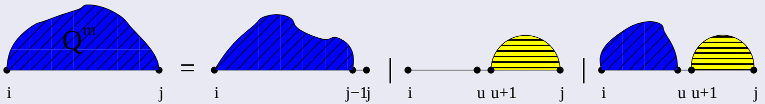

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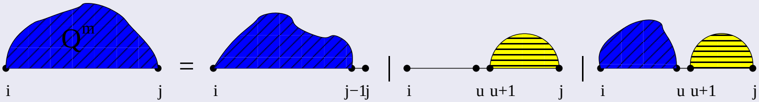

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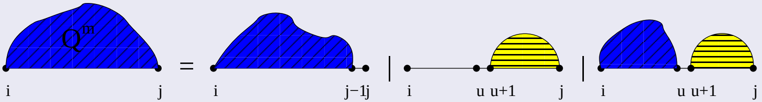

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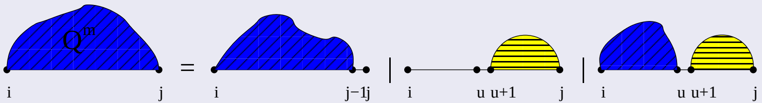

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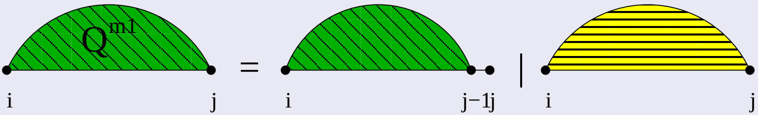
Decomposition of multiloop part with exactly one component



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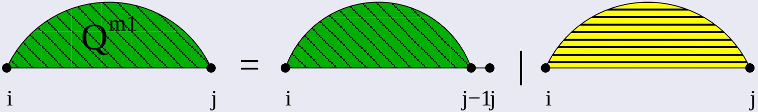

$$Q^{m1}_{i,j} = Q^{m1}_{i,j-1} \cdot e^{\frac{-c}{RT}} + Q^{b}_{i,j} \cdot e^{\frac{-b}{RT}} \tag{5}$$

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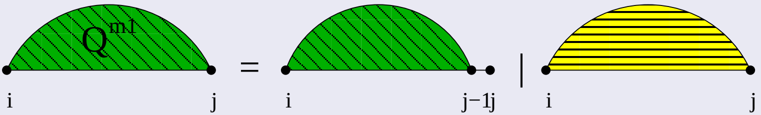

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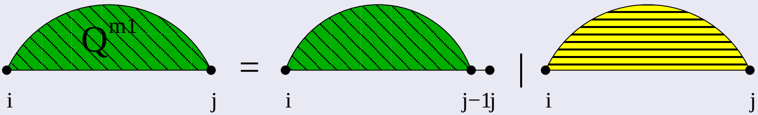

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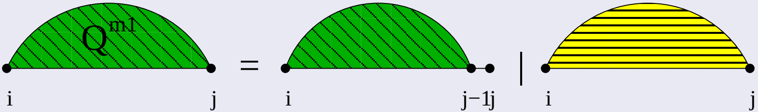

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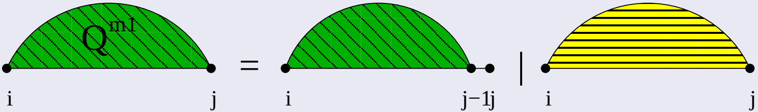

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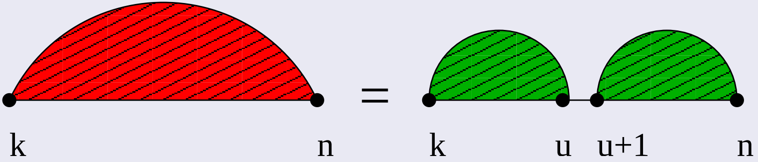
Difference:

- computing sum of weighted energies for all loops that contain bases x_n and x_1
- leads to the computation of so called ”exterior loops”

Computation of M^2 Array



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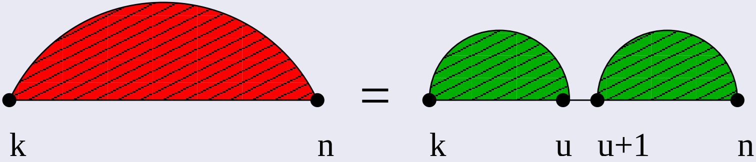
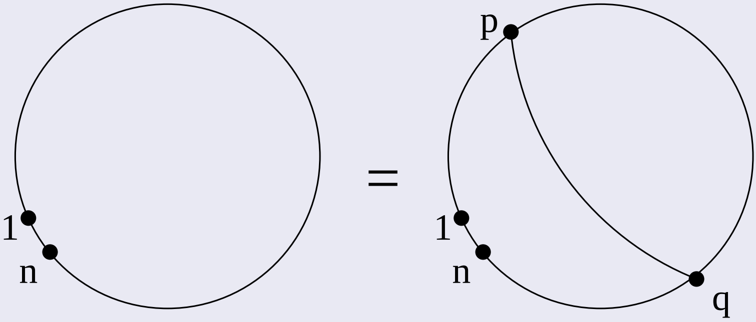


$$Q^{M2}_{k,n} = \sum_{k<u<n} Q^{m1}_{k,u} \cdot Q^{m1}_{u+1,n} \tag{6}$$

Computation of exterior hairpin loop



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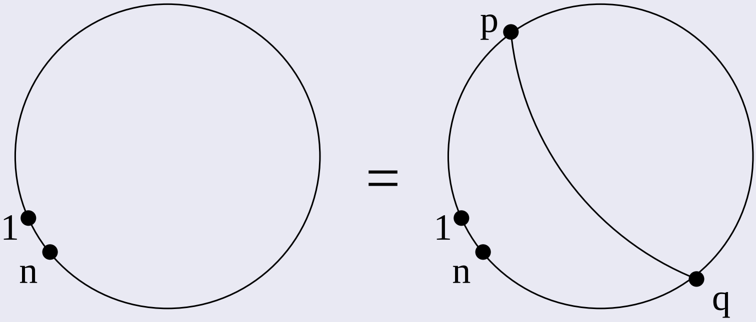

$$Q^o_H = \sum_{p<q} Q^b_{p,q} \cdot e^{\frac{-G^{H_{q,p}}}{RT}} \qquad (7)$$

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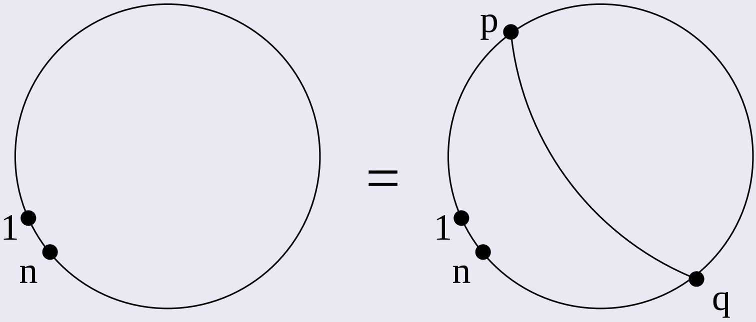

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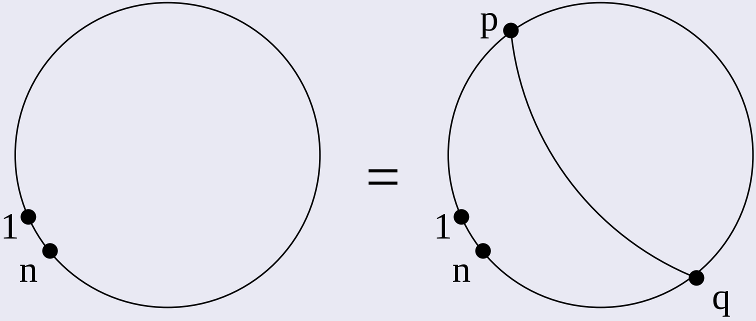

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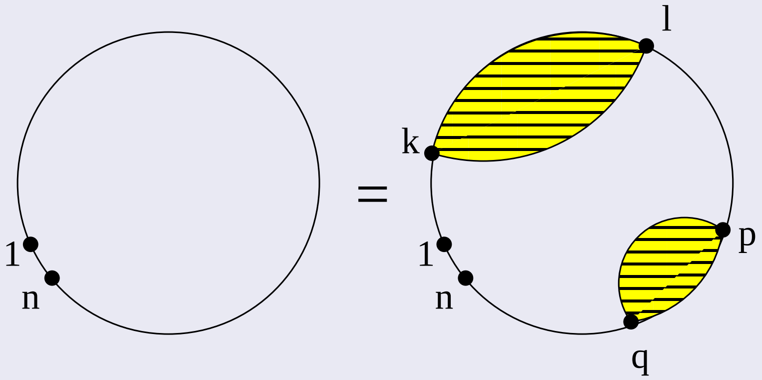
Computation of exterior interior loop



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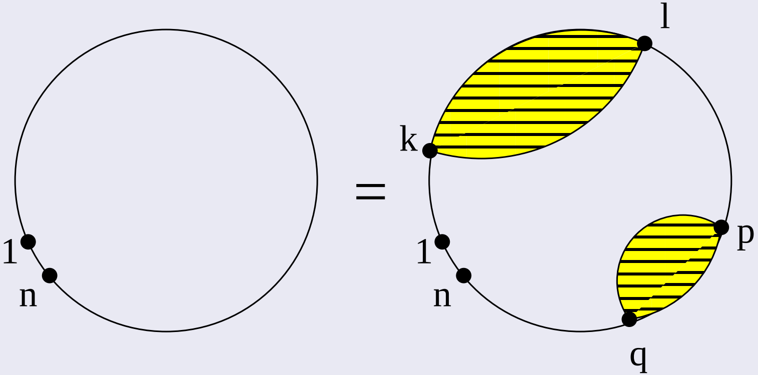
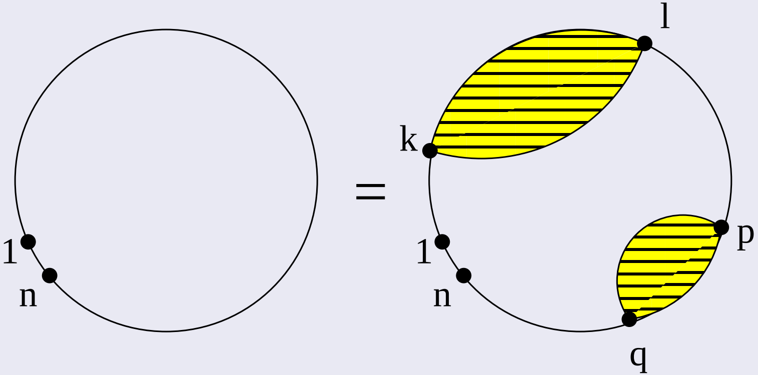


$$Q_I^o = \sum_{k<l<p<q} Q_{k,l}^b \cdot Q_{p,q}^b \cdot e^{\frac{-G^I k,l,p,q}{RT}} \quad (8)$$

Computation of exterior interior loop



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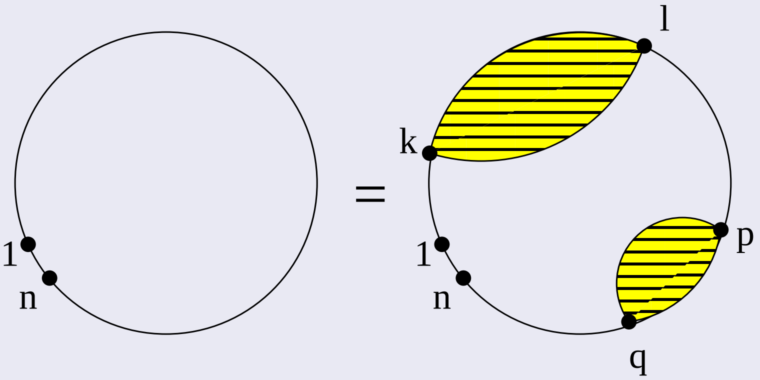

$$Q^o_I = \sum_{k<l<p<q} Q^b_{k,l} \cdot Q^b_{p,q} \cdot e^{\frac{-G^{I_{k,l,p,q}}}{RT}} \quad (8)$$

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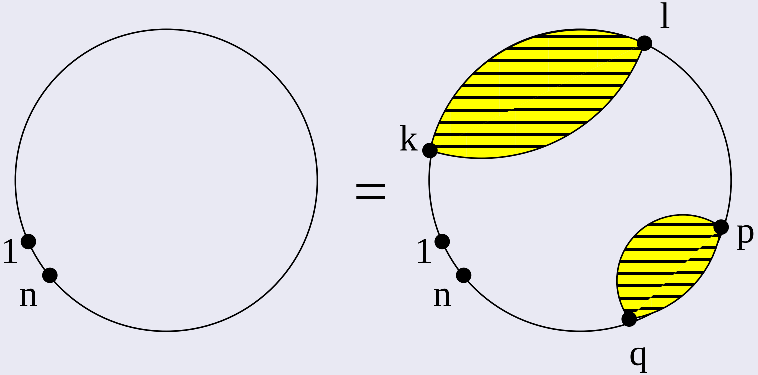

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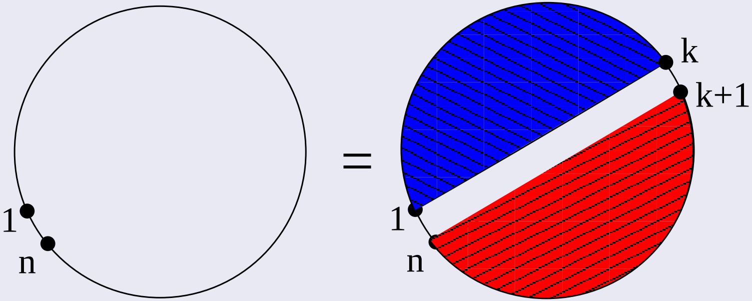
Computation of exterior multiloop



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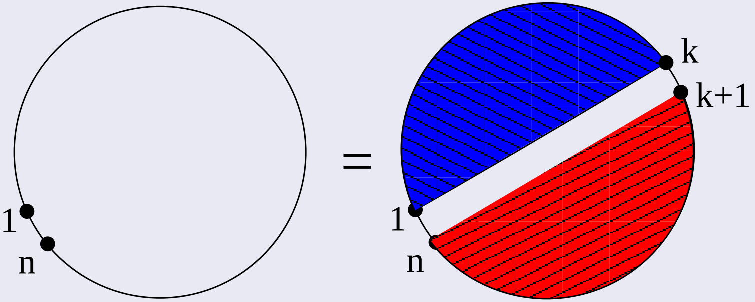
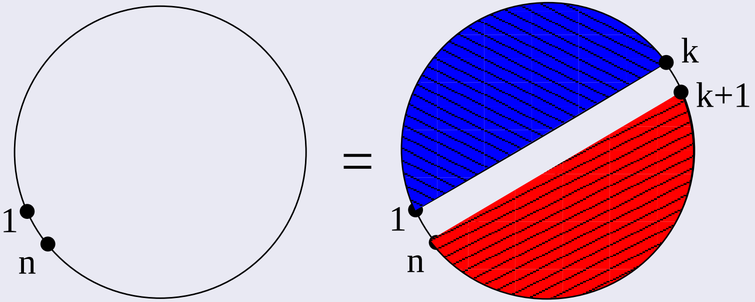


$$Q^o_M = \sum_{1<k<n} Q^m_{1,k} \cdot Q^{M2}_{k+1,n} \cdot e^{\frac{-a}{RT}} \qquad (9)$$

Computation of exterior multiloop



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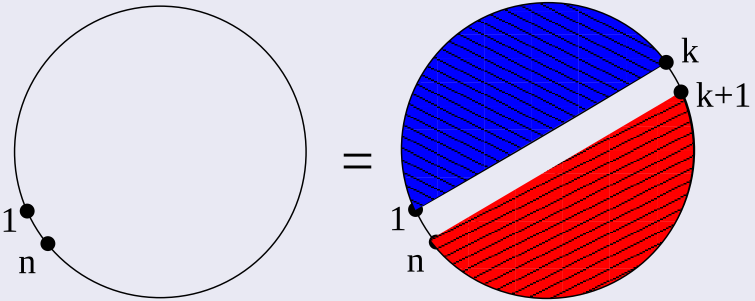
Computation of exterior multiloop



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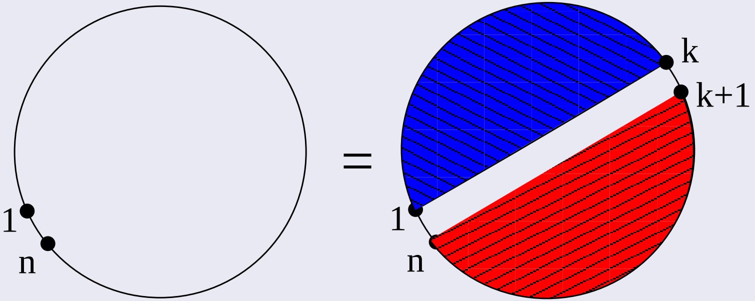

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Partition function for circular case:

$$Q^o = Q^o_H + Q^o_I + Q^o_M \tag{10}$$

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I will skip

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- equations for base pairing probability matrix

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- equations for base pairing probability matrix
- way of backtracking to obtain secondary structures

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In almost all cases, circular RNAs code not for any mRNA but for their own structure

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Thank you for your attention