

# 269020 VO Computational Concepts in Biology II

## Part: Biological Networks

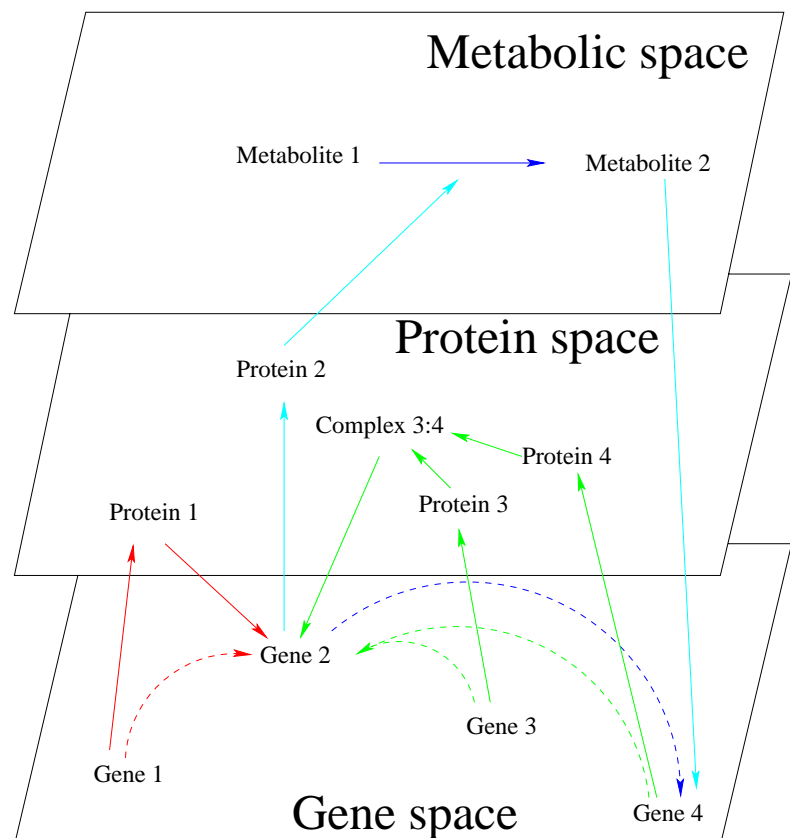
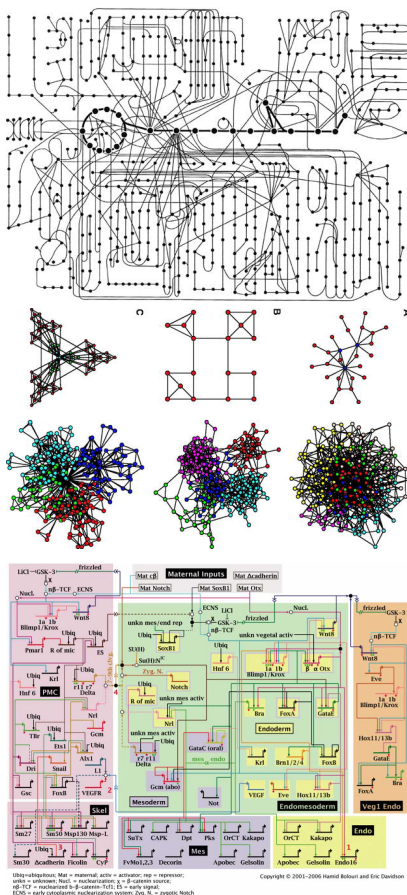
Stefanie Widder<sup>‡</sup> and Christoph Flamm<sup>†</sup>

<sup>†</sup>Institute for Theoretical Chemistry  
<sup>‡</sup>CeMM - Center for Molecular Medicine of the  
 Austrian Academy of Sciences

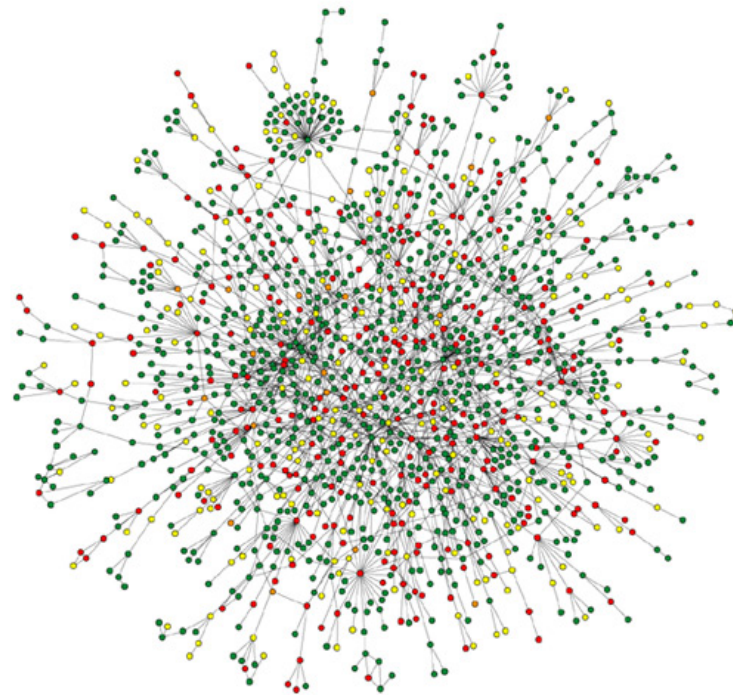
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 SS2019



## Biological Networks



# Yeast Protein Interaction Network



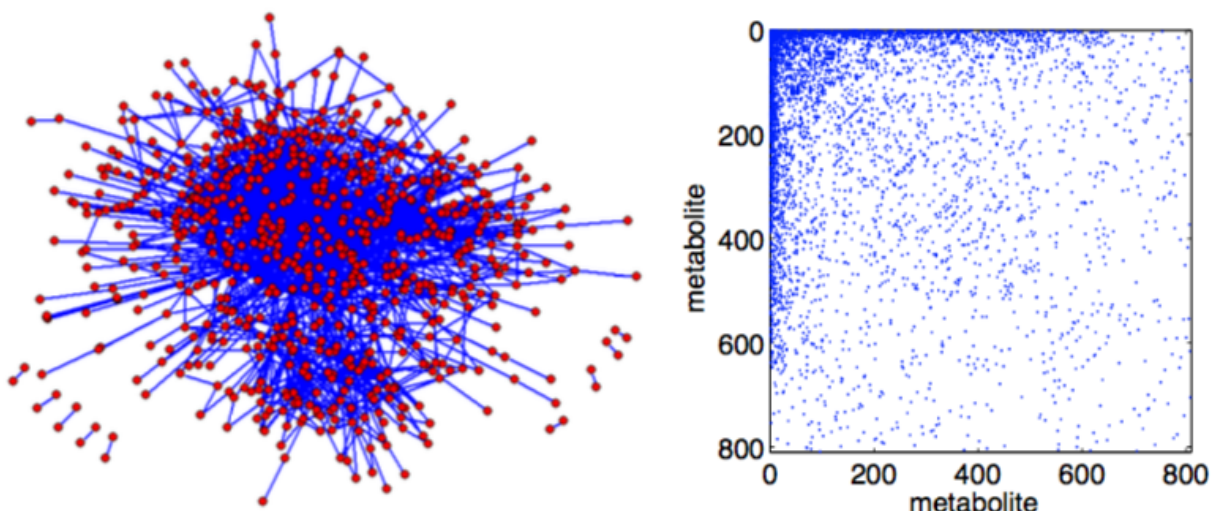
Nature Reviews | Genetics

The vertex color reflects the phenotypic effect of a gene deletion mutant (green nonlethal, red lethal, orange slow growth, yellow unknown).

Figure adapted from Jeong, H et al. (2001), Lethality and centrality in protein networks, *Nature* 411:41-42 | [doi:10.1038/35075138](https://doi.org/10.1038/35075138)

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# Yeast Metabolic Network



Reconstruction of the *S. cerevisiae* metabolic network consisting of 810 metabolites (vertices) and 843 reactions (3419 edges).

Yeast Consensus Reconstruction (Yeast 5): 1418 metabolites, 2110 reactions with 918 verified *S. cerevisiae* genes.

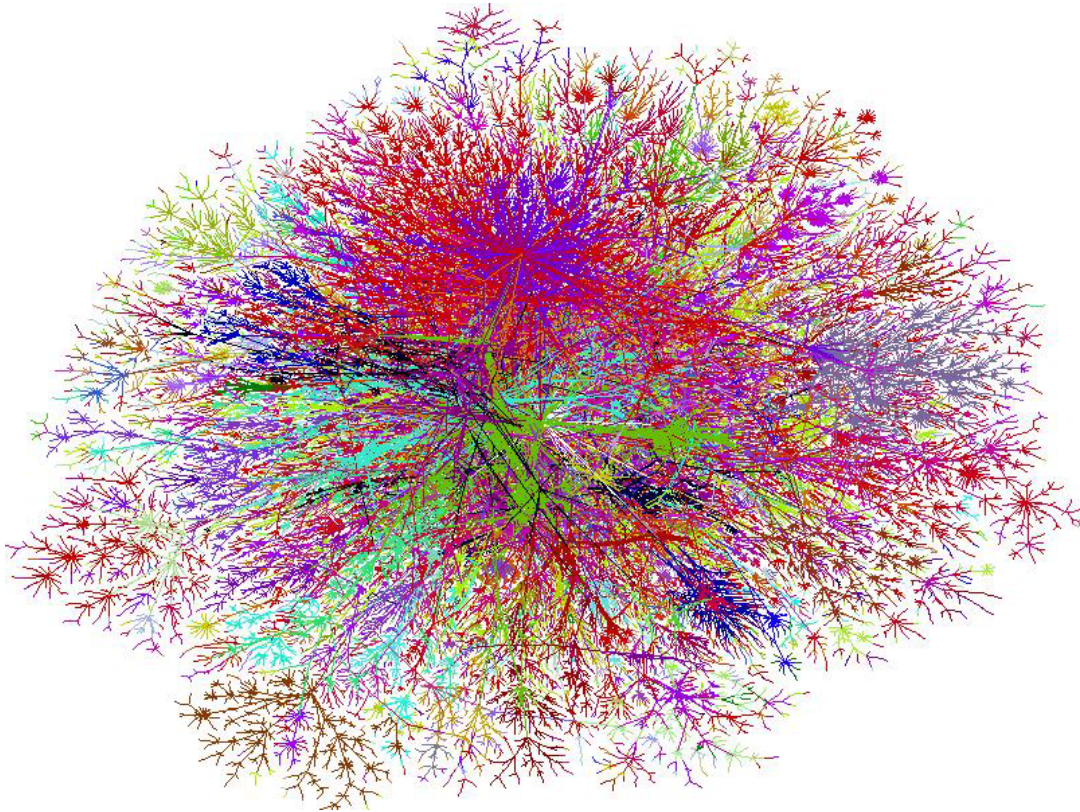
Figure adapted from Förster J et al. (2003), Genome-Scale Reconstruction of the *Saccharomyces cerevisiae* Metabolic Network, *Genome Res* 13:244-253 | [doi:10.1101/gr.234503](https://doi.org/10.1101/gr.234503)

Benjamin D Heavner BD et al (2012), Yeast 5 – an expanded reconstruction of the *Saccharomyces cerevisiae* metabolic network, *BMC Sys Biol* 6:55 | [doi:10.1186/1752-0509-6-55](https://doi.org/10.1186/1752-0509-6-55)

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# Map of the Internet

Analytic approaches are needed to “look” at complex networks!



Map of Internet December 1998: colored by IP addresses By William R. Cheswick  
<http://www.cheswick.com/ches/map/>

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## Language of Complex Networks

Complex dynamical systems are characterized by a large number of non-linearly interacting elements, giving rise to emergent properties (complication|complexity).

Translation of biological phenomena and processes into networks:

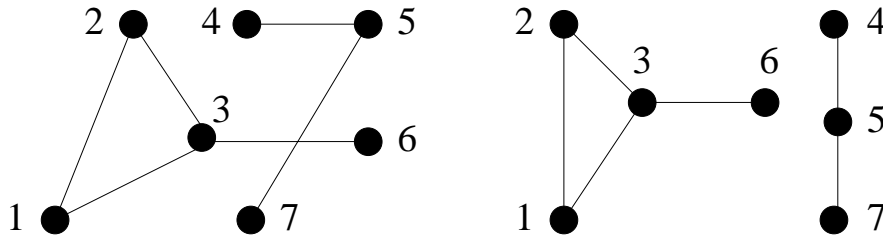
- ① makes problems mathematically tractable  
(e.g. transcriptional regulation, metabolism, protein configurations)
- ② uncovers
  - functional organization (network topology)
  - underlying design principles
  - unknown organizational principles
- ③ reveals crucial system properties
  - robustness, resilience
  - redundancy, modularity
  - functional dependencies among network elements
  - systems dynamics

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# Primer: Graphs

$$G = (V, E) \quad \text{where} \quad e_i = (v_i, v_j)$$

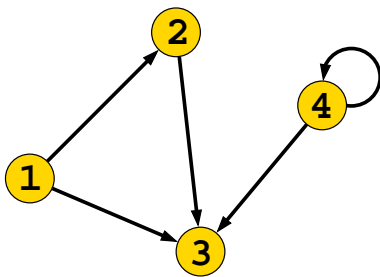
- A graph is a tuple of two sets, the **vertex set** and the **edge set**.
- The edge set members are tuples of vertex set members.
- Graphs preserve neighborhood relations.



$$\begin{aligned} \text{vertex set } V &= \{1, 2, 3, 4, 5, 6, 7\} \\ \text{edge set } E &= \{(1, 2), (2, 3), (1, 3), (3, 6), (4, 5), (5, 7)\} \end{aligned}$$

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## Primer: Graph Representations



graph drawing

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

adjacency matrix

$$\begin{aligned} (1, & \longrightarrow (2, 3) \\ 2, & \longrightarrow (3) \\ 3, & \longrightarrow () \\ 4) & \longrightarrow (3, 4) \end{aligned}$$

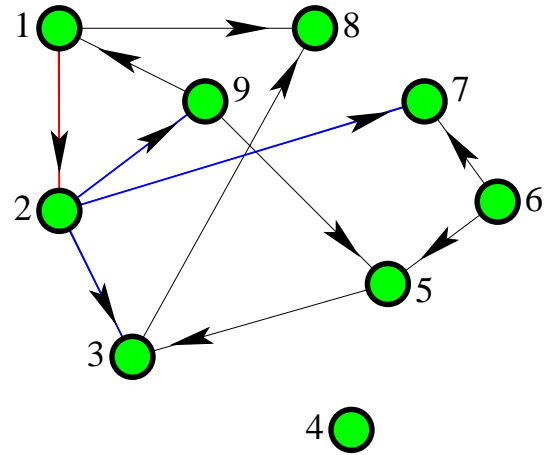
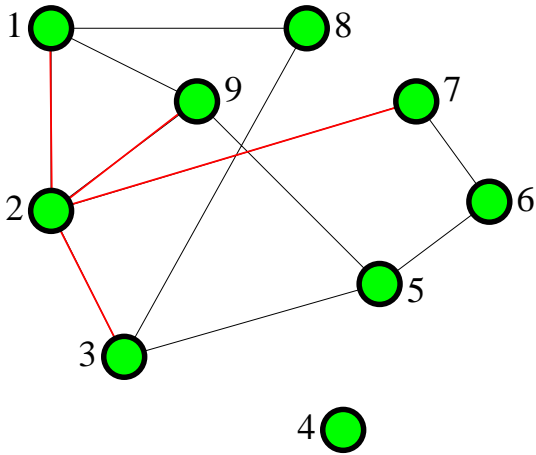
adjacency list

While the adjacency list is very memory efficient, it is in general quicker to use the  $|V| \times |V|$  adjacency matrix to determine if an edge  $(u, v)$  is present/absent in the graph.

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# Network Measure: Degree distribution

directed | undirected, in-degree | out-degree



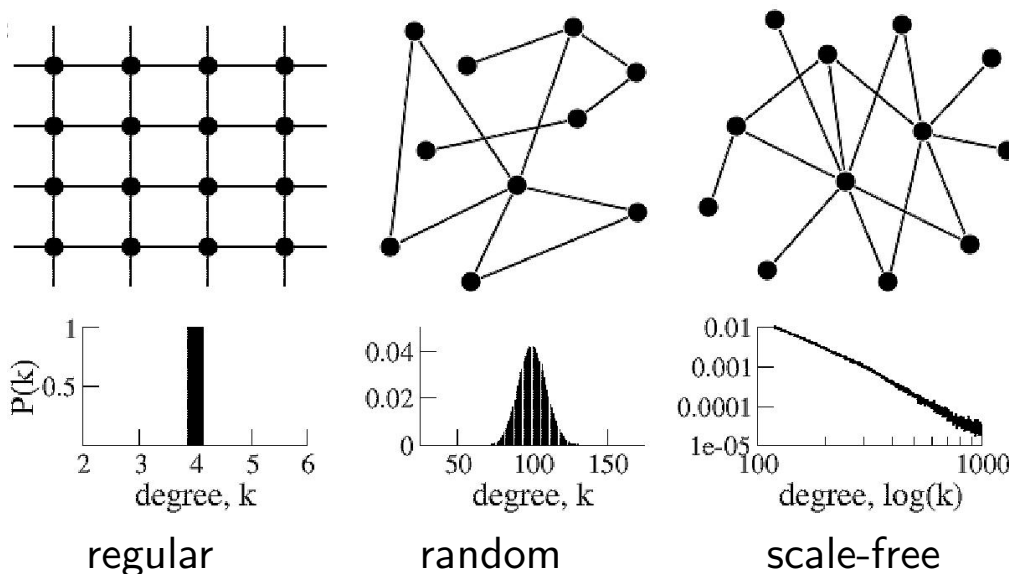
$$p(k) = \frac{|\{v | d(v) = k\}|}{N}$$

The degree distribution  $p(k)$  measures the probability that a randomly chosen node in the network has degree  $k$ ,  $d(v)$  is the degree of node  $v$ ,  $N$  is the total number of nodes in the graph.

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## Classifying Networks into Categories

In graphs without self-loops, the degree equals the number of neighbors of a vertex.



$$p(k) = k^{-\gamma}, \quad \gamma \in \mathcal{R}^+$$

[Note:  $\log\{p(k)\} = \log\{k^{-\gamma}\} = -\gamma \cdot \log\{k\}$ ].

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# Power Laws are self-similar

$$f(x) = K \cdot x^\alpha$$

**Scale-free** denotes the lack of a meaningful average as power laws are self-similar. Overall system properties are similar at all levels of organization.

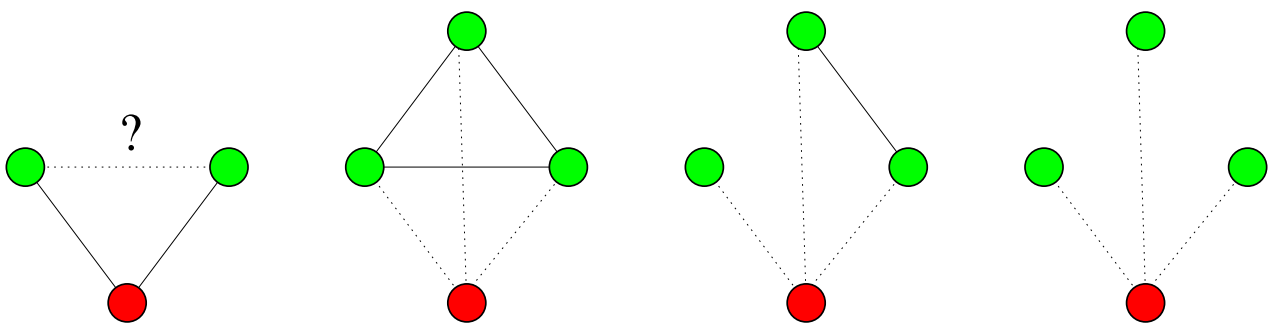
$$x \mapsto c \cdot x, \quad f(x) \mapsto f(c \cdot x) = K \cdot c^\alpha \cdot x^\alpha = c^\alpha f(x) \propto f(x)$$

A scale transformation leaves the form of the function invariant, it results in a proportional scaling of the function itself. Power laws are the only solution to this functional equation.

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## Network Measure: Clustering coefficient

Measures the probability that two vertices with a common neighbor are connected.



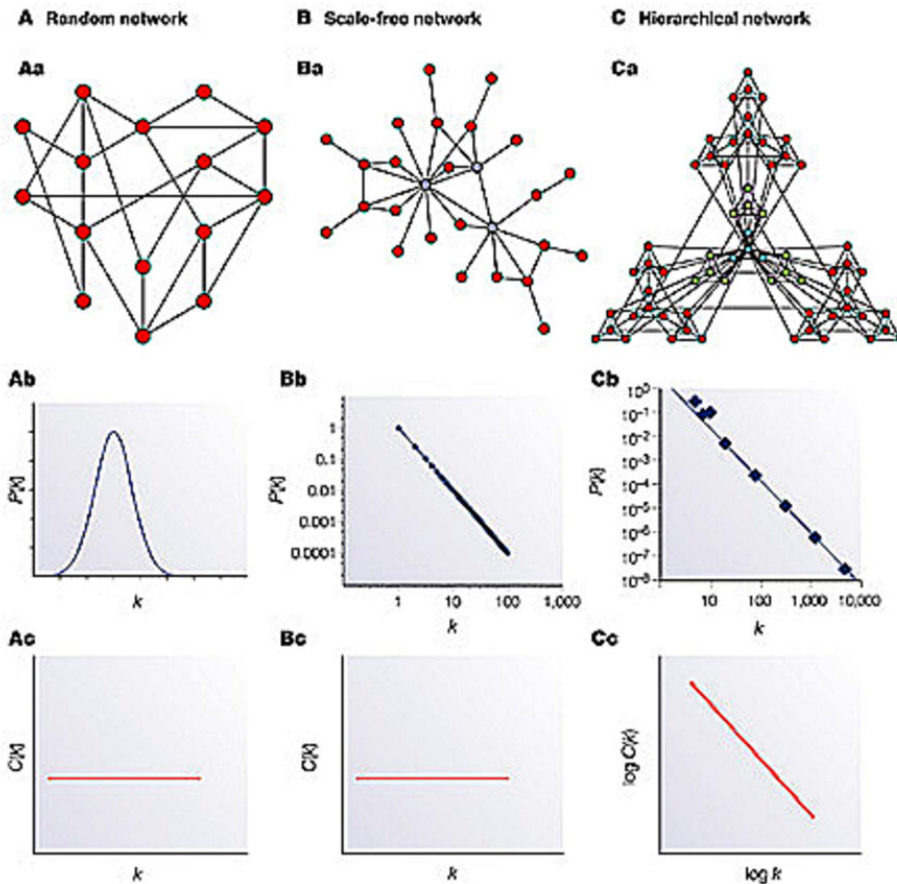
$$C_i = \frac{2 \cdot E_i}{k_i(k_i - 1)}$$

$E_i$  is the number of all observed edges between  $k_i$  neighbors of vertex  $n_i$ .  $C_i$  quantifies the local order (local structure) in the network.

[Note: The maximum number of possible edges between  $k_i$  neighbors of vertex  $n_i$  is  $E_{\max} = k_i(k_i - 1)/2$ ].

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# Discriminating between different Networks



Barabási AL and Oltvai ZN (2004), Network biology: understanding the cell's functional organization, *Nature Rev Gen* 5:101-113 | doi:10.1038/nrg1272

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## Network Prototype Models

Purpose of prototype models to provide

- insight how observed features arise from construction rules.  
network measures → prototype models → global features
- null models (statistic significance of observed features).

The most common prototype models are:

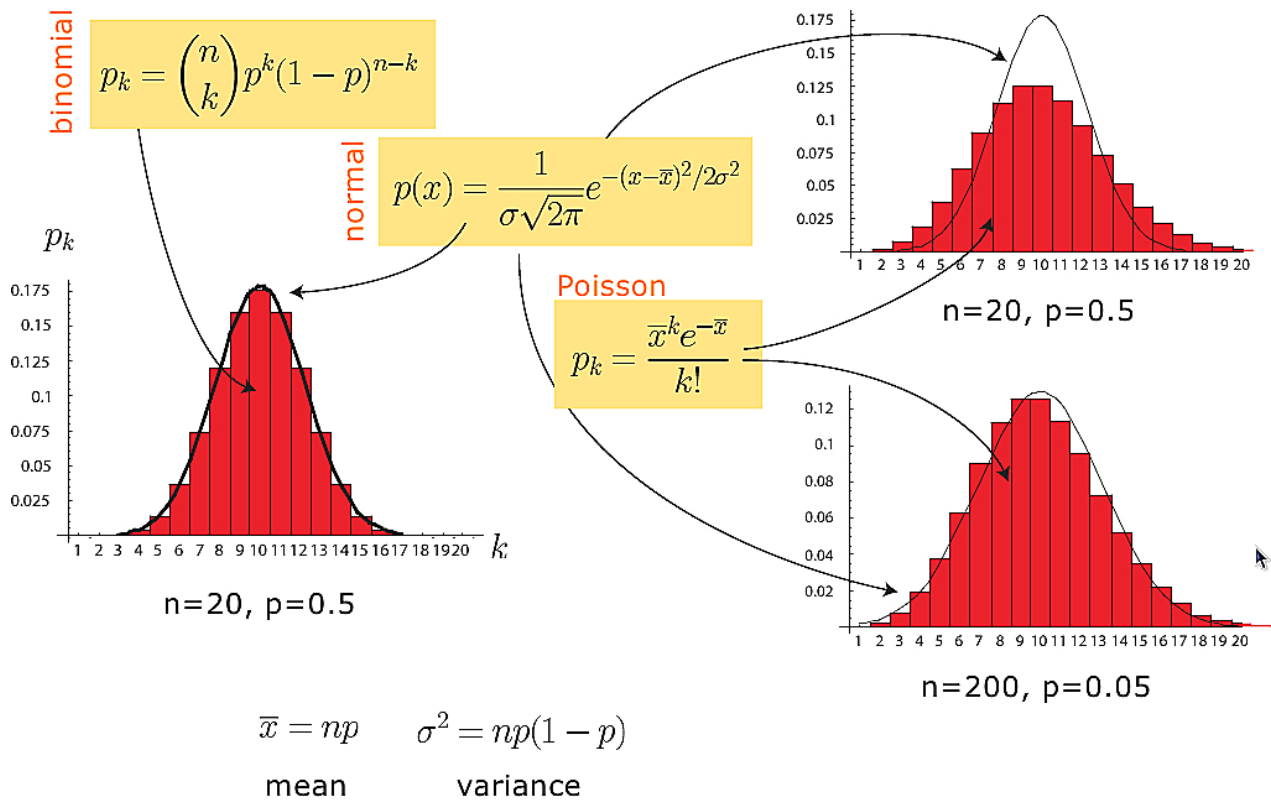
- ① Erdős-Rényi (ER) Model
- ② Watts-Strogatz (WS) Model
- ③ Barabási-Alberts (BA) Model

These are all random networks with particular features...

Albert R & Barabási A-L (2002), Statistical mechanics of complex networks, *Rev Mod Phys* 74:47-97 | doi:10.1103/RevModPhys.74.47

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# Primer: Statistical distributions



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## The Erdős-Rényi Model

Consists of  $N_V$  vertices connected by  $N_E$  undirected edges which are chosen randomly from the set of  $N_V \cdot (N_V - 1)/2$  possible edges.

The probability  $p$  that two randomly chosen vertices are connected is thus

$$p = \frac{2 \cdot N_E}{N_V \cdot (N_V - 1)}$$

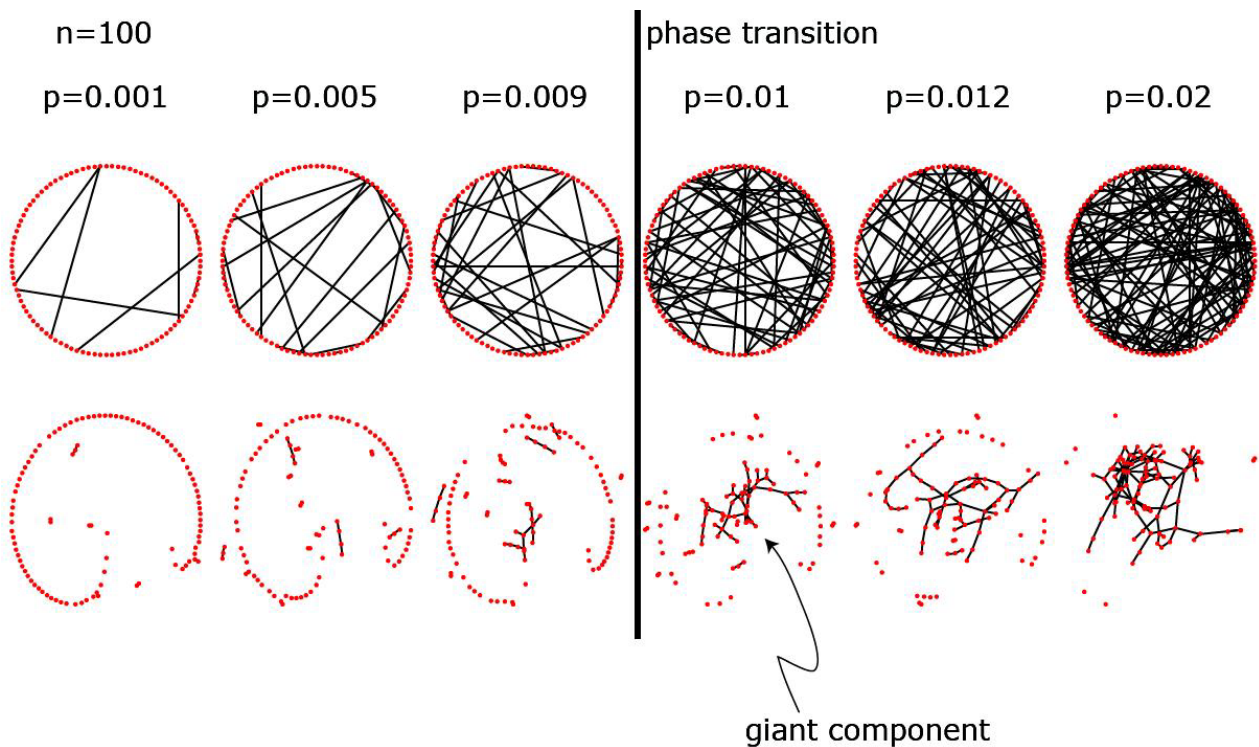
The degree distribution is given by a **binomial distribution** that becomes approximately **Poissonian** for large networks ( $N_V \rightarrow \infty$ ).

The ER model exhibits the **small-world** property, i.e. over the percolation threshold the average path length  $l$  between nodes scales as  $l \sim \log N_V$ .

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# Phase Transition in the Erdős-Rényi Model

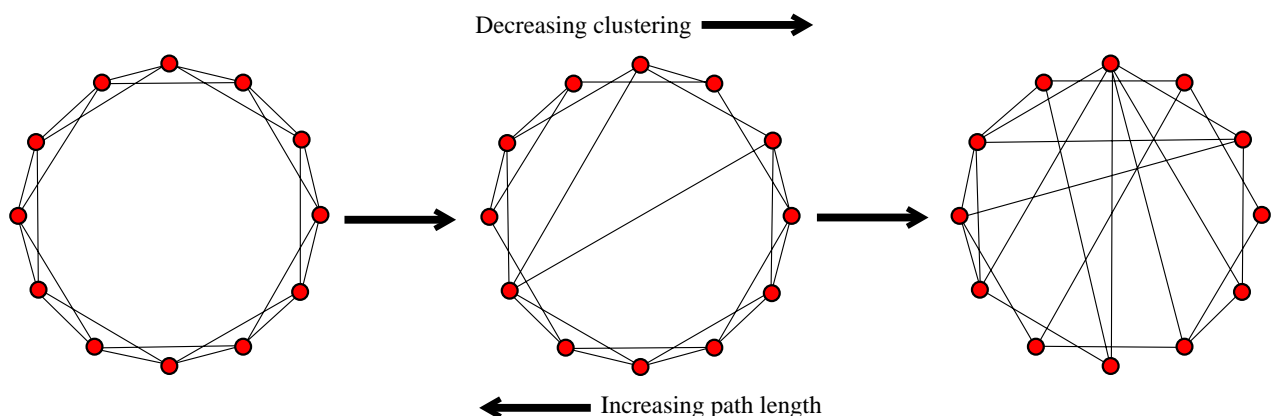


For small  $p$  the network is disconnected. At  $p \sim 1/N_V$  (i.e.  $\langle k \rangle \approx 1$ ) a phase transition to a **giant component** occurs. For  $p \geq \log(N_V)/N_V$  all vertices are connected.

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## The Watts-Strogatz Model

While the ER model correctly reproduces the **small world** property, it fails to account for local clustering.



- Start with regular lattice-like network with local clusters.
- Introduce **shortcuts** with probability  $p_{new}$ .
- For increasing  $p_{new}$  the WS model approaches the ER model (for  $p_{new} \rightarrow 1$  the ER model is recovered).

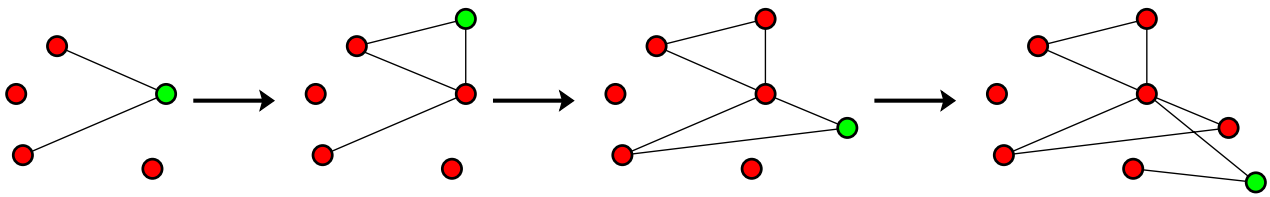
A very small number of shortcuts  $p_{new} \ll 1$  is sufficient to rapidly decrease the average path length and maintain local clustering.

Watts DJ & Strogatz SH (1998), Collective dynamics of 'small-world' networks, *Nature* 393:440-442 | doi:10.1038/30918

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# The Barabási-Albert Model

ER and WS model do not capture the inhomogeneous degree distributions found in empirical networks.



The BA model is based on two essential ingredients:

- Growth process (new vertex is added each time step).
- Preferential attachment:

The probability that a vertex  $n_i$  receives a new edge is proportional to its degree  $k_i$

$$p(n_i) = \frac{k_i}{\sum_j k_j}$$

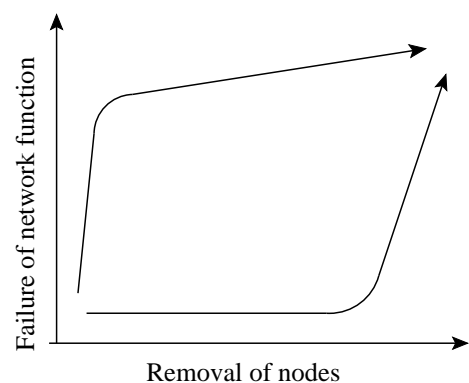
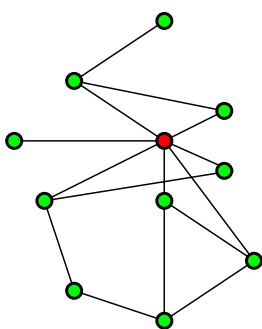
For  $t \gg 1$  the degree distribution follows a **power law**  $p(k) \sim k^{-\gamma}$ .

Barabási A-L & Albert R (1999), Emergence of Scaling in Random Networks, *Science* **286**(5439):509-512 | [doi:10.1126/science.286.5439.509](https://doi.org/10.1126/science.286.5439.509)

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## Tolerance against Node Removal

The topology of scale-free networks is dominated by a few highly connected **hubs**.



Therefore scale-free networks are robust against failures of arbitrary nodes but quite vulnerable if hubs are attacked.

Note that a comprehensive view of robustness must as well take **dynamic aspects** into account!

Barabási A-L (2012), Network science: Luck or reason, *Nature* **489**:507-508 | [doi:10.1038/nature11486](https://doi.org/10.1038/nature11486)

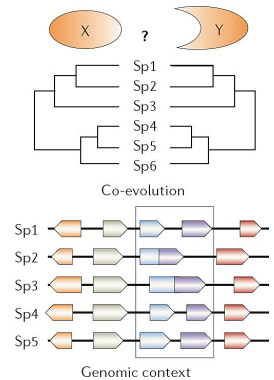
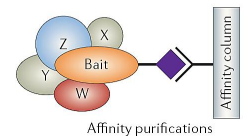
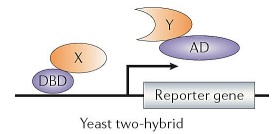
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# The Interactom: Who interacts with whom?

Provide a comprehensive list of protein-protein interactions.

## Experimental approaches:

- chemical crosslinking / footprinting
- protein arrays
- fluorescence resonance energy transfer (FRET)
- fluorescence cross-correlation spectroscopy
- yeast two-hybrid system
- affinity purification



## Computational approaches/predictions:

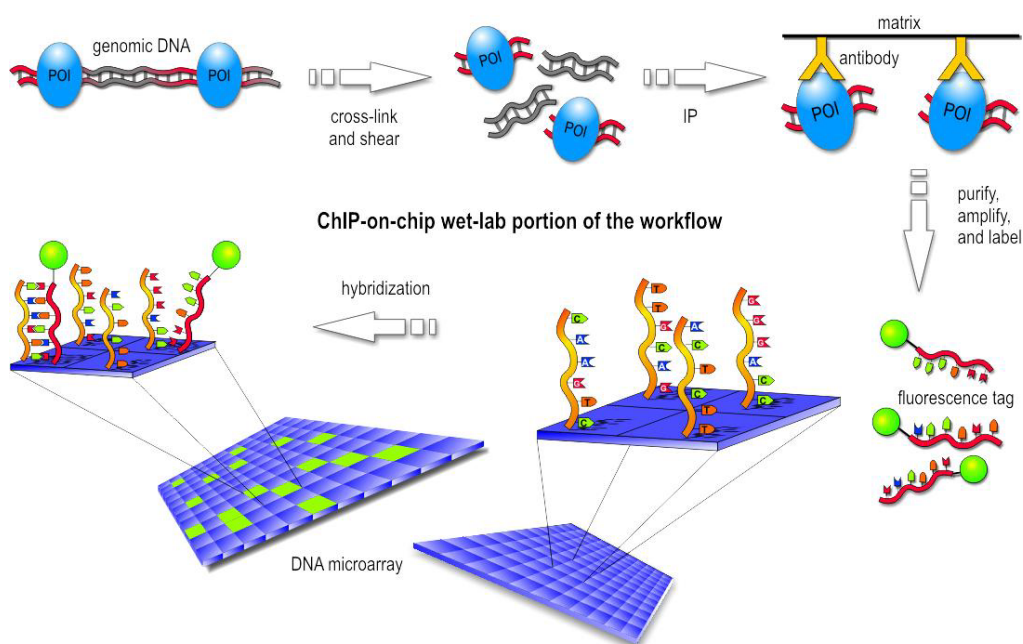
- Genomic context
- Co-evolutionary

Sardiu ME and Washburn MP (2011), Building Protein-Protein Interaction Networks with Proteomics and Informatics Tools, *J Biol Chem* **286**:23645-23651 | doi:10.1074/jbc.R110.174052

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## Identification of protein DNA interactions (GRNs)

The **ChIP-on-chip** technique combines chromatin immunoprecipitation ("ChIP") with DNA microarray technology ("chip").



ChIP-on-chip allows to study binding sites of DNA-binding proteins on a genome-scale basis. Other methods: DNase-seq, ATAK-seq distinguishes binding sites and open chromatin regions

# Weaknesses of Interactom Screens

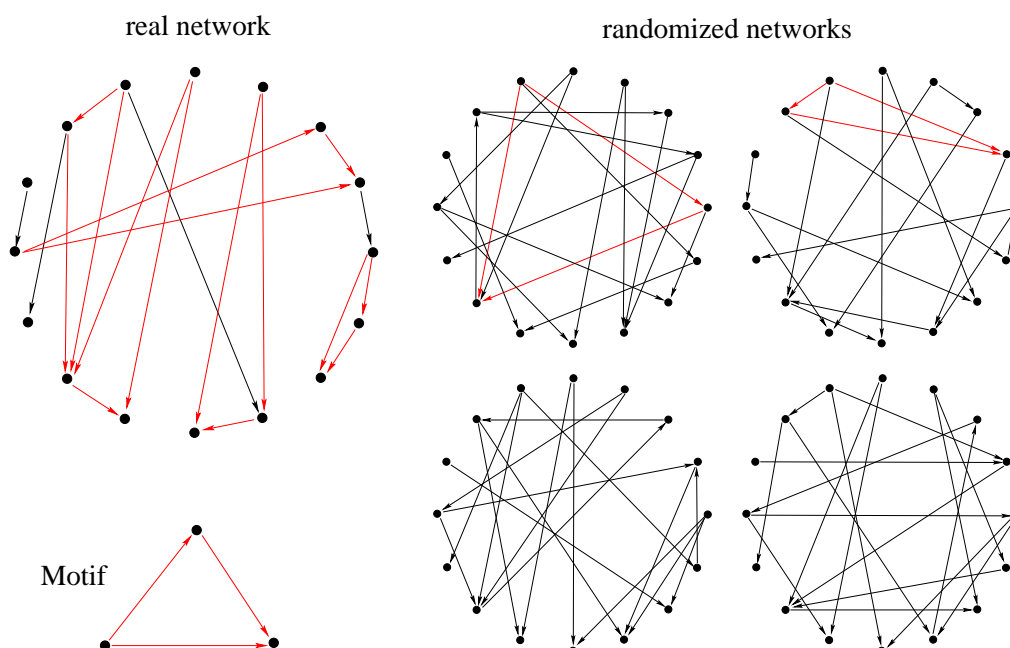
- 1 A **poor overlap** was observed between screens run by different groups on the same organism.
- 2 High numbers of false positives (30-60%) and false negatives (40-80%) in the screens.
- 3 All biophysically possible interactions are discovered not necessarily only those occurring in biology (Q: What constitutes a false positive?).
- 4 Many already known interactions are missed. (This is due to under representation, mainly owed to the biological properties of proteins, e.g. aggregation upon overexpression).
- 5 Transient interactions or interactions depending on post-transcriptional modification events (e.g. phosphorylation, ubiquitination) are not captured by the current methods.

Russell RB and Aloy P (2008), Targeting and tinkering with interaction networks, *Nature Chem Biol* 4(11):666-673 | [doi:10.1038/nchembio.119](https://doi.org/10.1038/nchembio.119)

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## Motif Detection and Network Dynamics

- 1 Represent the network of interest as a directed graph.
- 2 Determine the frequency of particular subgraphs.
- 3 Compare against ensemble of randomized networks.

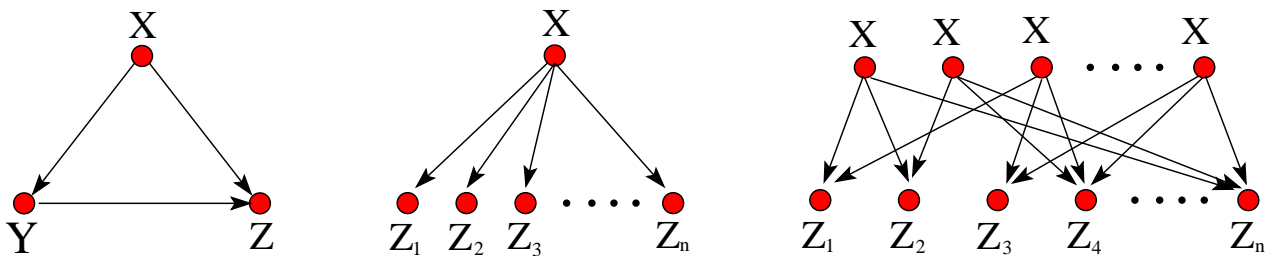


Shen-Orr SS et al (2002), Network motifs in the transcriptional regulation network of *Escherichia coli*, *Nature Genetics* 31:64-68 | [doi:10.1038/ng881](https://doi.org/10.1038/ng881)

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# Transcription regulation motifs in *E. coli*

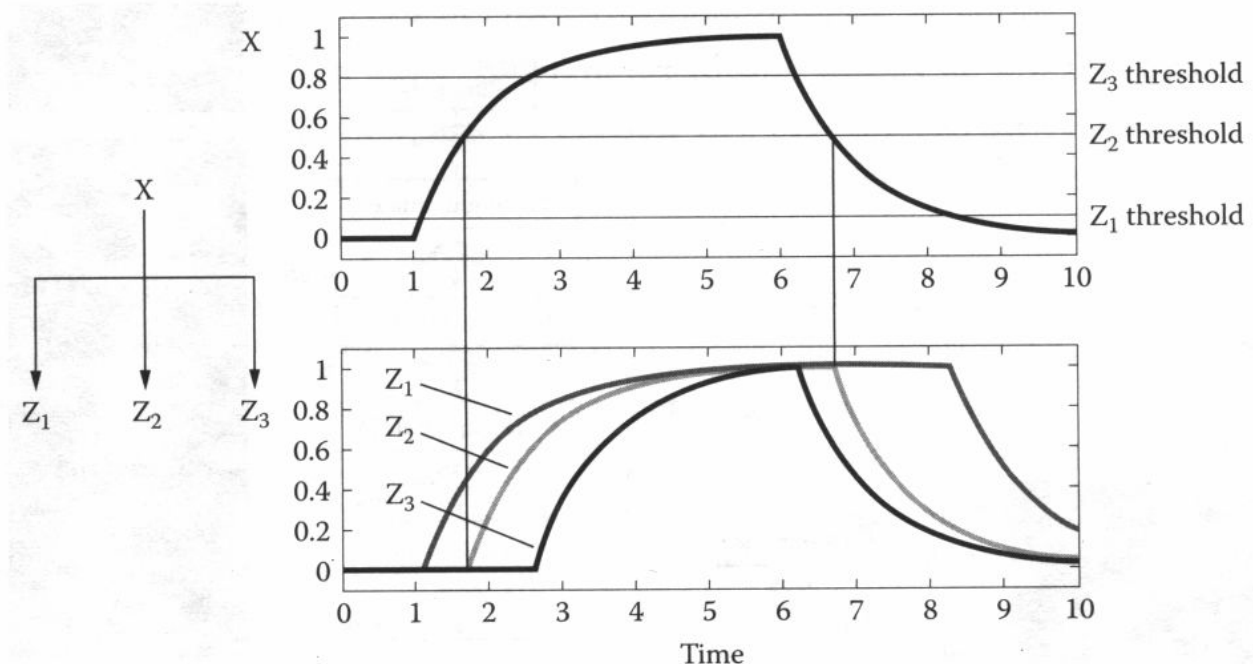
Motif	Freq <sub>real</sub>	Freq <sub>rand</sub>	P-value
coherent FFL	34	4.40 ± 3	$P < 0.001$
incoherent FFL	6	2.50 ± 2	$P \sim 0.03$
SIM	68	28.00 ± 2	$P < 0.01$
DORs	203	57.00 ± 14	$P < 0.01$
cycles	0	0.18 ± 0.6	$P \sim 0.8$



Shen-Orr SS et al (2002), Network motifs in the transcriptional regulation network of *Escherichia coli*, *Nature Genetics* 31:64-68 | doi:10.1038/ng881

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## SIMs generate a temporal program of expression

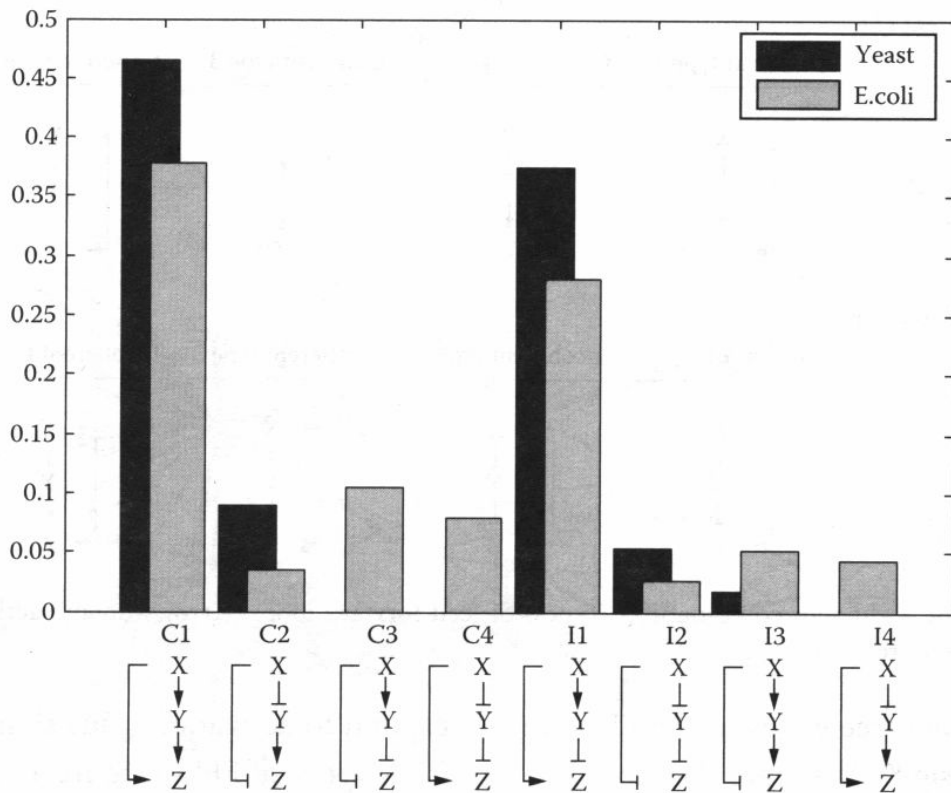


- ① Frequent in metabolic pathways regulation (e.g. arginine system).
- ② Temporal order of genes matches their functional order.
- ③ Economic design! Proteins are not produced before they are needed.

Zaslaver A et al (2004), Just-in-time transcription program in metabolic pathways, *Nature Genetics* 36:486-491 | doi:10.1038/ng1348

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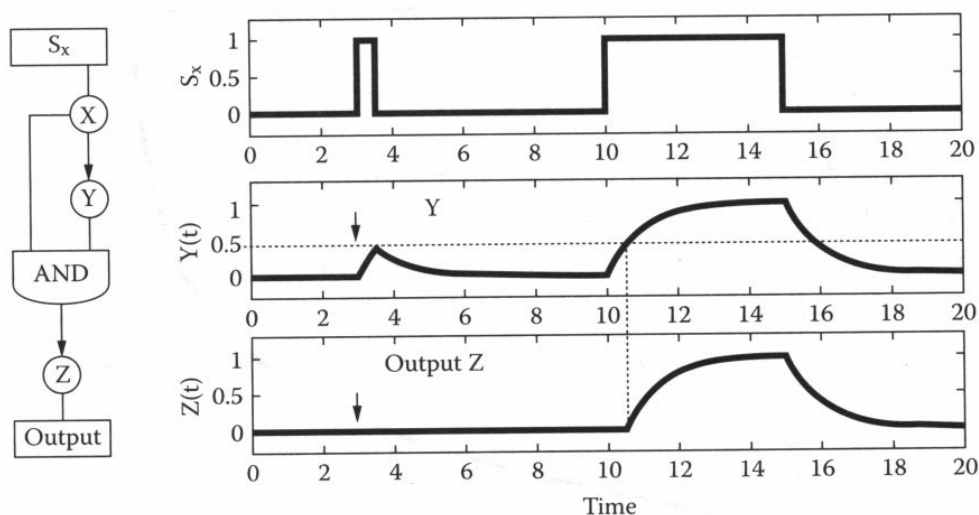
## Relative abundance of FFL types



Mangan S et al (2006), The incoherent feed-forward loop accelerates the response-time of the gal system of Escherichia coli, *J Mol Biol* **356**:1073-1081 | doi:10.1016/j.jmb.2005.12.003

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## Dynamics of the coherent feed-forward-loop motif



The motif function: filter out brief spurious pulses of signal

- Sign-sensitive delay function (signal on/off  $\Rightarrow$  delay/no delay).
- Responds only to persistent signals

Alon, U (2007), Network motifs: theory and experimental approaches, *Nature Rev Genetics* **8**:450-461 | doi:10.1038/nrg2102

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# Further Reading



Alon U

Network motifs: Theory and experimental approaches.  
*Nature Reviews Genetics*, **8**:450–461, 2007.



Albert R, Barabási A-L

Statistical mechanics of complex networks.  
*Reviews of Modern Physics*, **74**:47–97, 2002.



Barabási A-L, Zoltán NO

Network Biology: Understanding the cell's functional organization.  
*Nature Reviews Genetics*, **5**:101–113, 2004.



Newman MEJ

The structure and function of complex networks.  
*SIAM Review*, **45**(2):167–256, 2003.



Watts DJ, Fell DA

Collective dynamics of 'small-world' networks.  
*Nature*, **393**:440–442, 1998.